Authority and Promotion Tournaments in Organizations

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Abstract

This study explores the interaction between authority and promotion tournament in firms. We argue that the two authority schemes at middle management positions—centralization and decentralization—give rise to different effects between incentive and selection in a promotion tournament involving heterogeneous agents. While under centralization both effects are caused by the firm’s information, under decentralization, these are caused by the agents’ own information. We show that the optimal authority structure is not monotonic and that in particular, both incentive and selection effects make the firm less likely to delegate authority to agents, even if the agents have better information.

Keywords: Centralization, Decentralization, Internal labor market, Promotion tournament.

JEL Classification: D23, J41, M51

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1 Introduction

In most internal labor markets, the prospect of promotion tournament is strong motivator for competing workers. In particular, current amounts of their efforts are considerably influenced by what they can do after promoting: some workers, those higher up in the hierarchy, may be given more authority to make decisions, others may still need to obey the order of firm. Firms thus have to matter decision-making authority allocation taking into account the workers’ dynamic concerns in promotion competitions.

In this article, we study the interaction between the structure of authority and the promotion incentives. We consider a two lowest hierarchical ties of a firm: production and lower management. the firm hires two risk-neutral workers to conduct a production task. We assume that the performance of each production worker is observable yet unverifiable: as a result, the firm is unable to write a contract contingent on the worker’s absolute performance. We therefore focus on a situation where the firm must rely on a promotion-based tournament scheme in its internal labor market to provide proper incentives and to fill vacancies in management with suitable candidates.

At management position, a promoted manager conducts one of projects. Before conducting it, a decision-maker chooses one of two projects, and an outcome of the project depends on state of nature. In the choice of project, both the firm and the manager independently receive informative signal associated with the state. Moreover, besides the state of the world, workers may have a biased preference over the project selection, which belongs a private benefits to them irrespective of nature.

The workers share the same abilities in production but may differ in their skills for the management task. Before contracting, there is symmetric uncertainty about a workers’ management skills, that is, no party can observe a worker’s suitability for the management task. Each worker then learns about his individual suitability for the management position in the firm. Consequently, They become able to assess their individual valuation of being employed as a manager, which affects their incentives to compete for promotion.

In this environment, the firm decides either to keep the authority of project selection at management task, or to delegate the authority to the promoted manager, that is, either centralization or decentralization. We first argue that how the incentive effect and selection effect in the promotion tournament interact with each authority structure. Under decentralization, prize for managerial success in the tournament depends on each worker’s type, which induces them to make less efforts and to decrease the success probability of the project. Under centralization, however, both workers’ effort and the success probability are mainly influenced by perception of the firm, which leads more efforts among workers and constant success proba-
bility of the project. We then shows that both incentive and selection effects make the firm less likely to delegate authority to the manager, even if the heterogeneous workers have better information than the firm’s one.

The paper is related to two strands of literature. One strand is the issue of optimal authority allocation problem influenced by Aghion and Tirole (1997) and Dessein (2002). Aghion and Tirole (2002) suggest that the key trade-off in determining the decentralization lies in the loss of the worker’s information acquisition incentives under the firm’s authority and the loss of the firm’s control under the worker’s authority. They then show that a firm may delegate formal authority to an worker in order to give the worker’s better incentives to acquire information. Dessein (2002) analyses whether the firm should keel the authority and communicate with the worker, or delegate the authority to the worker. He shows that, as long as the worker’s bias is sufficiently small, the firm prefers delegation to communication under some conditions. However, They don’t consider the situation where (heterogeneous) workers are competing in promotion tournament.

Other strand is the issue of tournaments as both incentive and selection devices. Current paper is closest to Schottner and Thiele (2010). They analyze the optimal interaction of several incentive schemes, including promotion tournaments, and provide an explanation as to why individual performance pay is less prevalent than promotion-based incentives.

The remainder of this paper is organized as follows. The model is introduced in Section 2. In Section 3, we derive workers’ effort levels at the production stage given any structure of authority. The equilibrium effort and selection effects given any structure of authority are characterized in Section 4 and Section 5, respectively. Section 6 derives firm’s optimal authority structure. Section 7 concludes.

2 The model

We consider two representative periods in a firm’s business. A risk-neutral firm owns a firm in which two types of tasks need to be conducted: production tasks at stage 1 and management tasks at stage 2.

