Fair Transfers between Generations: Some (Im)Possibility Results

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Abstract: In a productive economy, there is a generic conflict of interest between generations: On the one hand, there should be savings to achieve economic progress and at least to avoid non-sustainability, i.e. a decline of well-being over time. On the other hand, the burden that is inflicted on earlier generations through their savings should not become too high. Against this background, the question arises whether a unique social welfare ordering is capable of achieving a fair distribution of well-being between generations in any case, i.e. for any given technology. In a simple two-generation model we show that the answer to this question is ambiguous depending on the underlying normative objective. While social welfare orderings can be found that ensure sustainability and avoid excessive thus exhibiting flexibility in the sense of Asheim (2017), the inequality of well-being between generations cannot be kept limited by as single social welfare ordering for any given technology.

Keywords: Intergenerational equity, sustainability, utilitarianism, sufficientarianism.

JEL Classification: D63, D64, D71.
1. **Motivation**

With regard to equity between generations, there is a double-sided fairness problem, which is reflected by two conflicting normative postulates.

**Postulate 1**: There should be human development (“growth of well-being”): Later generations should have a higher welfare than the earlier ones. This requires investments from the earlier generations, which in this way are making transfers to the later ones.

The *minimum condition* for satisfying Postulate 1 is that later generations should not be worse off than the earlier ones, which leads to the *sustainability postulate* as a pre-condition for economic progress: “(T)he no-gift baseline is pure sustainability” (Llavador, Roemer and Silvestre, 2015, p. 38, and in a similar vein Cowen, 2018).

**Postulate 2**: Earlier generations should not be overburdened through their transfers to the later generations, which in the literature is known as the *excessive savings problem* (e.g. Arrow, 1999, or Dasgupta, 2008).

These two postulates (only) become meaningful when the technology that is available for the economy under consideration is *productive*, which means that savings of an earlier generation increase welfare of the later generation by a higher amount so that, literally speaking, there is an “incubation bucket” for the intergenerational transfers (Schelling, 1995). A sustainable development then trivially is possible. The productivity condition usually is assumed to hold in reality – with “doomsday prophets” as the exception.

Against the background of this twofold fairness problem, it has to be asked from the perspective of an “Ethical Observer” (as in Llavador, Roemer and Silvestre, 2015), i.e. from the viewpoint of ethical social choice theory, which intertemporal evaluation criteria are able to simultaneously meet the two postulates in line with ethical intuition.\(^2\) This means that it has to be examines which evaluation criteria lead to a distribution of well-being between generations that is skewed in favour of the future generations but not skewed too much: Transfers from the earlier to the later generations should be made so that the potential welfare gains from productivity are not wasted. Yet at the same time these transfers should not become too

\(^2\) The problem of finding an ethically appealing balance of interests between generations satisfying the two postulates listed above was already addressed by the philosophers Kant (1784) and Rawls (1972).
high in order to protect the interests of the earlier generations and to prevent their overburdening. In the following it will be analysed whether evaluation criteria exist that satisfy these two postulates for any productive technology thus avoiding that social preferences have to be adapted to the specifically given technology in order to attain an ethically sensible balance of interest between generations. In particular, it will be shown

- that it is easy to find criteria that avoid violation of Postulate 1 for any productive technology, i.e. to exclude a non-sustainable development.
- that it is only possible to a limited extent to find such criteria that satisfy Postulate 2.

Our arguments can be presented in an elementary two-period model similar as in Buchholz (2003), Llavador, Roemer and Silvestre (2015, pp. 44 – 47) and Roemer (2019). Problems of evaluation of utility streams over an infinite time horizon that since Koopmans (1960) usually play a central role in the context of intertemporal evaluation are of not much relevance here and thus are neglected.

2. The Framework

There are two generations 1 and 2 of equal size. Well-being (= consumption in a comprehensive sense including e.g. environmental benefits, health, ...) of generation 1 is denoted by $c_1$ while that of generation 2 is $c_2$. A decreasing and weakly concave (and differentiable) function $c_2 = T(c_1)$ describes the transformation curve between the consumption levels in the two periods, i.e. the given technology. The transformation curve $T(c_1)$, which is defined on an interval $[0, \bar{c}_1)$ with $\bar{c}_1 < \infty$ and has $T(0) = 0$ and $\bar{c}_2 := T(0)$ describes all combinations of consumption levels $(c_1, c_2)$ that are feasible and Pareto optimal for the given technology (see Figure 1). The technology is productive if $-T'(c_1) > 1$ holds for all $c_1 \in [0, \bar{c}_1]$ so that any consumption sacrifice (= savings) by generation 1 leads to a higher increase of consumption for generation 2.

