Indexing public pensions in progress to wages or prices *

by

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Abstract

While indexing initial public pensions mostly follows economy-wide wages, that of pensions in progress may follow either wages or prices or their combination. We create a simple overlapping cohorts framework to study a neglected issue in indexation: close contribution paths should imply close benefit paths even at real wage shocks. This robustness criterion of an equitable pension system is only satisfied by wage indexing, which in turn requires the adjustment of the accrual rate (connecting individual wages and benefits). To minimize the redistribution from low-earning short-lived citizens to high-earning long-lived ones, progression should be introduced.

JEL codes: D10, H55

Keywords: public pensions, indexation, fairness

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1. Introduction

Since the 1970s, all over the developed world, *initial public pensions* have been indexed (valorized) to the economy-wide average wages, but *pensions in progress* have been indexed to prices, average wages and various combination of prices and average wages—varying across countries and periods. Indexing pensions is only a technical measure in the short run, but it can be very important in the long run. This is especially true when the public pension (paid as unisex indexed life annuity) replaces a large part of the previous earnings. In my opinion, in the pension literature, indexing pensions has not received the attention which it deserves. Perhaps it is not an accident that in his excellent Presidential lecture, Peter Diamond had to relegate the problem into a footnote (Diamond 2004, p. 7. ftnt. 24):

“Mandatory annuitization in a social security program raises the interesting question of how a monthly benefit should vary over time—with prices, wages, and possibly other variables such as rates of return. Relevant for this issue are the age structure of optimized expenditures, the relative importance of both real and relative consumption, and the allocation of risk bearing between the elderly and the rest of population. Currently [in the US, A.S.], the benefits in force are increased for inflation as measured by the CPI. While this is a reasonable solution, I suspect it would be better, on a revenue neutral basis, to have lower initial benefits that then grew faster (for example as a weighted average of prices and wages). This would help more the longer-lived than the shorter-lived but the effect on expected lifetime income distribution could be partially adjusted by changing the benefit formula.”

Barr and Diamond (2008) devoted a whole chapter (Chapter 5) to this multi-dimensional problem: they separately discussed indexing initial and continued benefits. Very few countries use and very few economists favor indexing initial pensions to prices (cf. President’s Commission, 2001, p. 84, model 3 and Biggs, Brown and Springstead, 2005). On the other hand, opinions are divided whether pensions in progress should be indexed to prices or wages or their combination. Therefore we confine our attention to benefits in progress and study the problem with a multicohort rather than a two-generation overlapping model. Apart from Theorem A.1, in this paper we avoid combined indexation, and suggest that indexation to wages (similar to a point system—see Appendix A—or Nonfinancial Defined Contribution) is superior to indexation to prices, especially when a real wage shock hits the economy. Writing this paper I have been heavily influenced by the turbulent years 1993-2018 of the Hungarian pension policy but I am convinced that the analysis of this erratic process highlights a number of important points, therefore it is of a general interest.

In the short run, in a country with smooth real wage dynamics, indexing pensions in progress is almost irrelevant. With a consumer price index of 102 and a nominal wage index of 104, at first sight it is not too interesting if the nominal pensions are increased by 2 or 4%, or their arithmetic average, by 3%. A typical pensioner, however, spends about 20 years in retirement, therefore the annual 1–2% differences become 20–40% differences at the end and 10–20% deviations during the whole period. The latter difference manifests itself both at the macro and the micro levels. But even in the well-designed US Social Security system, the correction of an earlier indexation error created so-called notch babies: cohorts retiring just after a certain year received much lower benefits than slightly earlier cohorts (e.g. Krueger and Pischke, 1992).
Table 1. *Output, real wage and real pension dynamics: Hungary: 1993–2018*

<table>
<thead>
<tr>
<th>Year</th>
<th>Real growth rate of GDP [100(g^y - 1)]</th>
<th>Real growth rate of net wage [100(g^v - 1)]</th>
<th>Real growth rate of pension [100(g^b - 1)]</th>
<th>Net replacement rate [\gamma_t]</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1993</td>
<td>-0.8</td>
<td>-3.9</td>
<td>-4.6</td>
<td>0.603</td>
<td></td>
</tr>
<tr>
<td>1994</td>
<td>3.1</td>
<td>7.2</td>
<td>4.7</td>
<td>0.594</td>
<td>E: change in PIT</td>
</tr>
<tr>
<td>1995</td>
<td>1.5</td>
<td>-12.2</td>
<td>-10.1</td>
<td>0.619</td>
<td>change in delay</td>
</tr>
<tr>
<td>1996</td>
<td>0.0</td>
<td>-5.0</td>
<td>-7.9</td>
<td>0.593</td>
<td></td>
</tr>
<tr>
<td>1997</td>
<td>3.3</td>
<td>4.9</td>
<td>0.4</td>
<td>0.563</td>
<td></td>
</tr>
<tr>
<td>1998</td>
<td>4.2</td>
<td>3.6</td>
<td>6.2</td>
<td>0.578</td>
<td>E</td>
</tr>
<tr>
<td>1999</td>
<td>3.1</td>
<td>2.5</td>
<td>2.1</td>
<td>0.592</td>
<td></td>
</tr>
</tbody>
</table>

Swiss indexation (half wage+half price)