At the beginning of stage 1, the firm hires two risk-neutral agents to conduct a production task. At the beginning of stage 2, there is a vacant management position, and the manager will be recruited from one of the agents hired in the stage 1. We assume that the prospective manager requires a sufficient level of firm-specific human capital to conduct the corresponding task effectively. Therefore, the manager will be recruited from the internal pool of agents, that is, from the two production agents hired at the beginning of stage 1.
There are two different types of agents in internal labor market, denoted type $t \in \{L, H\}$. They are equally skilled in the production task, but differ in their abilities for the management task. Type $H$ worker can conduct the management task more efficiently than Type $L$ worker. Prior to the contracting stage, neither the firm nor the agents observe the types. It is, however, common knowledge that an worker is of type $H$ with probability $0.5$, and of type $L$ with $0.5$. After accepting the contract offered by the firm, each worker learns his own type and observes the type of his coworker, while the firm never observes their types. This assumption captures the fact that workers, who work closely together, usually possess better information about one another’s talents and ambitions than the firm. For simplicity, all parties’ reservation payoffs are equal to zero.

At stage 1, worker $i, i = H, L$, chooses an effort level $e_i \geq 0$, which are measured by his cost. worker $i$’s promotion probability is then described by

$$P_i(e_i, e_j) = \begin{cases} e_i & \text{if } e_i + e_j \neq 0, \\ \frac{e_i}{e_i + e_j} & \text{if } e_i + e_j = 0. \end{cases}$$

This functional form was employed Tullock (1980) and the ensuing voluminous literature on rent-seeking.

At stage 2, a project from two projects, $X$ and $Y$, will be chosen, and conducted. The project’s payoff depends on the state of the world: there are two equally probable states of the world, denoted by $s \in \{X, Y\}$. Project $x \in \{X, Y\}$ pays $B_P > 0$ to the firm and $B_M$ to the manager if $s = x$, and it pays zero otherwise.

The firm and the agents have different abilities to distinguish which is profitable project. After nature decides which state occurs at the beginning of stage 2, all parties receive imprecise and independent signals about the state that was realized. If state $s$ occurs, type $H$ (resp. type $L$) manager receives signal $s_H = s$ (resp. $s_L = s$) with probability $p_H \in (1/2, 1]$ (resp. $p_L \in (1/2, 1]$), and $s \neq s_H$ (resp. $s \neq s_L$) with remaining probability, where $p_H > p_L$. Similarly, the firm receives $s_P = s$ with probability $q \in (1/2, 1]$, and $s_P \neq s$ with remaining probability. The prior probability of $1/2$, as well as the signal’s perceptions, are common knowledge to both parties.

Besides the state of the world, agents may have a biased preference over the project selection. We assume that irrespective with manager’s type, he prefers project $X$ to project $Y$. We formulate their preference as follows: if project $X$ (resp. $Y$) is conducted, he enjoys private benefit $b > 0$ (resp. $0$).

Agents’ employment contracts are characterized by an authority structure, which is committed by the firm at the contracting stage. We focus on the following authority structure:
1. Centralization: the firm makes the project choice without consulting the manager.
2. Decentralization: the project choice is delegated to the manager with no interference from the firm.

The timing of the model is summarized as follows:
1. The firm commits an authority structure and offers two agents employment contracts.
2. Each worker learns his type and that of coworker, and chooses their respective effort $e_i$.
3. Based on the efforts and the probability structure, promotion occurs, and winner will be promoted to manager.
4. State $s$ occurs, and the firm and the manager receive independent signal.
5. Based on authority structure, authority holder chooses a project $x \in \{X, Y\}$.
6. Project revenue and benefit are generated and paid to all parties.

3 Effort in the Production Stage

3.1 Centralization

Under centralization, the firm can choose project based only on a signal, $s_P$, which she received at the beginning of stage 2. Note that she makes the project choice without consulting the manager. Then, taking the effort of worker $j$ as given, worker $i$ chooses $e_{ic}$ to solve the following optimization:

$$
e_{ic} \in \arg\max_{e'_{ic}} \left\{ P_i(e'_i, e_j)Z_c - e'_{ic} \right\},
$$

