Gender bias and women political performance*

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PRELIMINARY VERSION

Abstract

We model voters’ gender bias as a prejudice on women’s competence coming from a distorted prior. We analyse the effect of this bias in a two-period two-party election in which voters care about both ideology and competence. We find that female politicians are less likely to win office but if elected have higher competence on average. As a consequence they choose to seek re-election more often. We also show that if parties endogenously select candidates, the effect of gender bias is stronger, in that we observe fewer female candidates and elected politicians, and of higher competence. This holds even when parties are not biased.

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1 Introduction

The results of US midterm elections in November 2018 have already granted to 2018 the name of “The Year of the Woman” for the unprecedented number of female candidates in both the House, Senate and gubernatorial elections. In the American political system being candidate implies winning some primary election, showing that this pink wave had already gained voter’s approval. But the true elections brought this phenomenon a step forward and now in the House of Representatives a record number of women will serve as legislators.

The fact that such a heavy presence of women among the elected candidates made it on the front pages of newspapers across the globe should be strong enough evidence that under-representation of women in parliaments worldwide is a relevant issue, one that has been puzzling political scientist and political economist for a while. The puzzle comes from the fact that elected women tend to be more qualified than men and while in office seem to be better representatives for their district. If, given this superiority, women are still such a minority among legislators it must be because voters hold some kind of gender bias when evaluating candidates. Studies have shown that voters hold women to a higher standard, and vote them only if they are both capable and likeable. Also, there is evidence of the belief that women are more suitable for dealing with some issues like healthcare and education and less with others like homeland security.

According to pre-elections polls the issues that were deemed salient by the electorate in 2018 (with a growing economy) were healthcare and education, topics that are quite easily associated with women competences. The election results may therefore be evidence of a reduced bias or of a persistent bias that has somehow played in favour of a women prone election.

Female representation in the world. Major improvements have been done in reducing the gender gap in terms of equality of sexes in human capital investments and economic opportunities, and yet female under-representation persists in political leadership positions. Women are still a minority in elected legislatures, around
22.6% of the members of parliaments worldwide, in spite of being roughly 59% of the population. This deficit of representation is of concern because women policy preferences receive more attention when a larger percentage of women sit in elected legislatures.

According to the Inter Parliamentary Union, the top five motivations that globally keep women from entering politics are: domestic responsibilities, prevailing cultural attitudes regarding the role of women in society, lack of support from family, lack of confidence and lack of finance.

A large survey evidence gathered by psychologists and political scientists suggests that voters largely think that “men are better suited emotionally”, “men make better leaders” and that there are circumstances in which being “tough” is really important (see Dolan 2004, and references therein). In fact, when elections focus on terrorism, defence and homeland security women do worse.

Yet, information (or the lack of it) may play a relevant role in determining gender bias. Sanbonmatsu (2002) shows how voters may use gender as a low-information short-cut to make decisions at the poll station. As a matter of fact, voters who pay little attention to politics and do not gather enough information may vote according to heuristics, the first being party affiliation, and the second one being the candidate’s gender. This suggests that gender stereotypes affect voting behaviour because they influence, more or less consciously, the way in which candidates are evaluated. The influence is stronger the lower the information level, as the effect of gender attitudes can be attenuated by providing more information on the candidates’ qualifications and past experiences (see Mo, 2015).

Intuitively, if voters hold a bias unfavourable to female candidates, only the most talented and hard working women will succeed in the electoral process. The evidence is indeed consistent with the fact that women in office are on average better than the men in the same elected body. More precisely, there is evidence that women tend to have greater prior political experience (Pearson and McGhee, 2013), that they deliver more federal funds for their district (Anzia and Berry, 2011), that they put more bills through the legislative process (Wolde, Wiseman and Wittmer, 2012) and that they deliver more speeches on the house floor (Pearson and Dancey, 2011).
If information is the key explanation, being exposed to very talented and qualified female politicians may actually reduce the sex bias with which voters evaluate candidates when making their voting decision. In this respect, quotas and other affirmative action policies may speed up the process as institutional changes are a useful method to modify cultural and social norms that otherwise evolve extremely slowly. Regarding this, it is worth noting that this policies may work even when they are temporary (De Paola et al. 2010) and may even cause an increase in the quality of male politicians through some sort of competition effect (Besley et al. 2017).