<table>
<thead>
<tr>
<th>Year</th>
<th>Real growth rate of GDP [100(g^y - 1)]</th>
<th>Real growth rate of net wage [100(g^v - 1)]</th>
<th>Real growth rate of pension [100(g^b - 1)]</th>
<th>Net replacement rate [\gamma_t]</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>4.2</td>
<td>1.5</td>
<td>2.6</td>
<td>0.591</td>
<td></td>
</tr>
<tr>
<td>2001</td>
<td>3.8</td>
<td>6.4</td>
<td>6.6</td>
<td>0.591</td>
<td>+ raise</td>
</tr>
<tr>
<td>2002</td>
<td>4.5</td>
<td>13.6</td>
<td>9.8</td>
<td>0.573</td>
<td>E++ raise</td>
</tr>
<tr>
<td>2003</td>
<td>3.8</td>
<td>9.2</td>
<td>8.5</td>
<td>0.568</td>
<td>+ 1 week pension</td>
</tr>
<tr>
<td>2004</td>
<td>4.9</td>
<td>-1.1</td>
<td>3.9</td>
<td>0.600</td>
<td>+ 2 weeks pension</td>
</tr>
<tr>
<td>2005</td>
<td>4.4</td>
<td>6.3</td>
<td>7.9</td>
<td>0.611</td>
<td>+ 3 weeks pension</td>
</tr>
<tr>
<td>2006</td>
<td>3.8</td>
<td>3.6</td>
<td>4.5</td>
<td>0.623</td>
<td>E + 4 weeks pension</td>
</tr>
<tr>
<td>2007</td>
<td>0.4</td>
<td>-4.6</td>
<td>-0.3</td>
<td>0.668</td>
<td></td>
</tr>
<tr>
<td>2008</td>
<td>0.8</td>
<td>0.8</td>
<td>3.4</td>
<td>0.691</td>
<td></td>
</tr>
<tr>
<td>2009</td>
<td>-6.6</td>
<td>-2.3</td>
<td>-5.7</td>
<td>0.672</td>
<td>no 13th month benefit</td>
</tr>
</tbody>
</table>

Indexation to prices

<table>
<thead>
<tr>
<th>Year</th>
<th>Real growth rate of GDP [100(g^y - 1)]</th>
<th>Real growth rate of net wage [100(g^v - 1)]</th>
<th>Real growth rate of pension [100(g^b - 1)]</th>
<th>Net replacement rate [\gamma_t]</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>0.7</td>
<td>1.8</td>
<td>-0.9</td>
<td>0.651</td>
<td>E</td>
</tr>
<tr>
<td>2011</td>
<td>1.8</td>
<td>2.4</td>
<td>1.2</td>
<td>0.647</td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>-1.7</td>
<td>-3.4</td>
<td>0.1</td>
<td>0.670</td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>1.9</td>
<td>3.1</td>
<td>4.5</td>
<td>0.678</td>
<td>overindexation**</td>
</tr>
<tr>
<td>2014</td>
<td>3.7</td>
<td>3.2</td>
<td>3.2</td>
<td>0.675</td>
<td>E+ overindexation</td>
</tr>
<tr>
<td>2015</td>
<td>2.9</td>
<td>4.3</td>
<td>3.5</td>
<td>0.668</td>
<td>overindexation</td>
</tr>
<tr>
<td>2016</td>
<td>2.1</td>
<td>7.4</td>
<td>1.4</td>
<td>0.631</td>
<td>start of wage explosion</td>
</tr>
<tr>
<td>2017</td>
<td>4.1</td>
<td>10.2</td>
<td>3.0</td>
<td>0.583</td>
<td>wage explosion continued</td>
</tr>
<tr>
<td>2018*</td>
<td>4.0</td>
<td>8.0</td>
<td>2.0</td>
<td>0.550</td>
<td>wage explosion ends?</td>
</tr>
</tbody>
</table>

Source: ONYF (2016, Table 1.3, p. 16), new data are added, *: forecast, E = election, **: when the inflationary forecast was higher than the actual, the additional benefit rise was not deducted.

In another country where average real wages may increase or decrease by 5–10% a year, with a relative freedom from the GDP’s growth rate, indexation matters even in the short-run. Table 1 above displays the historical time series of the Hungarian developments during 1993–2018, the more so because the GDP and the net wage growth rates were very turbulent, and indexation rules to wages, to wages and prices and to prices followed each other. We inform the reader on other factors in the last column.
of Table 1. Just concentrating on the latest development, from 2015 to 2018, real net average wage grew by an astonishing 28% while the GDP only grew by 10%, pensions (mostly price-indexed pensions in progress) only by 6%. Those who retire in 2017–2019 receive pensions higher by 7–18–28% than those, who—with similar wage paths—retired in 2016. This issue does not only arise in Hungary but also in a number of other countries with wildly oscillating growth rates of real wages, therefore indexing pensions deserves the attention of both theoretical and applied economists.

The only method to avoid the above-mentioned anomaly is to raise pensions in progress by the economy-wide wage growth rate, shortly: indexing to wages. At the same time, this type of indexation makes the necessary restraint with the initial benefits more visible and prefers those living longer (females and higher earners) and weakens the incentives to retire later. Finally, in a country, where the pension system is proportional (equivalently: earnings-related), some form of a flat component is inevitable. At this point I must admit that between 2010 and 2017 I also accepted the prevailing wisdom in Hungary: only pure price indexing is feasible in the long run. The preservation of combined indexation or the return to pure indexation to wages would have required the simultaneous reduction of accrual rates!

In this paper, we use the framework of overlapping cohorts. Each cohort is represented by a single individual, whose real super-gross earnings (more precisely: total labor cost) vary with years. After a transition period, with indexation to wages, the benefits become independent of the age of the beneficiary and follow the wage dynamics through the (total) accrual rate.

What happens under the more popular indexation to prices? Referring to the ratio of the average benefits to the average wages as average replacement ratio (sometimes called benefit ratio), we show that the higher the real wage growth rate, the lower the average replacement ratio. A related consequence is that a real wage hike separating two subsequent cohorts, may transform two mostly overlapping wage paths into distant benefit paths.

Furthermore, the deliberate reduction of the employer’s contribution rate (partly executed, partly planned in Hungary between 2016 and 2022) accelerates the growth of real net wages with respect to that of the total labor cost. If the benefits are related to net wages rather than to gross ones (like in Hungary), then this manipulation further reduces the older benefits with respect to newer benefits.

Returning to wage indexing, note that the representative individuals can be simply replaced by multi-type cohorts with varying wages, life expectancies and fragmented labor careers but by assuming time-invariant and type-invariant growth rates of labor costs. (Arbitrary wage paths are relegated to Appendix A.)

cohort-specific NDC rules.