where $Z_c = qB_M + b/2$. Solving for $e_{ic}$ yields

$$
e_{ic} = e_{jc} \equiv e_c = \frac{Z_c}{4}.
$$

Note that in equilibrium, $P_i(e_c, e_c) = P_j(e_c, e_c) = 1/2$ because any types of managers’ benefit from promotion are same under centralization. The principal’s expected payoff, denoted by $\pi_c$, is given by

$$
\pi_c = 2e_c^* + qB_P.
$$

3.2 Decentralization

Under decentralization, a promoted manager has right to choose one of projects and conduct it. For type $t \in \{H, L\}$, type-$t$ manager will choose project $x \in \{X, Y\}$ based on $s_t$ if and only if

$$
b \leq B_M(2p_t - 1) \equiv b_t.
$$
Note that \( b_L < b_H \), which implies that each type of manager has different threshold value over project choice. Figure 1 illustrates that three combinations about how each type of manager make the choice.

![Figure 1: Agents’ bias](image)

### 3.2.1 Homogeneous agents

As a benchmark, we start our analysis of the homogeneous agents’ effort choices in the production stage under decentralization. To begin with, We suppose that a homogeneous manager receives signal which is coincident with the state with probability \( p \in (1/2, 1] \). Then the manager will choose project \( x \in \{X, Y\} \) based on signal if \( b \leq BM (2p - 1) = \bar{b} \). Therefore, for agents’ optimization problems, we need to account for two cases of agents’ private benefits: \( b \leq \bar{b} \) (case 1) and \( b > \bar{b} \) (case 2).

**Case 1 \((b \leq \bar{b})\)** Taking the effort of worker \( j \) as given, worker \( i \) chooses \( \bar{e}_{1i} \) to solve the following optimization:

\[
\bar{e}_{1i} \in \arg\max_{\bar{e}_{1i}} \left\{ P_i(\bar{e}'_{1i}, \bar{e}_{1j}) \bar{Z} - \bar{e}'_{1i} \right\},
\] (5)

where \( \bar{Z}_1 = pBM + b/2 \). Solving for \( \bar{e}_{1i} \) yields

\[
\bar{e}_{1i} = \bar{e}_{1j} = \bar{e}_1 = \frac{\bar{Z}_1}{4}.
\] (6)

Note that in equilibrium, \( P_i(\bar{e}_1, \bar{e}_1) = P_j(\bar{e}_1, \bar{e}_1) = 1/2 \). The principal’s expected payoff, denoted by \( \pi_1 \), is given by

\[
\pi_1 = 2\bar{e}_1 + pB_P.
\] (7)
Case 2 ($b > \bar{b}$) Taking the effort of worker $j$ as given, worker $i$ chooses $e_{2i}$ to solve the following optimization:

$$e_{2i} \in \arg\max_{e_{2i}} \{P_i(e_{2i}', e_{2j})Z_2 - e_{2i}'\},$$

(8)

where $Z_2 = B_M/2 + b$. Solving for $e_{2i}$ yields

$$e_{2i} = e_{2j} \equiv e_2 = \frac{Z_2}{4}.$$  

(9)

Note that in equilibrium, $P_i(e_i, e_j) = P_j(e_i, e_j) = 1/2$. The principal’s expected payoff, denoted by $\pi_2$, is given by

$$\pi_2 = 2e_3 + \frac{1}{2}BP.$$  

(10)

Together with the case of centralization, We now compare two schemes under homogeneous agents and determine which scheme yields the highest expected payoff.

Lemma 1: Suppose that agents are homogeneous.

(i) Suppose that $b \geq \bar{b}$. Then if $q \leq p$, it is optimal for the firm to choose centralization. Otherwise, decentralization is optimal for her.

(ii) Suppose that $b > \bar{b}$. Then there exists a threshold

$$\overline{q} = \frac{b}{2(B_M + 2BP)} + \frac{1}{2},$$

such that it is optimal for the firm to choose centralization for $q \geq \overline{q}$. Otherwise, decentralization is optimal for her.