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3 “Consumption” $c_i$ could (as, e.g. in Adler, 2012, or Adler and Treich, 2015) be identified with cardinally measurable utility $v(\xi_i)$ where $\xi_i$ is the “outcome” for agent/generation $i$. 
Consumption paths are evaluated by an intertemporal Social Welfare Relation $SWR$, i.e. a complete, reflexive, transitive and symmetric binary relation, which indicate whether one consumption path is deemed at least as good than another one. By $P$ we denote its asymmetric part, i.e. strong preference, and by $\sim$ its symmetric part, which indicates equivalence. The SWR $R$ is assumed to have the following three properties

- **Strong Pareto SP**: Given $(c_1', c_2')$ and $(c_1'', c_2'')$ with $c_1'' \geq c_1'$ and $c_2'' \geq c_2'$ and $c_1'' > c_1'$ or $c_2'' > c_2'$. Then $(c_1'', c_2'') P (c_1', c_2')$.

- **Continuity CON**: Let $(c_1^{(n)}, c_2^{(n)})_{n\in\mathbb{N}}$ be a sequence of consumption paths that converges to some $(c_1', c_2')$. If for some $(c_1'', c_2'')$ we have $(c_1'', c_2'') R (c_1^{(n)}, c_2^{(n)})$ for all $n \in \mathbb{N}$ then $(c_1'', c_2'') R (c_1', c_2')$.

- **(Strict) Quasi-Concavity QC**: Given any $(c_1', c_2')$ and $(c_1'', c_2'')$ with $(c_1'', c_2'') R (c_1', c_2')$ then $(\lambda c_1' + (1-\lambda)c_1'', \lambda c_2' + (1-\lambda)c_2'') P (c_1', c_2')$ holds for all $\lambda \in [0,1]$. 

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**Figure 1**

- $c_2$
- $c_1$
- $T(.)$

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When these assumptions are given then for any transformation curve $T(.)$ there exists a unique $R$-optimal consumption path $(c^*_1, c^*_2)$, which means that $(c^*_1, c^*_2) \ P (c_1, c_2)$ holds for all $c_2 = T(c_1)$.

3. Preventing Unsustainability

A SWR $R$ does not violate Postulate 1 if the $R$-optimal path is sustainable (= non-decreasing), i.e. $c^*_2 \geq c^*_1$ holds for any productive technology. This is ensured if $R$ is (weakly) friendly towards the future FTF. This means that for any consumption profile $(c_1, c_2)$ with $c_1 \geq c_2$ we have $(c_2, c_1) R (c_1, c_2)$ so that society does not become worse off if the consumption profile is permuted so that the later generation benefits from the higher consumption level.

**Proposition 1**: Given FTF then for any productive technology the $R$-optimal consumption path $(c^*_1, c^*_2)$ is sustainable.

**Proof**: Assume that some $R$-optimal path $(c^*_1, c^*_2)$ on the transformation curve is non-sustainable, i.e. $c^*_1 > c^*_2$, so that $(c^*_2, c^*_1) R (c^*_1, c^*_2)$ holds by FTF. Observing $c^*_2 = T(c^*_1)$ we get

$$1) \quad c^*_1 = c^*_2 + \int_{c_2^*}^{c_1^*} dc < c^*_2 + \int_{c_2^*}^{c_1^*} (-T'(c))dc = T(c^*_1) + (T(c^*_2) - T(c^*_1)) = T(c^*_2)$$

SP implies $(c^*_2, T(c^*_1)) \ P (c^*_2, c^*_1) R (c^*_1, c^*_2)$. Hence, $(c^*_1, c^*_2)$ cannot be $R$-optimal, which is a contradiction. QED