Since 2000, several economists have documented that the apparently progressive US Social Security system (with steeply declining marginal accrual rates) is weakly progressive on a lifetime basis. In fact, life expectancy at retirement is a steeply increasing function of the lifetime wages (e.g. Liebmann, 2002). Recently there is a growing concern for this tendency which is strengthening all over the world. Among others, Whitehouse and Zaidi (2008), The National Academy ... (2015), Auerbach et al. (2017); Ayuso, Bravo and Holzmann (2017) reconsidered this problem on newer data. Simonovits (2018a, Section 14.4) returned to Diamond’s concern: the impact of wage index weight (i.e. the share of the wage index in the combined wage–price index) on the redistribution from the short-lived low-paid to the long-lived high-paid.

Augusztinovics and Köllő (2008) studied the negative impact of fragmented careers on benefits in Hungary. Czeglédi, Simonovits, Szabó and Tir (2017); Simonovits (2018a, Section 9.4) and Granseth, Keck, Nagl, Simonovits and Tir (2019) combined the modeling of the choice of retirement age and of the length of fragmented contributions, and discovered surprising connections: for example, seniority pension schemes may result in negative correlations between retirement age and length of contributions. Obviously, the form of indexation influences the choice of the retirement age. For example, Simonovits (2018b) modeled how Female40, a seniority retirement system in force in Hungary since 2011 allowed females with eligibility of 40 years to retire without actuarial deduction. From the start this system unduly punished females with slightly shorter careers. Moreover, it became a boomerang due to price indexing and real wage hike: a large share of beneficiaries would have received greater lifetime benefits if they had retired later.

Schookkaert, Devolder, Hindriks and Vandbrucke (2018) discussed a related model of the point system which is equitable and sustainable. Their model is more elaborate than ours, especially that it contains a general demographic block and a sophisticated blend of DB and DC principles for Belgium. But the foregoing model neglects a basic concern of the present paper: fragmented careers make the correlation between retirement age and length of career loose. Future research should combine the two approaches.

The structure of the present paper is as follows. Sections 2 and 3 discuss the wage and price indexing rules at a macrolevel, respectively. Section 4 generalizes the wage indexing to microlevel and Section 5 concludes. Appendix A considers the combined indexation and the point system for individual real wage paths differing from the average path. Appendix B explains the calibration of the parameter values used in the calculations.

2. Indexing to wages (macro)

All wages and benefits are calculated at constant prices. We work with a very simple dynamic macro framework of overlapping cohorts, where each cohort is represented by a single person. Cohort’s real super-gross wage is independent of her age but depends on the calendar year. We shall denote the length of employment by \( S \) and the length of retirement period by \( T > 1 \), both integers (measured in years). Assuming that every cohort’s representative retires at the same age and dies at the same age we avoid the very difficult problem of redistribution among cohorts due to population aging.

The simplest assumption on benefits is as follows: the initial pension benefit is
proportional to the current super-gross wage \(w_t\):

\[ b_{0,t} = \beta w_t, \quad t = 1, 2, \ldots, \] (1)

where \(\beta\) is called the total accrual rate (or alternatively, replacement ratio). For the sake of utmost simplicity, we neglect the usual one-year-lag in valorization [(1)]. We shall derive the general formula for arbitrary individual wage paths in Appendix A. Note that various national public pension systems more-or-less differ from (1) but this is neglected here.

In this Section, the benefits in progress are indexed to wages, i.e. every year the government raises these benefits according to the rationally expected time-variant wage growth factor \(g_t = w_t/w_{t-1}\). Distinguishing various benefits in progress by the number of years \(k\) elapsing since their start, denoted by \(b_{k,t}\), wage indexing is described as

\[ b_{k,t} = g_t b_{k-1,t-1}, \quad k = 1, 2, \ldots, T - 1, \quad t = 1, 2, \ldots. \] (2)

To start system (1)–(2) in \(t = 1\), the predetermined values have to be given:

\[ b_{k,0}, \quad k = 0, 1, 2, \ldots, T - 1. \] (3)

To simplify the analysis, we shall assume that the system has been working since \(t = -T + 1\), therefore the ‘initial values’ are already independent of \(k\):

\[ b_{k,0} = g_0 \cdots g_{b-k} w_{-k} = \beta w_0, \quad k = 0, 1, 2, \ldots, T - 1. \] (4)

We have

**Theorem 1.** Under wage indexing (1)–(2) and (4), the benefits are independent of the age of the retirees and proportional to the current super-gross wage:

\[ b_{k,t} = \beta w_t, \quad k = 0, 1, 2, \ldots, T - 1, \quad t = 1, 2, \ldots. \] (5)

**Proof.** For \(t = 1\), (2) and (4) imply \(b_{k,1} = g_1 b_{k-1,0} = g_1 \beta w_0 = \beta w_1\). By mathematical induction, if \(b_{k,t} = \beta w_t\), then \(b_{k+1,t+1} = g_{t+1} b_{k,t} = g_{t+1} \beta w_t = \beta w_{t+1}\).

To simplify notations, when indexing to wages we shall write \(b_t\) for the common values of \(b_{k,t}\): \(b_t = \beta w_t\).

Without assuming (4), we would only obtain (5) for \(t \geq T\). The left half of Table 2 displays the life paths of two cohorts starting to work in years 0 and 1, respectively; under wage indexing (the right half will be used in Section 3). Their wage and benefit paths only differ at the start and the end, namely in cohort 0’s first wage \(w_0\) and benefit \(b_{S-1}^w\) and in cohort 1’s last wage \(w_S\) and benefit \(b_{S+T}^w\), otherwise the corresponding benefits are equal. To avoid confusion, here we distinguish the variables of wage- and price-indexed systems by superscripts \(w\) and \(p\), respectively; but otherwise we may drop the superscripts.
The balance condition of a pay-as-you-go wage-indexed pension system is as follows:

\[ \tau S w_t = T \beta w_t, \]  

where \( \tau \) is the pension contribution rate. This implies

**Theorem 2.** In a wage-indexed public pension system, the balanced accrual rate is given by the product of the contribution rate and the dependency ratio:

\[ \beta = \tau \mu, \]  
\[ \text{where} \quad \mu = \frac{S}{T}. \]

Unfortunately, indexing to wages does not prevent the decline of the real value of the benefit, when the real wage drops: if \( w_t < w_{t-1} \), then \( b_t < b_{t-1} \). To avoid this, between 1975 and 1980 the UK government chose a strange index: the maximum of 1 and the real wage growth factor \( g_t \). But this rule overindexed the pensions in progress and was terminated (Barr–Diamond, 2008, Box 5.8, p. 77).

