Cooperative approach to a plant location problem with positive externalities

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Abstract

We consider that a firm is planning to open a new plant in certain country. This country is divided into different regions where the plant could be installed. On these regions are located a finite number of firms, which obtain some benefit from this new incorporation. We assume that a planner should decide where to locate the new plant maximizing the global benefit and providing a compensating scheme such that the incoming firm gets more than in the decentralized mechanism and the other firms are not worst off. In order to do this, we associate to any problem a utility transferable cooperative game, considering that cooperation on a region exists if and only if all firms already installed on that region belong to the coalition. We study the core of the game and propose two solutions belonging to the core. We also provide characterizations for these solutions.

The model

A problem is a tuple \((0, N, P, b)\) where

- \(0\) is the firm which will open a plant in the country.
- \(N = \{1, ..., n\}\) is the set of firms already located in the country.
- \(R = \{1, ..., r\}\) is the set of regions in the country and \(P = \{P_1, ..., P_r\}\) is a partition of \(N\). \(P_k\) denotes the set of firms located in region \(k\).
- \(b = \{b^k_i : i \in N_0 = N \cup \{0\}\text{ and } k \in R\}\). \(b^k_i \geq 0\) denotes the benefit obtained by firm \(i\) when \(0\) locates the plant at region \(k\).

We assume that if an existing firm is outside the region where the new plant locates, this firm does not obtain any benefit, this is, for all \(k \in R\) and all \(i \notin P_k\), \(b^k_i = 0\).

In a decentralized mechanism, firm \(0\) will be located in \(\ell = \arg\max_{k \in R} \{b^0_k\}\).

We assume that a planner should decide where to locate firm \(0\) maximizing the global benefit and providing a compensating scheme such that firm \(0\) gets more than in the decentralized mechanism and the other agents are not worst off.
In order to do this, we associate to any problem the following utility transferable cooperative game. For every \( S \subseteq N_0 \),
\[
v(S) = \begin{cases} 
\max_{k \in R} \left\{ b^k_0 + \sum_{i \in U(S)} b^k_i \right\}, & \text{if } 0 \in S \\
0, & \text{otherwise,}
\end{cases}
\]
where \( U(S) = \{ i \in S : P_k \subseteq S \} \) is set of firms whose region is contained in \( S \) (\( k_i \in R \) is the region where the firm \( i \) is located).

Given a problem \((0, N, P, b)\) we say that \( k^* \) is an optimal location if firm 0 could be located at region \( k^* \) in order to maximize the total profit.

Let \( L_0 \) be as the maximum utility that can be achieved when firm 0 when it does not cooperate with the firm located at region \( k^* \). Namely,
\[
L_0 = \max_{k \in R} \left\{ \sum_{i \in N_0 \setminus P_k} b^k_i \right\}.
\]

Main results

The core

We prove that the core of the game \((N_0, v)\) is non empty and it is given by
\[
C(N_0, v) = \left\{ x \in \mathbb{R}^{n+1} : \sum_{i \in N_0} x_i = v(N_0), L_0 \leq x_0 \leq v(N_0), \\
x_i = 0, \forall i \notin P_{k^*}, 0 \leq x_i \leq v(N_0) - L_0, \forall i \in P_{k^*} \right\}.
\]

Allocation rules

The nucleolus:

\[
\eta_i(v) = \begin{cases} 
L_0 + \frac{v(N_0) - L_0}{|P_{k^*}| + 1}, & \text{if } i = 0 \\
v(N_0) - L_0 \frac{|P_{k^*}|}{|P_{k^*}| + 1}, & \text{if } i \in P_{k^*} \\
0, & \text{otherwise.}
\end{cases}
\]

Weighted optimal location rule:

\[
\psi_i(v) = \begin{cases} 
L_0 + \frac{b^k_{0^*}}{v(N_0)} (v(N_0) - L_0), & \text{if } i = 0 \\
\frac{b^k_{i^*}}{v(N_0)} (v(N_0) - L_0), & \text{if } i \in P_{k^*} \\
0, & \text{otherwise.}
\end{cases}
\]
Characterization

Core selection. A rule gives an allocation belonging to the core.

Equal treatment. If the value of the optimal location increases, then all firms of such region and firm 0 should gain the same.

Weighted treatment. If the value of the optimal location increases, then all firms of such region and firm 0 should gain in proportion of the value they generate on the original problem.

Symmetry. If there are firms on the same region that obtain the same benefits when firm 0 opens a new plant at their region, then they receive the same.

Weighted symmetry. If firms on the same region receive proportionally the benefits they obtain when firm 0 opens a new plant at their region.

The nucleolus of the game $v$ is the unique rule satisfying core selection and equal treatment.

The weighted optimal location rule $\psi$ is the unique rule satisfying core selection and weighted treatment.