Proof of Lemma 1: (i) It follows from the above that

$$\pi_c - \pi_1 = \frac{1}{2}(B_M + 2BP)(q - p),$$

which implies that $\pi_c - \pi_1 \geq 0$ (resp. $\pi_c - \pi_1 < 0$) for $q \geq p$ (resp. $q < p$). ■

(ii) It follows from the above that

$$\pi_c - \pi_2 = \frac{1}{4}\{(2q - 1)(B_M + 2BP) - b\},$$

which implies that $\pi_c - \pi_2 \geq 0$ (resp. $\pi_c - \pi_2 < 0$) for $q \geq \overline{q}$ (resp. $q < \overline{q}$). ■

It is clear that under case 1, if the firm is better informed than the homogeneous worker, it is optimal for her to choose centralization; otherwise decentralization is optimal. On the other hand, under case 2, the promoted manager under decentralization will choose project $X$ irrespective of his posterior belief, while the firm under centralization will choose a project based on its own belief. Therefore, optimal authority scheme is determined by incentive effect, which means that a scheme which induces more efforts is optimal.
3.2.2 Heterogeneous agents

We now move on the heterogeneous agents’ effort choice in the production stage under decentralization. For \( t \in \{H, L\} \), type-\( t \) manager will choose project \( x \in \{X, Y\} \) based on signal if \( b \leq B_M(2p_t - 1) \equiv \bar{b}_t \). Therefore, for agents’ optimization problems, we need to account for three cases of agents’ private benefits.

**Case 1** \((b \leq b_L)\) in case 1, type-\( t \) manager prefers to choose project \( x = s_t \) if \( s_t \) is realized, which means that the agents’ ex post preference is congruent with that of the firm.

Type \( H \)'s and type \( L \)'s respective optimization problems are given by

\[
\begin{align*}
    e_H & \in \arg\max_{e_H} \{ P_H(e_H', e_L)Z_H - e_H' \} \\
    e_L & \in \arg\max_{e_L} \{ P_L(e_H, e_L')Z_L - e_L' \},
\end{align*}
\]

(11)  

(12)

where \( Z_H = p_H B_M + b/2 \) and \( Z_L = p_L B_M + b/2 \). The first-order conditions of (11) and (12)
implies that \( e_H = (Z_H e_L)/Z_L \). Substituting this into (11) and (12), we obtain

\[
e^*_H = \frac{Z^2_H Z_L}{(Z_H + Z_L)^2},
\]

(13)

\[
e^*_L = \frac{Z_H Z^2_L}{(Z_H + Z_L)^2}.
\]

(14)

It is straightforward to verify that \( e^*_H > e^*_L \) because \( p_H > p_L \). Let denotes \( \Delta_1 = e^*_H - e^*_L \). The principal’s expected payoff, denoted by \( \pi_1 \), is given by

\[
\pi_1 = e^*_H + e^*_L + \left\{ P_H(e^*_H, e^*_L) p_H + P_L(e^*_H, e^*_L) p_L \right\} B_P.
\]

(15)

\[
\pi_1 = \frac{Z_H Z_L}{Z_H + Z_L} + \frac{p_H Z_H + p_L Z_L}{Z_H + Z_L} B_P.
\]

Case 2 (\( b_L < b \leq b_H \)) in case 2, while type-\( H \) manager prefers to choose project \( x = s_t \) if \( s_t \) is realized, type-\( L \) manager will choose project \( X \) irrespective of his signal, \( s_L \). Type \( H \)’s and type \( L \)’s respective optimization problems are given by

\[
e_{HS} \in \arg \max_{e'_{HS}} \left\{ P_H(e'_{HS}, e_{LY}) Z_H - e'_{HS} \right\}
\]

(16)

\[
e_{LX} \in \arg \max_{e'_{LX}} \left\{ P_L(e_{HS}, e'_{LX}) Z_X - e'_{LX} \right\},
\]

(17)

where \( Z_X = B_M/2 + b \). The first-order conditions of (16) and (17) implies that \( e_{HS} = (Z_H e_{LX})/Z_X \). Substituting this into (16) and (17), we obtain

\[
e^*_{HS} = \frac{Z^2_H Z_X}{(Z_H + Z_X)^2},
\]

(18)

\[
e^*_{LX} = \frac{Z_H Z^2_X}{(Z_H + Z_X)^2}.
\]

(19)

Let denotes \( \Delta_2 = e^*_H - e^*_L \).