A sensible solution is as follows. The above rule is only a conditional plan:

\[ \tilde{b}_t = \min(\beta w_t, \tilde{b}_{t-1}). \]

To avoid excessive benefit rises, the government sets up a trust fund, the capital of which is equal to \( F_t \) at the end of year \( t \). The government introduces the following conditional feedback rule with an appropriately chosen feedback coefficient \( \kappa > 0 \):

\[ \tilde{b}_t = \left\{ \begin{array}{ll}
\tilde{b}_t + \kappa F_{t-1} & \text{if } \tilde{b}_{t-1} = \tilde{b}_{t-2}; \\
\tilde{b}_t & \text{otherwise}.
\end{array} \right. \]

The trust fund’s dynamics is as follows:

\[ F_t = F_{t-1} + \tau S w_t - T \tilde{b}_t, \quad F_0 = 0. \]
To illustrate the operation of our rule, we use the following parameter values as of Hungary, 2016 (see Appendix B). Pension contribution rate: \( \tau = 0.213 \); a special form of old-age dependency ratio: \( \mu = T/S = 20/31.2 = 0.64 \), the super-gross average replacement ratio is equal to \( \beta = 0.407 \).

Table 3 displays two wage and benefit paths when the real wage growth factor wildly oscillates: \( g_t = 1.02 + (-1)^{t+1}0.04 \), i.e. it alternates between 0.98 and 1.06, their geometric average being close to 1.02. Note that without the trust fund, from year 2 to year 3, in terms of the initial super-gross wage \( w_0 = 1 \), the benefit drops from 0.393 to 0.385, etc. In the modified system with \( \kappa = 0.1 \), in odd years, the benefit remains the same as previously, but in even years, its value is diminished with respect to the simple rule, e.g. in year 4, \( 0.393 < 0.408 \). The trust fund’s capital oscillates with narrow bounds.

### Table 3. Wage-indexed pensions without or with a trust fund

<table>
<thead>
<tr>
<th>Year ( t )</th>
<th>Total labor cost ( w_t )</th>
<th>Simple benefit ( b_t )</th>
<th>Modified benefit ( \tilde{b}_t )</th>
<th>Trust fund ( F_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.980</td>
<td>0.370</td>
<td>0.370</td>
<td>0.000</td>
</tr>
<tr>
<td>2</td>
<td>1.039</td>
<td>0.393</td>
<td>0.393</td>
<td>0.000</td>
</tr>
<tr>
<td>3</td>
<td>1.018</td>
<td>0.385</td>
<td>0.393</td>
<td>-0.157</td>
</tr>
<tr>
<td>4</td>
<td>1.079</td>
<td>0.408</td>
<td>0.392</td>
<td>0.157</td>
</tr>
<tr>
<td>5</td>
<td>1.058</td>
<td>0.400</td>
<td>0.400</td>
<td>0.157</td>
</tr>
<tr>
<td>6</td>
<td>1.121</td>
<td>0.424</td>
<td>0.424</td>
<td>0.157</td>
</tr>
<tr>
<td>7</td>
<td>1.099</td>
<td>0.415</td>
<td>0.424</td>
<td>-0.012</td>
</tr>
<tr>
<td>8</td>
<td>1.164</td>
<td>0.440</td>
<td>0.439</td>
<td>0.012</td>
</tr>
<tr>
<td>9</td>
<td>1.141</td>
<td>0.431</td>
<td>0.439</td>
<td>-0.139</td>
</tr>
</tbody>
</table>

Finally, returning to the simple indexation system, we calculate the undiscounted lifetime benefits of the two cohorts introduced in Table 2:

\[
C_0 = \sum_{t=0}^{T-1} b_{S+t} \quad \text{and} \quad C_1 = \sum_{t=1}^{T} b_{S+t},
\]

hence their difference is equal to

\[
C_1 - C_0 = b_{S+T} - b_S = \beta (w_{S+T} - w_S) = \beta (G_S - 1) w_S, \quad \text{where} \quad G_S = w_S/w_0.
\]

### 3. Indexing to prices (macro)

In this Section, we investigate the pensions in progress under price indexing. Its analysis is more complex than that of the wage indexing because we have to distinguish the benefits of pensioners retired in different years even after the initial transition is over. (Here we already allow for the one-year-lag in valorization.)
Newly awarded benefits with delay:

\[ b_t = \beta w_{t-1}, \quad t = 1, 2, \ldots . \]  

(7)

Invariant value of benefit, started in year \( t - k \):

\[ b_{t-k} = \beta w_{t-k-1}, \quad k = 1, 2, \ldots , T - 1, \quad t = 1, 2, \ldots . \]  

(8)

Again, the predetermined benefits are set as if the system started in \( t = -T + 1 \):

\[ b_{-k} = \beta w_{-k-1}, \quad k = 0, 1, 2, \ldots , T - 1. \]  

(9)

To achieve revenue-neutrality, now the accrual rate \( \beta \) can differ from the previous one: \( \beta^p > \beta^w \). Recall that we only distinguish explicitly the two systems’ variables and parameters if its lack would cause confusion. The right half of Table 2 above displays the life paths of the same two cohorts as in the left part, but now under price indexing. Note the inconsistency of price indexing: an extra year’s earning \( w_S \) rather than \( w_0 \) yields a stream \( (b^p_S) \) rather than \( (b^p_{S-1}) \)—both being flat which significantly differ if a wage shock arises in \( S \).