In the $c_1$-$c_2$-diagram (see Figure 2) inequality (1) means that the permuted consumption profile $(c^*_2, c^*_1)$ lies below the transformation curve $T(.)$. 
FTF is ensured in the important case when the SWR $R$ has the anonymity $AN$ property so that generations are treated equally by $R$, which seems to be morally required to attain a fair non-discriminatory treatment of generations.\textsuperscript{4} Formally, $AN$ means that $(c_1, c_2) \sim (c_2, c_1)$ holds for all $(c_1, c_2)$.\textsuperscript{5}

$AN$ (and $SP$, $CON$ and $QC$) holds for undiscounted utilitarian criteria $R$ which, for a given strictly monotone increasing, differentiable and concave utility function $u(c)$, rank consumption paths $(c'_1, c'_2)$ and $(c''_1, c''_2)$ as follows:

$$(c''_1, c''_2) \sim (c'_1, c'_2) \text{ if and only if } u(c''_1) + u(c''_2) \geq u(c'_1) + u(c'_2).$$

A SWR that is weakly future friendly could in an analogous way given by a social welfare function $SWF u(c_1) + \delta u(c_2)$ where $\delta \geq 1$ represents an ‘upgrading factor’ for the future generation’s utility.

\textsuperscript{4} Impartiality towards different generation is not only postulated by philosophers but also by economists. See, e.g., Stern (2007), Roemer (2011, 2019) and Adler and Treich (2015) as recent prominent examples.

\textsuperscript{5} See Asheim, Buchholz and Tungodden (2001) for a generalization of Proposition 1 under the $AN$ assumption.
4. Ensuring Sustainability through Amending a Given SWR

Starting from an arbitrary SWR $R$ AN (and thus FTF) can be attained by constructing new “hybrid” SWRs. As a first approach let for any consumption profile $(c_1, c_2)$ be $\bar{c}_1 := \min\{c_1, c_2\}$ and $\bar{c}_2 := \max\{c_1, c_2\}$. Then define a ranking between consumption profiles $(c_1', c_2')$ and $(c_1'', c_2'')$ as follows:

$$(c_1'', c_2'') \, \bar{R} \, (c_1', c_2') \text{ if and only if } (c_1'', c_2'') \, R \, (c_1', c_2').$$

This construction means that consumption paths $(c_1, c_2)$ with $c_2 < c_1$ are permuted before $R$ is applied (see Zuber and Asheim, 2012) so that only the ranking given by $R$ in the upper region $\{(c_1, c_2) : c_2 \geq c_1\}$ is relevant. In a $c_1$-$c_2$-diagram (see Figure 3) the entire indifference curves for $\bar{R}$ then are obtained by mirroring the upper part of the $R$-indifference curves, which lies above the 45°-line, on this line.

![Figure 3](image-url)
A second approach for getting FTF starting from some given SWR $R$, which is analogous to Asheim and Mitra (2010), is based on a partial inclusion of the maximin-criterion in the following way. For this construction we start with a binary relation $\tilde{R}$ on subsets of $\mathbb{R}_+^2 \times \mathbb{R}_+^2$:

- For consumption paths $(c_1', c_2')$ and $(c_1'', c_2'')$ in $\{(c_1, c_2) : c_2 \geq c_1\}$ we have
  $$(c_1'', c_2'') \tilde{R} (c_1', c_2') \text{ if and only if } (c_1'', c_2'') R (c_1', c_2').$$

- For consumption paths $(c_1', c_2')$ and $(c_1'', c_2'')$ in $\{(c_1, c_2) : c_2 \leq c_1\}$ we have
  $$(c_1'', c_2'') \tilde{R} (c_1', c_2') \text{ if and only if } c_2'' \geq c_2'.$$

This binary relation can be completed to a SWR straightforwardly: For two consumption paths $(c_1', c_2') \in \{(c_1, c_2) : c_2 \leq c_1\}$ and $(c_1'', c_2'') \in \{(c_1, c_2) : c_2 \geq c_1\}$ we let $(c_1'', c_2'') \tilde{R} (c_1', c_2')$ if and only if $(c_1'', c_2'') R (c_1', c_2')$. The properties required for a SWR are easily established for $\tilde{R}$.

The indifference curves for $\tilde{R}$ (see Figure 4) coincide with those of $R$ above the $45^\circ$-line and are flat straight lines below the $45^\circ$-line. For $\tilde{R}$ the property SP therefore does not hold below the $45^\circ$-line.