The principal’s expected payoff, denoted by \( \pi_2 \), is given by

\[
\pi_2 = e^*_H + e^*_L + P_H(e^*_H, e^*_L) p_H B_P + \frac{1}{2} P_L(e^*_H, e^*_L) B_P.
\]

(20)

Case 3 (\( b_H < b \)). in case 3, Both types of manager no longer choose project \( x = s_t \), but project \( X \). Their optimization problems are given by

\[
e_{iX} \in \arg \max_{e'_{iX}} \left\{ P_i(e'_{iX}, e_{jX}) Z_X - e'_{iX} \right\},
\]

(21)

We then obtain

\[
e^*_{iX} = e^*_{jX} = e^*_X = \frac{r Z_X}{4}.
\]

(22)

\footnote{The equilibrium efforts follow NTI (1999).}
4 Effort effects

Before preceding Proposition 1, we compare the induced efforts under both authority schemes. Intuitively, for sufficient large posterior belief of the firm, $q_c^*$ is larger than the induced efforts under decentralization. However, it should be noted that even if her belief is smaller than the manager’s one, $e_c^*$ can be larger than the effort under decentralization.

Lemma 2: Suppose that agents are heterogeneous, and that the firm chooses decentralization. (i) If $b \leq b_L$, then (1) for any given $p_H \leq 1$, there exist values for $q$ such that $q < p_H$ and $e_c > e_H$; (2) for any given $p_L \leq 1$, there exist values for $q$ such that $q < p_L$ and $e_c > e_L$. (ii) If $b_L < b \leq b_H$, then for any given $p_H \leq 1$, there exist values for $q$ such that $q < p_H$ and $e_c > e_{HS}$.

Proof: See Appendix.

Under centralization, since the prize for the success depends on the firm’s signal, workers face same situation where they are homogeneous, which in turn induce them to make more effort. Under decentralization, however, prize for managerial success depends on each worker’s type, which induce them to make less efforts. Consequently, increasing effort is less beneficial for both types of workers under the heterogeneous tournament.

We next move on Proposition 1. Proposition 1 shows that optimal relationship between the induced efforts in any authority schemes and the private benefit.

Proposition 1: (i) The induced effort levels in equilibrium are increasing in $b$. (ii) If $b \leq b_L$, the induced effort difference $\Delta e_1^*$ is increasing in $b$. (iii) If $b_L < b \leq b_H$, the induced effort difference $\Delta e_2^*$ is decreasing in $b$.

Proof: See Appendix.

Figure 3 illustrates Proposition 1 in the $(b, e)$ space for given $b = b_L, b_H$. In the figure, the solid line represents the induced efforts under decentralization and dotted line represents the induced efforts under centralization. Intuitively, an increase in the private benefit ($b$) raises the prize ($Z_i$) for management task, and consequently induces more effort of both players in manufacturing task. It should be noted that when $b \leq b_L$, an increase in the prize of type $H$ worker harms the effort of type $L$ worker in the heterogeneous tournament, which leads to $\Delta e_1^* > 0^2$. Moreover, when $b_L < b \leq b_H$, an increase in $b$ consequently induces the effort of

\footnote{NTI’s (1999) Proposition 4 shows similar effects as our Proposition 1 that competition becomes more intense as the valuation of the underdog increases, and competition becomes less intense as the valuation of the favored player increases.}
type L worker more, which leads to $\Delta e^*_2 < 0$.

![Figure 3: Effort comparison ($q = p_H, p_L$)](image)

5 Selection effects

We next turn to optimal selection effects in each authority schemes. To this end, we define accuracy of selection as the success probability of a project. Let $S_y$ denote the accuracy of selection in case $y \in \{1, 2, 3\}$ under decentralization. We note that under centralization, the accuracy of selection is $q$. Each case's accuracy of selection is given by

$$S_1 \equiv P_H(e^*_H, e^*_L)p_H + P_L(e^*_H, e^*_L)p_L = \frac{(p^2_H + p^2_L)B_M + (p_H + p_L)b/2}{b + (p_H + p_L)B_M}, \quad (23)$$

$$S_2 \equiv P_H(e^*_HS, e^*_LX)p_H + P_L(e^*_HS, e^*_LX)\frac{1}{2} = \frac{(2p^2_H + 1/2)B_M + (1 + p_H)b}{3b + (2p_H + 1)B_M}, \quad (24)$$

$$S_3 \equiv P_H(e^*_X, e^*_X)\frac{1}{2} + P_L(e^*_HS, e^*_LX)\frac{1}{2} = \frac{1}{2}. \quad (25)$$

We thus obtain the following proposition.