To determine the new balance conditions, we introduce

Total expenditures in year \( t \):

\[ B_t = \sum_{k=0}^{T-1} b_{t-k}. \]

Then the average benefit and average replacement ratio respectively are equal to

\[ \bar{b}_t = \frac{B_t}{T} \quad \text{and} \quad \gamma_t = \frac{\bar{b}_t}{w_t}. \]  

(10)

We shall see that the average replacement ratio depends on the real wage paths, transforming the underlying defined contribution system to a defined benefit system with time-variant pension contribution rate \( \tau_t \).

First we illustrate theoretically and numerically the dependence of the average replacement ratio on the time-invariant wage growth factor \( g \). We shall need the concept of equivalent number of years in retirement:

\[ T_g = \sum_{k=1}^{T} g^{-k} = \frac{1 - g^{-T}}{g - 1}, \quad g > 1 \quad \text{and} \quad T_1 = T. \]

Equivalence means that indexing to prices during \( T \) years costs the same as indexing to wages during \( T_g \) years. With \( T_g \)'s help, we have

**Theorem 3.** For a price-indexed pension system and for a time-invariant real wage growth factor \( g \), the corresponding time-invariant average replacement ratio is given by

\[ \gamma = \beta \frac{T_g}{T}. \]  

(11)
A well-known disadvantage of price indexing for the pensioners (which is an advantage for the government) is as follows: the higher the real wage growth rate, the lower the average replacement ratio with respect to the accrual rate. The lag in valorization (7) causes a small part of the drop, and the lagging of pensions in progress behind the initial one in indexation (8) causes the large part of the drop. Quantitatively, with $T = 20$ years, Table 4 demonstrates how the replacement ratio—in parallel with the equivalent $T_g$—drops from $\gamma(0) = \beta = 0.407$ through 0.333 to 0.254 as the growth rate of the real wages rises from 0 through 2 to 5%. Since pensions are often lighter taxed than gross wages (or not taxed at all), the net replacement ratio ($\gamma^n$) is more relevant than the gross one, and the former is also reported below the latter.

**Table 4. Average replacement ratio and growth rate of real wage with price indexing**

<table>
<thead>
<tr>
<th>Growth rate of super-gross real wage $100(g - 1)$</th>
<th>Equivalent number of years $T_g$</th>
<th>Net average replacement ratio $\gamma^n$</th>
<th>Super-gross average replacement ratio $\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>20,000</td>
<td>0.772</td>
<td>0.407</td>
</tr>
<tr>
<td>1</td>
<td>18.046</td>
<td>0.696</td>
<td>0.367</td>
</tr>
<tr>
<td>2</td>
<td>16.351</td>
<td>0.631</td>
<td>0.333</td>
</tr>
<tr>
<td>3</td>
<td>14.877</td>
<td>0.574</td>
<td>0.303</td>
</tr>
<tr>
<td>4</td>
<td>13.590</td>
<td>0.524</td>
<td>0.277</td>
</tr>
<tr>
<td>5</td>
<td>12.462</td>
<td>0.481</td>
<td>0.254</td>
</tr>
</tbody>
</table>

**Remark.** $\beta = 0.407$

We turn now to the dynamics of the average replacement ratio when the real wage path is uneven. Starting with the tautological approach, (10) yields a trivial formula:

$$\frac{\gamma_t}{\gamma_{t-1}} = \frac{\bar{b}_t}{\bar{b}_{t-1}g_t}.$$  

In words: the growth factor of the average replacement ratio is equal to the ratio of the growth factors of the average benefits and of the wages. (Note that Table 1 above presented net rather than super-gross wages and replacement ratios but the relation remains valid.) While the formula holds for any type of indexation, in pure indexing to wages, both sides simplify to 1.

Digging deeper, we can express the dynamics of the average benefits and of the average replacement ratio as functions of the underlying wage growth factors, respectively. The following recursion is to be used:

$$B_t = B_{t-1} + b_t - b_{t-T}. \quad (12)$$

Hence relying on (9), (10) and (12), the new average replacement ratio is given by

$$\gamma_t = \frac{\bar{b}_t}{w_t} = \frac{\bar{b}_{t-1}}{g_tw_{t-1}} + \beta \frac{w_{t-1} - w_{t-T-1}}{Tw_t}. \quad (13)$$

To simplify (13), we use again the cumulated real wage growth factor between years $t - T$ and $t$: $G_t = w_t/w_{t-T}$ which is also equal to the ratio of the next year’s youngest and oldest pensions: $G_t = b_{t+1}/b_{t-T+1}$.
Theorem 4. For time-variant wage growth rates, the dynamic of average replacement ratio is given by

\[ \gamma_t = \frac{\gamma_{t-1}}{g_t} + \beta \frac{1 - G_{t-1}}{g_t T}, \quad t = 1, 2, \ldots \] (14)

Remark. (14) remains valid for net rather than gross categories, only \( w_t / w_{t-1} \) and \( \beta \) should be replaced by \( v_t / v_{t-1} \) and \( \beta^n \), respectively.

It is worth adding some explanation to (14). The first, dominant term represents the past average replacement ratio, mildly scaled-down by the current real wage growth factor. The second term is typically less than \( \beta / T = 0.03 \) in modulus, which pushes up or down the first term.

Having this formula, we model the impact of the extraordinary real wage hike occurring in Hungary during 2016–2018 on the average replacement ratio. We assume that there are two values of the real wage growth factors (1 <) \( g_m < g_M \), the greater is reached in year \( t_0 - 1, t_0, t_0 + 1 \):

\[ g_t = \begin{cases} g_m & \text{if } t < t_0 - 1 \text{ or } t > t_0 + 1; \\ g_M & \text{otherwise}. \end{cases} \]

Assume \( S = 32 \) and \( T = 20 \) years. Noting that \( G_0 = g_m^T \) and \( \gamma_0 = \gamma(g_m) = 0.333 \) and working with \( g_m = 1.02 \) and \( g_M = 1.08 \), Table 5 depicts a stylized process, odd years are not shown from \( t = 4 \). Starting from a steady state, the real wage explosion reduces \( \gamma_0 = 0.333 \) to \( \gamma_3 = 0.283 \) and then \( \gamma_t \) slowly returns to the start.