**Figure 4**
On the given transformation curve $T(\cdot)$ now define $\hat{c}$ as the constant consumption level that is technically feasible for both generations, i.e. $\hat{c} = T(\hat{\epsilon})$. The $\bar{R}$- and $\hat{R}$-optimal consumption paths then can be characterized as follows:

**Proposition 2**: (i) If the $R$-optimal consumption path $(c_1^{R*}, c_2^{R*})$ is sustainable then it is also $\bar{R}$- and $\hat{R}$-optimal.

(ii) If the $R$-optimal consumption path $(c_1^{R*}, c_2^{R*})$ is non-sustainable then $(\hat{c}, \hat{\epsilon})$ is $\bar{R}$- and $\hat{R}$-optimal.

**Proof**: We provide a proof for $\hat{R}$. For $\bar{R}$ the proof is analogous.

(i) $(c_1^{R*}, c_2^{R*}) P (c_1, T(c_1))$ holds for all $c_i \leq \hat{c}$ since $\bar{R}$ coincides with $R$ in this region. Now assume that there is some $\tilde{c}_1 > \hat{c}$ with $(\tilde{c}_1, T(\tilde{c}_1)) \bar{R} (c_1^{R*}, c_2^{R*})$ and thus, by definition of $\bar{R}$, $(T(\tilde{c}_1), \tilde{c}_1) R (c_1^{R*}, c_2^{R*})$. Productivity of the technology implies that $(T(\tilde{c}_1), \tilde{c}_1)$ lies below $T(\cdot)$. Then there exists some consumption level $\tilde{c}_1 < \hat{c}$ for which $(\tilde{c}_1, T(\tilde{c}_1)) P (T(\tilde{c}_1), \tilde{c}_1) R (c_1^{R*}, c_2^{R*})$ holds, which is a contradiction.

(ii) Step 1: If $c_2^{R*} < c_1^{R*}$ then $(\hat{c}, \hat{\epsilon}) R (c_1, T(c_1))$ holds for all $c_i < \hat{c}$. Otherwise, if there were some $\tilde{c}_1 < \hat{c}$ with $(\tilde{c}_1, T(\tilde{c}_1)) P (\hat{c}, \hat{\epsilon})$ QC would imply that for the point $(\tilde{c}_1, \tilde{\epsilon})$ at which the straight line connecting $(c_1^{R*}, c_2^{R*})$ and $(\tilde{c}_1, T(\tilde{c}_1))$ intersects the $45^0$-line we would have $(\tilde{c}_1, \tilde{\epsilon}) P (c_1^{R*}, c_2^{R*})$ (see Figure 5).
Concavity of $T(.)$ gives $c < \hat{c}$ so that $(\hat{c}, \hat{c}) P (c, c) P (c_1^{R^*}, c_2^{R^*})$ by SP. This, however, is a contradiction to the $R$-optimality of $(c_1^{R^*}, c_2^{R^*})$.

**Step 2:** $(\hat{c}, \hat{c}) P (c_1^{R^*}, c_2^{R^*})$ holds since otherwise we had $(c_2^{R^*}, c_1^{R^*}) R (\hat{c}, \hat{c})$ by definition of $R$.

Since productivity of the technology implies that $(c_2^{R^*}, c_1^{R^*})$ lies below the transformation curve $T(.)$ then there would exist some consumption level $\tilde{c}_1 < \hat{c}$ with $(\tilde{c}_1, T(\tilde{c}_1)) P (\hat{c}, \hat{c})$, which contradicts the result obtained in Step 1.

**Step 3:** Combining Step 1 and Step 2 gives the assertion (ii). QED

Ensuring sustainability through such amendments is of special importance when the basic criterion $R$ is given by a discounted utilitarian SWF of the type $u(c_1) + \delta u(c_2)$ with $\delta < 1$. Note in this context that for any given productive technology a non-sustainable development, i.e. $c_2^{R^*} < c_1^{R^*}$, would result as the social optimum if the discount factor $\delta$ becomes small enough.
5. Preventing an Unequal Distribution between Generations: An Impossibility Result

The reasoning so far has shown: Avoiding violation of Postulate 1 can be guaranteed by a fixed SWR $R$ for any given productive technology, which means that $R$ exhibits flexibility in the sense of Asheim (2017). We now ask whether such flexibility of a SWR can also exist w.r.t. Postulate 2, i.e. in particular when “the Ethical observer (...) (is, WB.) interested in reducing intergenerational inequality” (Llavador, Roemer and Silvestre, 2015, p. 47). The question thus is whether an uneven distribution of well-being be avoided for any given productive technology by a unique and fixed SWR $R$. The answer to this question is ambiguous depending on the assumed properties of the SWR and the exact specification of the meaning of “uneven”.