**Proposition 2:** (i) Under centralization, the accuracy of selection is constant: $q$. (ii) Under decentralization, accuracy of selection is strictly increasing in $p_H$, $p_L$, and $B_M$, and (weakly) decreasing in $b$. 

10
Proof: See Appendix.

\[ q = p_H \]

Figure 4: Selection comparison \((q = p_H)\)

Figure 4 illustrates Proposition 2 for given \(q = q_H\). In the figure, the solid line represents the accuracy of selection under decentralization and dotted line represents the one under centralization. Proposition 2 says that while under centralization, the selection effect is only influenced by perception of the firm and thus constant with increase of private benefit, under decentralization an increase of private benefit interfere with the selection effect of promotion tournament.

## 6 Optimal policy

Proposition 1 and 2 provided how the incentive effect and selection effect in the promotion tournament interact with each authority structure. We finally obtain the optimal structure of authority as follows.

**Proposition 3:** (i) Suppose that \(b \leq b_L\). Then if

\[
q \leq q_1 = \frac{(p_H + p_L)(B_M + 2B_P)b + 4B_M(pHpLB_M + (p_H^2 + p_L^2)B_P)}{2(B_M + 2B_P)b + 2B_M(p_H + p_L)(B_M + 2B_P)},
\]

it is optimal for the firm to choose decentralization; otherwise, centralization is optimal for her. Moreover, \(q_1\) is increasing in \(p_H\), \(p_L\), and \(B_P\). In particular, \(q_1\) is increasing (resp. decreasing)
in b if and only if $B_M > 2B_P$ (resp. $B_M < 2B_P$).

(ii) Suppose that $b_L < b \leq b_H$. Then if

$$q \leq \bar{q}_2 = \frac{b^2 + \{(1 + 6p_H)B_M + 4(1 + p_H)B_P\}b + 2B_M\{2p_HB_M + (1 + 4p_H^2)B_P\}}{2(B_M + 2B_P)\{(2p_H + 1)B_M + 3b\}}, \quad (27)$$

it is optimal for the firm to choose decentralization; otherwise, centralization is optimal for her.

(iii) Suppose that $b_H < b$. Then if

$$q \leq \bar{q}_3 = \frac{b}{2(B_M + 2B_P)} + \frac{1}{2}, \quad (28)$$

it is optimal for the firm to choose decentralization; otherwise, centralization is optimal for her.

Figure 5: Optimal policy $(p_L \in \left(\frac{6p_H + 1}{8}, p_H\right), B_M \in (2B_P, \frac{B_P(4p_L - 2p_H - 1)}{2(p_H - p_L)})$)

Figure 5 illustrates an example of Proposition 3 for given $p_L \in \left(\frac{6p_H + 1}{8}, p_H\right), B_M \in (2B_P, \frac{B_P(4p_L - 2p_H - 1)}{2(p_H - p_L)})$. In the figure, the black line represents optimal thresholds when workers are heterogeneous and gray line represents the threshold followed by Lemma 1, that is, when workers are homogeneous. Proposition 3 shows that when workers are heterogeneous, the optimal authority structure is not simple as case of homogeneous workers, and that in particular, both incentive and selection effects make the firm less likely to delegate authority to agents, even if the agents have better information.
7 Conclusion

We explore the interaction between the structure of authority and the promotion incentives. The focus is placed on the different effects between incentive and selection under each authority schemes. We shows that both incentive and selection effects make the firm less likely to delegate authority to the manager, even if the heterogeneous workers have better information than the firm’s one.

References


8 Appendix

Proof of Lemma 2: (i) for $t, -t \in \{H, L\}$ and $t \neq -t$, suppose that $q = p_t - \epsilon$, where $\epsilon > 0$. Then, we have

$$e_c - \epsilon_t^* = \frac{1}{4(Z_t + Z_{-t})^2} \{Z_c(Z_t + Z_{-t})^2 - 4Z_t^2Z_{-t}\}$$

$$= \frac{1}{4(Z_t + Z_{-t})^2} \{Z_t(Z_t - Z_{-t})^2 - \epsilon B_M(Z_t + Z_{-t})^2\}.$$ 

$$= \frac{1}{4(Z_t + Z_{-t})^2} \{B_MZ_t(p_t - p_{-t})^2 - \epsilon(Z_t + Z_{-t})^2\}.$$
Note that \( Z_c = Z_t - \epsilon B_M \). Hence, we conclude that \( e_c > e_t^* \) if
\[
\frac{B_M(b + 2p_B M)(p_t - p_{-t})^2}{2(b + B_M(p_t + p_{-t}))^2} > \epsilon. \quad \Box
\]