**Table 5.**

<table>
<thead>
<tr>
<th>Year</th>
<th>Average replacement ratio</th>
<th>Year</th>
<th>Average replacement ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>( \gamma_t )</td>
<td>( t )</td>
<td>( \gamma_t )</td>
</tr>
<tr>
<td>0</td>
<td>0.333</td>
<td>10</td>
<td>0.304</td>
</tr>
<tr>
<td>1</td>
<td>0.314</td>
<td>12</td>
<td>0.309</td>
</tr>
<tr>
<td>2</td>
<td>0.298</td>
<td>14</td>
<td>0.314</td>
</tr>
<tr>
<td>3</td>
<td>0.283</td>
<td>16</td>
<td>0.319</td>
</tr>
<tr>
<td>4</td>
<td>0.286</td>
<td>18</td>
<td>0.324</td>
</tr>
<tr>
<td>6</td>
<td>0.292</td>
<td>20</td>
<td>0.328</td>
</tr>
<tr>
<td>8</td>
<td>0.298</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

It is natural that the temporary drop of the average replacement ratio makes room for a similar temporary reduction of the contribution rate. But as the replacement ratio eventually returns to its former value, so does the contribution rate. To show this, we introduce

Total contributions in year \( t \):

\[ I_t = \tau_t S w_t. \]

The new balance condition \((I_t = B_t)\) is as follows:

\[ \tau_t S w_t = T \bar{b}_t = T \gamma_t w_t. \] (15)

We have arrived to
Theorem 5. Under price-indexation and dynamic (14), the balanced pension contribution rate is given by the product of the dependency ratio and the average replacement ratio:

\[ \tau_t = \mu \gamma_t, \quad \text{where} \quad \mu = \frac{T}{S}. \quad (16) \]

Finally, we return to the undiscounted lifetime benefits of the two cohorts introduced in Table 2 but now for the price-indexed ones.

\[ C_0 = \sum_{t=0}^{T-1} b_{S+t} = Tb_S \quad \text{and} \quad C_1 = \sum_{t=1}^{T} b_{S+t} = Tb_{S+1}, \]

hence their difference is equal to

\[ C_1 - C_0 = T\beta(w_S - w_{S-1}) = T\beta(g_S - 1)w_{S-1}. \]

Table 6 presents the differences of lifetime benefits in terms of the initial total labor cost, arising for a real wage hike under wage- and price indexing rules, respectively. The wage hike runs from 0 to 10% in year \( S \), otherwise real wages grow by 2%. Note that under wage indexing, the difference (given in terms of the initial total labor cost \( w_0 = 1 \)) grows moderately, while under price indexing, the unadjusted difference rises rather steeply.

<table>
<thead>
<tr>
<th>Real wage hike 100((g_S - 1))%</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wage-indexed diff ( C^w_1 - C^w_0 )</td>
<td>0.226</td>
<td>0.231</td>
<td>0.235</td>
<td>0.240</td>
<td>0.244</td>
<td>0.249</td>
</tr>
<tr>
<td>Price-indexed diff ( C^p_1 - C^p_0 )</td>
<td>0.000</td>
<td>0.186</td>
<td>0.372</td>
<td>0.552</td>
<td>0.745</td>
<td>0.921</td>
</tr>
</tbody>
</table>

4. Indexing to wages (micro)

In Sections 2 and 3, we demonstrated that at macrolevel, the only consistent method is indexing to wages. This requires, however, some political restraint from the government to (i) reduce the accrual rate \( \beta \) appropriately and (ii) forsake temporary reduction of the contribution rate \( \tau_t \) during a real wage hike (see above). Moreover, at microlevel, wage indexing has an unpleasant side effect: since the life expectancies of various income groups widely differ, namely higher earners live longer, then the faster the benefits increase, the stronger the income redistribution from the shorter-lived to the longer-lived. We shall show how this can be mitigated by the introduction of pension progression.

Working out the necessary changes, for the sake of simplicity, we neglect now the time-variance of real wage growth rates, and relax the assumptions of homogeneous wages and life expectancies. Let \( i \) be the index of a wage group, \( i = 1, 2, \ldots, n \), \( f_i > 0 \) be its frequency: \( \sum_{i=1}^{n} f_i = 1 \). We assume that the real wage of each income group
grow at the same rate, therefore the corresponding type-specific wage in year $t$ is equal to

$$w_{i,t} = w_{i,0} g^t,$$

where $w_0 = \sum_{i=1}^{n} f_i w_{i,0} = 1.$ (17)

(In Appendix A, we shall cover the general case of type- and time-variant growth factors $g_{i,t}$.)

By assumption, everybody retires at age $R = Q + S$, type $i$ lives until $D_i = R + T_i$; $w_{0,i}$ as well as $D_i$ is increasing, and $\sum_{i=1}^{n} f_i D_i = \bar{D}$.

It would be a simple solution to have type-specific accrual rates $\beta_i$ ($i = 1, 2, \ldots, n$) but it would be politically untenable. Rather, we rely on progression or equalize part of the DC benefits. We shall denote average total labor cost and net wage in year $t$ by $w_t$ and $v_t$, respectively, the share of proportional benefits by $\alpha$, $0 \leq \alpha \leq 1$. Then

$$w_0 = \sum_{i=1}^{n} f_i w_{0,i}.$$ 

The mixed benefits of wage class $i$ are given by

$$b_{i,t} = \beta_i [\alpha w_{i,t} + (1 - \alpha) w_t], \quad i = 1, \ldots, n.$$ (18)

(Make distinction between $b_{i,t}$ and $b_{k,t}$ defined in (2) above!) Simplifying the calculations, we retain stationary population. The balance condition is now

$$\tau S w_t = \sum_{i=1}^{n} f_i T_i b_{i,t}.$$ 

Take the weighted and doubly weighted average times spent in retirement, respectively:

$$\bar{T} = \sum_{i=1}^{n} f_i T_i \quad \text{and} \quad T_w = \sum_{i=1}^{n} f_i T_i w_{i,0}.$$ (19)

Obviously, $T_w > \bar{T}$. Substituting (18) and (19) into the balance condition, yields another balance equation:

$$\tau S w_t = \sum_{i=1}^{n} f_i T_i \beta_i [\alpha w_{i,t} + (1 - \alpha) w_t].$$ (20)

Thus we have arrived to the generalization of Theorem 2.