We start with a negative result:

**Proposition 3:** If $R$ satisfies SP, CON, and QC then we can always find a productive technology $T(.)$ so that for the $R$-optimal consumption path $(\hat{c}_1^*, \hat{c}_2^*)$ the ratio $\rho^* = \frac{\hat{c}_2^*}{\hat{c}_1^*}$, i.e. the degree of inequality of consumption between the two generations, becomes arbitrarily large.

**Proof:** Take any $\bar{\rho} > 0$ and any consumption levels $\tilde{c}_1$ and $\tilde{c}_2$ with $\bar{\rho} = \frac{\tilde{c}_1}{\tilde{c}_2}$ and choose some $\tilde{c}_2' > \tilde{c}_2$. Then there is a $\tilde{c}_1' < \tilde{c}_1$ so that $(\tilde{c}_1', \tilde{c}_2') P (\tilde{c}_1, \tilde{c}_2)$. Otherwise, there would exist a sequence $c_1^{(n)}$ of consumption levels with $\lim_{n \to \infty} c_1^{(n)} = \tilde{c}_1$ and $(\tilde{c}_1, \tilde{c}_2) R (c_1^{(n)}, \tilde{c}_2')$ for all $n \in \mathbb{N}$. CON then would yield $(\tilde{c}_1, \tilde{c}_2) R (\tilde{c}_1', \tilde{c}_2')$ and thus a contradiction to SP. Now define a linear technology $T(c_1) := m\tilde{c}_1 + \tilde{c}_2 - m\tilde{c}_1'$ with the slope $m = \frac{\hat{c}_2' - \hat{c}_2}{\hat{c}_1' - \hat{c}_1}$ and $\tilde{c}_1 = \tilde{c}_1 - \frac{\hat{c}_2 - \hat{c}_2'}{\hat{c}_1 - \hat{c}_1'}$ as generation 1’s maximum consumption, i.e. its “income” in standard microeconomic household theory (see Figure 6).
The \( R \)-optimal solution \((c_1^*, c_2^*)\) lies on \( T(.) \) left to \((\tilde{c}_1, \tilde{c}_2)\) so that \( \rho^* = \frac{\tilde{c}_2}{\tilde{c}_1} > \frac{\hat{c}_2}{\hat{c}_1} = \hat{\rho}. \) (That \((c_1^*, c_2^*)\) lies right to \((\tilde{c}_1, \tilde{c}_2)\) is impossible: If \((c_1^*, c_2^*)\) \( P \ (\tilde{c}_1, \tilde{c}_2) \) and \((\tilde{c}_1', \tilde{c}_2')\) \( P \ (\hat{c}_1, \hat{c}_2)\) QC would yield \((\tilde{c}_1, \tilde{c}_2)\) \( P \ (c_1^*, c_2^*), \) i.e. a contradiction.)

QED

This result says that, because \textbf{SP}, \textbf{CON} and \textbf{QC} are not exceptional properties of a SWR, standard SWRs are not able to prevent an uneven intergenerational distribution for any productive technology, i.e. flexibility in the sense of Asheim (2017) cannot be attained w.r.t. to equality of distribution as a normatively appealing goal. For the special case of utilitarian SWFs \( u(c_1) + \delta u(c_2)\) this implies that neither adapting the elasticity of marginal utility \( \eta = -\frac{u''(c) c}{u'(c)} \), i.e. the degree of inequality aversion of the underlying utility function \( u(c) \), nor adapting the discount factor \( \delta \) can lead to a balanced intergenerational distribution in

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\(^6\) From the viewpoint of prioritarianism \( u(c) \) then is denoted not as a “utility function” but as “transformation function” (see, e.g., Adler, 2012) expressing the ethical observer’s preferences for equality of distribution. In the context of this paper this otherwise important distinction (see, e.g., Kaplow, 2010) is not of importance.
any case, i.e. only for all technologies. This insight was already anticipated by Rawls (1971, p. 262) by stating that “we may obtain a more plausible result if the welfare of future generations is weighted less heavily; and the most suitable discount rate may depend upon … productivity of capital....”