(ii) suppose that \( q = p_H - \epsilon, \) where \( \epsilon > 0. \) Moreover, without loss of generality, we assume that \( r = 1. \) Then, we have
\[
e_c - e_{HS}^* = \frac{1}{4(Z_H + Z_X)^2}\{Z_c(Z_H + Z_X)^2 - 4Z_H^2Z_X\}
= \frac{1}{4(Z_H + Z_X)^2}\{Z_H(Z_H - Z_X)^2 - \epsilon B_M(Z_H + Z_X)^2\}
= \frac{1}{4(Z_H + Z_X)^2}\{Z_H(Z_H - Z_X)^2 - \epsilon(Z_H + Z_X)^2\}.
\]

Note that \( Z_c = Z_H - \epsilon B_M. \) Hence, we conclude that \( e_c > e_{HS}^* \) if
\[
\frac{Z_H(Z_H - Z_X)^2}{(Z_H + Z_X)^2} > \epsilon. \quad \Box
\]

**Proof of Proposition 1:** In equilibrium, the induced effort differences are given by
\[
\Delta e_1^* = \frac{B_M(b + 2p_B M)(b + 2p_B M)(p_H - p_L)}{4\{b + B_M(p_H + p_L)\}^2},
\Delta e_2^* = \frac{(2b + B_M)(b + 2p_B M)\{B_M(2p_H - 1) - b\}}{2\{3b + (2p_H + 1)B_M\}^2}.
\]

Note that when \( b = b_H, \) \( \Delta e_2^* = 0. \) Then we have the following:
\[
\frac{\partial \Delta e_1^*}{\partial b} = \frac{B_M^2(p_H - p_L)^3}{2(b + B_M(p_H + p_L)^3)} > 0, \quad (29)
\frac{\partial \Delta e_2^*}{\partial b} = -\frac{6b^3 + 6B_M(1 + 2p_H)b^2 + 3B_M^2(1 + 8p_H^2)b}{2\{3b + M_B + 2p_H B_M\}^3} + \frac{B_M(-1 + 6p_H - 24p_H^2 + 16p_H^3)}{2\{3b + M_B + 2p_H B_M\}^3} < 0. \quad (30)
\]

**Proof of Proposition 2:**
\[
\frac{\partial S_1}{\partial b} = -\frac{B_M(p_H - p_L)^2}{2\{b + B_M(p_H + p_L)\}^2} < 0,
\frac{\partial S_2}{\partial b} = -\frac{B_M(4p_H - 1)(2p_H - 1)}{2\{3b + B_M(2p_H + 1)\}^2} < 0.
\]
\[
\frac{\partial S_1}{\partial p_H} = \frac{b^2 + 4p_B pM b + 2B_M^2(p_H^2 - p_L^2 + 2p_H p_L)}{2\{b + B_M(p_H + p_L)\}^2} > 0,
\]
\[
\frac{\partial S_1}{\partial p_L} = \frac{b^2 + 4p_L B_M b + 2B_M^2(p_L^2 - p_H^2 + 2p_H p_L)}{2\{b + B_M(p_H + p_L)\}^2} > 0.
\]

\[
\frac{\partial S_1}{dB_M} = \frac{b(p_H - p_L)^2}{2\{b + B_M(p_H + p_L)\}^2} > 0,
\]
\[
\frac{\partial S_2}{dB_M} = \frac{b(4p_H - 1)(2p_H - 1)}{2\{3b + B_M(2p_H + 1)\}^2} > 0.
\]

\[
\frac{\partial S_2}{dp_H} = \frac{3b^2 + (12p_H - 1)B_M b + B_M^2(4p_H^2 + 4p_H - 1)}{(3b + B_M + 2p_H B_M)^2} > 0.
\]

When \(b = b_L\), we have
\[
S_1 - S_2 = \frac{(4p_L - 1)(2p_L - 1)}{4(p_H + 3p_L - 1)} > 0.
\]

Similarly, when \(b = b_H\), we have
\[
S_2 - S_3 = \frac{(4p_L - 1)(2p_L - 1)}{4(4p_H - 1)} > 0.
\]