**Theorem 6.** For wage indexing of pensions in progress and heterogeneous wage profile ($w_{i,0}$) and times ($T_i$) spent in retirement, the balanced accrual rate is equal to

$$\beta_\alpha = \frac{\tau S}{\alpha T_w + (1 - \alpha) \bar{T}}.$$ (21)

**Remark.** As the proportional benefit’s share $\alpha$ decreases, so rises the balanced accrual rate. The proportion of the two extreme cases (0/1) is equal to $T_w/\bar{T}$, and this also increases with heterogeneity.

We shall analyze the income redistribution due to heterogeneous earnings and life expectancies. Corresponding to the logic of the pay-as-you-go system, the type-specific
lifetime balance in year 0 should be discounted by the real growth factor $g$, therefore it is defined by

$$z_{i,0} = \tau Sw_{i,0} - \sum_{k=R}^{D_i} g^{-(k-R)}b_{i,k}.$$  

Using (19), the type-specific balance is given by

$$z_{i,0} = \tau Sw_{i,0} - \beta [\alpha T_i w_{i,0} + (1 - \alpha) T_i]. \quad (22)$$

As an illustration, we consider the traditional homogeneous case, where $T_i \equiv \bar{T}$, i.e. $T_w = \bar{T}$, i.e. (21) is replaced by $\beta = \tau S/\bar{T}$, regardless of $\alpha$. The type-specific lifetime balance is equal to $z_{i,0} = \tau S(1 - \alpha)(w_{i,0} - 1)$, i.e. those who earn below the average ($w_{i,0} < 1$), gain ($z_{i,0} < 0$), the others lose ($z_{i,0} > 0$).

Table 7 presents a numerical illustration. There are three types: $w_{1,0} = 0.5$; $w_{2,0} = 1$ and $w_{3,0} = 1.5$; their frequencies are $1/3$. Let $Q = 28$ and $R = 60$ (to shorten the contribution period’s length to 32 years), the corresponding life expectancies be $D_1 = 75$, $D_2 = 80$, and $D_3 = 85$ years. Though their average is $\bar{D} = 80$ years, the lower the proportional part, the higher the accrual rate. In addition, we display the type-specific lifetime balances. For a proportional system ($\alpha = 1$), the higher earners and longer-lived are the gainers ($z_{3,0} < 0$), the others are the losers; this changes with decreasing $\alpha$ around 0.5.

<table>
<thead>
<tr>
<th>Proportional share</th>
<th>Lifetime balance with life expectancy</th>
<th>Accrual rates and type-specific balances</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>$\beta$</td>
<td>$z_{1,0}$</td>
</tr>
<tr>
<td>1.00</td>
<td>0.314</td>
<td>1.047</td>
</tr>
<tr>
<td>0.75</td>
<td>0.320</td>
<td>0.400</td>
</tr>
<tr>
<td>0.50</td>
<td>0.327</td>
<td>-0.272</td>
</tr>
<tr>
<td>0.25</td>
<td>0.333</td>
<td>-0.972</td>
</tr>
<tr>
<td>0.00</td>
<td>0.340</td>
<td>-1.701</td>
</tr>
</tbody>
</table>

Up to now we have left out the fragmentation of working careers (cf. Augusztinovics and Köllő, 2008) and here we make up this omission. A basic problem of most (but not so much the US) pension systems is that workers who have long lacunas in their labor histories will receive rather low benefits with respect to those who have no lacunas. We introduce the double-weighted expected contribution length:

$$S_w = \sum_{i=1}^{n} f_i w_{i,0} S_i.$$  

Now (21)–(22) modify respectively into

$$\beta_\alpha = \frac{\tau S_w}{\alpha T_w + (1 - \alpha) \bar{T}}.$$
and

\[ z_{i,0} = \tau S_i w_{i,0} - \beta_0 \{ \alpha T_i w_{i,0} + (1 - \alpha) T_i \}. \]

We could illustrate these formulas as well but we forsake it. We also skip the analysis of variable retirement ages.

5. Conclusions

At the end of the paper, we draw some conclusions. We have seen the relative simplicity and fairness of indexing public pensions to wages. Even the adverse impact of temporary drop of real wages on real benefits can be mitigated by a trust fund. We have also demonstrated that the apparently frugal price indexing has a number of pitfalls: In addition to reducing the relative value of old benefits to current wages, it also creates unjustified differences between pension paths of cohorts whose retirement is separated by a real wage shock. This anomaly favors wage indexing over price indexing, but then the government has to renounce the forced reduction of contribution rate during the real wage hike. Furthermore, the government has to weaken the side effect of wage indexing, namely maximizing the perverse redistribution from low-earning short-lived citizen to high-earning long-lived ones. To mitigate this pitfall, the public has to rely on progressive pensions.

Unfortunately, introducing progression often weakens the incentives to report wages. Together with wage indexing, both may undermine the incentives to work longer and strengthen those for early retirement. But the wage indexing is the smallest bad if supplemented with progression. The only remaining problem is: how to phase-in it?

References


Appendix A. Valorization, indexation and point system

In the main text we avoided wage paths with type-specific real wage growth rates. Now we make up this omission, also define combined wage-price indexing and the point system.