Proposition 3 also points at the boundaries of utilitarianism especially in the context of intertemporal evaluation as the main step in the proof of Proposition 3 is that, starting from the consumption profile \((\tilde{c}_1, \tilde{c}_2)\), some consumption sacrifice of generation 1 is deemed acceptable to make generation 2 better off. This, however, is a basic feature of utilitarianism, which is criticized by Rawls not only at a general level but also with a special focus on the intergenerational context: “(T)his calculus of advantages, which balances the losses of some against benefits of others, appears even less justified in the case of generations than among contemporaries.” (Rawls, 1971, p. 253). Here, “even less” refers to the fact that due to the productivity of savings the utilitarian “calculus of advantages” would too much harm the earlier generations (“some”) while favouring the later generations (“the others”) too much. In our terminology, Rawls thus was complaining the violation of Postulate 2. This confirms that “goodness orderings” given by utilitarian SWFs are not able to satisfy ethical intuition in any situation but should be replaced or at least be complemented by other types of evaluation criteria.

Against this background it might thus be obvious to use a criterion that reflects our “intuitive judgments” on an equitable intergenerational distribution in a direct way, i.e. by defining for some given \(\rho \geq 1\) a SWR \(R_\rho\) by letting \((c_1^*, c_2^*)\) \(R_\rho\) \((c'_1, c'_2)\) if and only if \(\min\{\rho c_1^*, c_2^*\} \geq \min\{\rho c'_1, c'_2\}\) (see Figure 7 for the \(R_\rho\)-indifference curves where \(\rho = \tan \alpha\)). The SWR \(R_\rho\) picks out that consumption path \((c_1^*, c_2^*)\) on some given \(T(.)\) for which \(\frac{c_2^*}{c_1^*} = \rho\) holds so that the desired degree of distributional equality is directly attained. For \(\rho = 1\) we in particular get the maximin-criterion, which entails equality of consumption for the two generations.

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7 For a discussion of “goodness orderings” in the intergenerational context see Broome (2012) and Greaves (2017) and especially for an assessment of their appropriate scope Kelleher (2017).
Application of $R_p$ clearly requires dropping SP. In our framework $R_p$ corresponds to the “growth sustainability approach” with $g = \rho - 1$ as the desired growth rate (see Llavador, Römer and Silvestre, 2011, 2015). This approach thus gets an additional justification.

6. Preventing Excessive Savings

Avoiding too much distributional inequality may not be the only objective with regard to protection of the interests of generation 1. A more modest target is the prevention of excessive savings PES, which is tantamount to postulating the prevention of immiseration of generation 1. In our formal setting the PES requirement for a SWR means that for any concave and productive technology $T(.)$ the $R$-optimal consumption path $(c_1^*, c_2^*)$ has to have $c_1^* \geq \alpha \bar{c}_1$ for some $\alpha \in ]0,1[\ so that in relation to its maximum attainable consumption level $\bar{c}_1$ generation 1’s consumption sacrifice should have an upper limit.
In order to find SWRs that have PES property we make the $R$-optimal consumption profile for linear technologies $T(c_1) = m(\overline{c}_1 - c_1)$ dependent on the productivity parameter $m$, i.e. $(c_1^*(m), c_2^*(m))$. Then a SWR $R$ is said to exhibit the NGC property if $c_1^*(m)$ and $c_2^*(m)$ are increasing in $m$, which in the standard microeconomics theory of the household corresponds to the Non-Giffen property for consumption of generation 1 and complementarity between consumption in both periods.

**Proposition 4:** If $R$ satisfies AN and NGC then PES holds for $\alpha = \frac{1}{2}$.

**Proof:** AN combined with QC yields $c_1^*(1) = c_2^*(1) = \frac{\overline{c}_1}{2}$. From NGC we get $c_1^*(m) > c_1^*(1) = \frac{\overline{c}_1}{2}$ for all $m > 1$. The assertion thus holds for all productive linear technologies. For a given non-linear concave technology $T(.)$ we define a hypothetical linear technology by $\tilde{T}(c_1) := -m^*c_1 + m^*c_1^* + c_2^*$ with $m^*$ = marginal rate of substitution between $c_1$ and $c_2$ in the $R$-optimal point $(c_1^*, c_2^*) (= \text{slope of } T(.) \text{ in } (c_1^*, c_2^*))$. The $R$-optimal consumption path for $\tilde{T}(.)$ also is $(c_1^*, c_2^*)$. Concavity of $T(c_1)$ implies that generation 1’s maximum consumption $\tilde{c}_1 = c_1^* + \frac{c_2^*}{m}$ under $\tilde{T}(c_1)$ is larger than $\overline{c}_1$ (see Figure 8). Thus $\frac{c_1^*}{c_1} > \frac{c_1^*}{\overline{c}_1} \geq \frac{1}{2}$ from Step 1. QED
A SWR that satisfies NGC and AN is, e.g., given by an undiscounted utilitarian SWF

\[ u(c_1) + u(c_2) = \frac{c_1^{1-\eta}}{1-\eta} + \frac{c_2^{1-\eta}}{1-\eta} \]

when \( \eta > 1 \). Consequently, there exist invariant social preferences, which provide protection for generation 1 (see Proposition 4) and at the same ensure sustainability thus giving protection also for generation 2 (see Proposition 1).

The lower bound of consumption that can be guaranteed for generation 1’s consumption in this way, however, is \( \alpha = \frac{1}{2} \), which may appear unduly low. Hence, undiscounted utilitarianism does not provide flexibility for securing a higher consumption level for generation 1.

Such flexibility, however, can be obtained through discounted utilitarian SWFs as, e.g.,

\[ u(c_1) + \delta u(c_2) = \ln c_1 + \delta \ln c_2 \]

with \( \delta < 1 \). Then, as shown by a similar argument for the case \( \delta = 1 \) before, \( \alpha = \frac{1}{1+\delta} \) results, which may take on any value in \( \left[ \frac{1}{2}, 1 \right] \) if the discount factor...

**Figure 8**

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varies in \([0,1]\). From the perspective of sufficientarianism (see, e.g., Grosseries, 2011) this result might give some novel justification for discounting because it helps to achieve \(\text{PES}\) with variable protection levels for the earlier generation. But this comes at a price since, as is easily shown, for \(\delta < 1\) there always exist technologies for which \(R\)-optimal paths are not sustainable. Sustainability nevertheless can be ensured for any productive technology by resorting to an amended criterion as presented in Section 4.

7. Conclusion

This paper has an ambiguous message since on the one hand there exist SWRs incorporating notions of distributional equity that are flexible enough to provide protection against impoverishment both for the future and the present generation irrespective of the specifically given productive technology: Sustainability is ensured and excessive savings can be avoided in any case irrespective of the underlying technology. Hence, concerning these objectives there is no need at all to adapt the social preferences to the specifically given technology in order to satisfy ethical intuition – which, according to the usual view in economics, does not seem to be legitimate as it would metaphorically speaking mean putting “the cart before the horse: Our ethics should (instead, W.B.) be stated prior to knowledge of what ... the constraints defining the feasibility set (are, W.B.).” (Llavador, Roemer and Silvestre, 2015, p. 47). To be able to procure different protection levels for the present generation while at the same time ensuring sustainability the amendments of discounted utilitarianism as described in this paper might provide helpful devices.

Yet on the other hand, it is not possible to find intertemporal SWRs with utilitarian features that are able to prevent extremely uneven distributions of well-being between the generations for any given technology. Utilitarianism and also prioritarianism thus show limitations when in a productive economy equality of well-being between generations is at stake. Other non-utilitarian criteria that – as the sustainabilitarian approach by Llavador, Roemer and Silvestre (2011, 2015) – are directly targeted at equality of distribution are required instead.

To bring the normative objectives that have played a role in this paper together SWRs leading to kinked indifference curve as depicted in Figure 9 might be conducive.
In Figure 9 $\rho = \tan \alpha$ again indicates the tolerated degree of inequality of consumption between the two generations, which determines the indifference curves’ upper kinks. The curved middle parts of the indifference curves stem from a discounted utilitarian SWF – and the steeper they are the higher is the protection level granted to the earlier generation. The lower kinks lying on the 45°-line finally represent the amendments to discounted utilitarianism that according to Asheim and Mitra (2011) ensure sustainability in any case.
References