Assume that a worker of type \( i \), born in year 0 enters work at age \( Q \) and earns a net real wage \( w_{i,a} \) at age \( a = Q, \ldots, R - 1 \). Her initial pension is given as

\[
b_{i,R} = \delta_R \sum_{a=Q}^{R-1} G_{R-1,a} w_{i,a},
\]

(A.1)

where the valorization multipliers from year \( a \) to year \( R \) in real terms are

\[
G_{R-1,a} = \frac{w_{R-1}}{w_a}, \quad a = Q, \ldots, R - 1,
\]

\( w_a \) being the economy-wide net real wage in year \( a \) and \( \delta_R \) denoting the marginal accrual rate with \( \beta_R = \delta_R S \).

Let \( \iota \) be a real number between 0 and 1, to be called wage index weight. Then raising the real wage growth factor to this power, the previous benefit is multiplied by this number yields the new benefit in progress:

\[
b_{i,a} = b_{i,a-1} g_{\iota}^t, \quad a = R + 1, \ldots, D - 1.
\]

(A.2)

The predetermined benefits are again given.

It is obvious that \( \iota = 1, 1/2, 0 \) represent wage, wage-price and price indexing rules, being in force in Hungary during the periods 1993–1999, 2000-2009 and 2010–, respectively, as described in Table 1.

In the framework of time-invariant wage growth rate, we can now formulate Diamond’s trade-off between faster indexation and lower initial benefit mentioned at the beginning. Introducing the wage-index-weight-dependent accrual rate \( \beta(\iota) \) and the generalized equivalent years spent in retirement

\[
T_{g,\iota} = \frac{1 - g^{(\iota-1)T}}{g^{1-\iota} - 1}, \quad \iota < 1 \text{ and } T_{g,1} = T,
\]

we can prove

**Theorem A.1.** For a time-invariant real wage growth factor \( g \) and a given contribution rate \( \tau \), there is a trade-off between wage index weight \( \iota \) and the accrual rate \( \beta(\iota) \):

\[
\beta(\iota) = \tau \frac{S}{T_{g,\iota}}.
\]

(A.3)

**Proof.** For \( g_t \equiv g \), (16) simplifies to

\[
\tau = \frac{T}{S} \gamma.
\]

(A.4)
Replacing $g^{-1}$ by $g^{-1}$, (11) becomes

$$
\gamma = \beta(\iota) \frac{T_{g,\iota}}{T}.
$$

(A.5)

Combining (A.4) and (A.5) yields

$$
\tau = \frac{\beta(\iota) T_{g,\iota}}{S}
$$

which implies (A.3).

Table A.1 displays the trade-off between the wage index weight and the accrual rate under a fixed contribution rate.

<table>
<thead>
<tr>
<th>Wage index weight $\iota$</th>
<th>0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total accrual rate $\beta(\iota)$</td>
<td>0.407</td>
<td>0.391</td>
<td>0.376</td>
<td>0.361</td>
<td>0.347</td>
<td>0.333</td>
</tr>
</tbody>
</table>

Finally we outline the point system. In year $t + a$, an $i$-type worker of age $a$ earns points

$$
p_{i,a,t+a} = \frac{w_{i,a,t+a}}{w_{t+a}},
$$

i.e. the ratio of her net wage to the economy-wide average. Her cumulated points earned up to retirement is equal to the sum of these points:

$$
P_{i,R,t+R} = \sum_{a=Q}^{R-1} p_{i,a,t+a}.
$$

The value of one point in year $t + a$, $x_{t+a}$ yields a benefit path

$$
b_{i,a,t+a} = p_{i,R} x_{t+a}, \quad a = R, \ldots, D_i - 1.
$$

Note that in the point system, there is neither wage indexing nor price indexing nor their mixture; for example, denoting the cross-sectional profile in year $t$ by $(b_{i,a,t})_a$, the point value $x_t$ is determined from a complex balance condition:

$$
\tau S w_t = \sum_{i=1}^{n} f_i \sum_{a=R}^{D_i} b_{i,a,0} = x_t \sum_{i=1}^{n} f_i T_i p_{i,R,t}.
$$
Appendix B. Calibrating the models, 2016

The traditional parameter values of the Hungarian pension and other transfer systems are calculated in terms of gross (and net) wages. We have difficulties to calibrate our model in a realistic way, but we do our best.

As a base, we use the data for year 2016. Employee’s contribution rates; pension: \( \tau^E = 0.1 \), health: \( \theta^E = 0.08 \), personal income tax rate: \( \sigma = 0.15 \). Employer’s contribution rates; pension: \( \tau^F = 0.17 \): health: \( \theta^F = 0.1 \) (to enhance realism, we shifted 3\% from the former to the latter).

Net wage and total labor cost \((u = 1)\):

\[
v = (1 - \tau^E - \theta^E - \sigma)u = 0.67 \quad \text{and} \quad w = (1 + \tau^F + \theta^F)u = 1.27
\]

Pension contribution rate in terms of total labor cost

\[
\tau = \frac{\tau^E + \tau^F}{1 + \tau^F + \theta^F} = \frac{0.1 + 0.17}{1.27} = 0.213
\]

The basic identity is as follows:

\[
\tau = \mu \gamma,
\]

where \( \mu = S/T \) stands for the old-age dependency ratio and \( \gamma \) stands for the replacement ratio. Transforming net into super-gross index,

\[
\gamma = \gamma^n v/w = 0.631 \times 0.67 / 1.27 = 0.333.
\]

Hence \( \mu = \tau / \gamma = 0.213 / 0.333 = 0.64 \). Since \( T = 20 \), therefore \( S = 20 / 0.64 = 31.3 \), rounded to 32, is low but acceptable.

We need the total net accrual rate \( \beta^n \). Denoting the effective time spent in retirement by

\[
T_g = \frac{1 - g^{-T}}{T(g - 1)},
\]

\( \gamma^n = T_g \beta^n / T \) [(11)] implies \( \beta^n = \gamma^n T / T_g \), which in turn, for \( g = 1.02 \) yields \( \beta^n = 0.631 \times 20 / 16.35 = 0.772 \). Hence the marginal accrual rate is equal to \( \delta = \beta^n / S = 0.772 / 32 = 0.024 \). Finally, \( \beta = \beta^n v / w = 0.772 \times 0.67 / 1.27 = 0.407 \).