**Modelling gender bias as an information bias.** In this work we model the presence of a gender bias without assuming explicit voters’ preference for male politicians and to study its implications on the probability of winning an election, politicians’ quality and implemented policies. In order to achieve this, a multi-period model is needed where political candidates have private information on two of their own characteristics: ideology and valence (see Bernhardt et al. 2011). Both voters and politicians have an ideology bliss point and their utility is increasing in the valence of the politician in power. The valence of male and female politicians is drawn from the same distribution but voters believe that women are drawn from a distribution where lower valences have higher probability than men. Voters observe a signal on the candidates’ valences before they run for the first time, while valences are observed during the first period in office (following Bernhardt et al. 2011).

Note that the bias against females is simply due to a misperception of the characteristics of the group. This, however, implies that if the voters observe the same signal on candidates’ valences they are going to expect that female candidates are of a worse type than male ones. The fact that valences are observed when in office, instead, implies that the bias against women can be reduced since voters update their prior on female valences.

Our results on the implication of such an informative gender bias are consistent with the evidence discussed above. We characterize the equilibrium of a two-period and two-party game in which ideology and competence are private information but are observed once a politician takes office. We find that women win elections less
often, but elected female politicians have higher competence (on average) than male
one. This implies also that female incumbents are re-elected more often than male
one, a result which is also consistent with empirical evidence.

Finally, the fact that voters update their prior is consistent with the evidence
that temporary measures in support of female candidacies have a persistent effect on
the presence of females in politics.

The structure of the paper is as follows: Section 2 introduces the model, Section
3 characterises the equilibrium and discusses its dynamics and implications, Section
4 extends the model to address endogenous candidacy, Section 5 concludes. An
Appendix contains the proofs.

2 The model

We consider a two-period model. In each period there is an electoral competition,
in which two candidates, one from party $L$ and one from party $R$, face each other,
and a policy period, in which the elected politician implements a policy. Politicians
are characterised by their ideology $x^k$, and their valence $v^k$, $k = L, R$, which is
essentially a measure of their competence. Voters are heterogeneous in their policy
preference, while they all prefer higher valence. The idea of the potential trade-off
between competence and ideology, and the shape of the utility function, is taken
from Bernhardt et al. (2011).

Politicians. Politicians are characterised by ideology and valence and they are
policy oriented. In every period they receive utility from the implemented policy,
$y_t \in \mathbb{R}$ and the valence of the elected politician, $v_t^P$, where $P = L, R$ is the identity
of the elected politician in period $t$. The utility of a politician from party $k$ in period
$t$ is:

$$u_t^k (y_t, v_t^P) = -(x^k - y_t)^2 + v_t^P.$$

A politician from party $L$ has ideology $x^L \sim U \left[-\frac{3}{2}, 0\right]$, and a politician from party
$R$ has ideology $x^R \sim U \left[0, \frac{3}{2}\right]$. The valence of a candidate is essentially a measure
of his competence. It is private information of each candidate before election, and 
v^k \sim U[0, 1] \text{ for } k = L, R. \text{ Note that while ideology and valence of a politician are constant across periods, the identity of each party’s candidate may vary across periods. Therefore we let } x^k_t \text{ and } v^k_t \text{ denote the ideology and valence of party } k \text{'s candidate in period } t. \text{ When a politician is elected his/her valence and ideology are observed\textsuperscript{1}.}

\textbf{Voters.} Each voter } i \text{ has ideological preferences characterised by a bliss point } x^i. \text{ Bliss points } x^i \sim U \left[ -\frac{3}{2}, +\frac{3}{2} \right], \text{ so that the median voter has bliss point } x^m = 0. \text{ Period } t \text{ utility of each voter depends on the implemented policy } y_t \text{ and on the valence of the elected politician } v^P_t \text{ as follows:}

\[ u^i_t \left( y_t, v^P_t \right) = - \left( x^i - y_t \right)^2 + v^P_t. \]

Note that voters and politicians have the same utility function.

\textbf{Gender and gender bias.} Candidates, the first time in which they run for office, are randomly selected from a gender-balanced population (male/female with equal probability). The gender of the candidate matters, in that there is a distortion in the voters’ perception of female candidates. Even though the valence of the candidates is } v^k_t \sim U[0, 1] \text{ regardless of the candidate’s gender, the voters’ prior belief on female candidates is that there is a probability } \phi_t \text{ that they come from a worse distribution, specifically } v^k_t \mid F \sim U \left[ 0, V \right] \text{ where } V \in \left( \frac{1}{3}, 1 \right), \text{ and from the true one } v^k_t \mid F \sim U[0, 1] \text{ with the complementary probability } (1 - \phi_t). \text{ Therefore, the higher is } \phi_t \text{ the higher is the gender bias. Period 1 belief } \phi_1 \text{ is taken as given, while period 2 belief } \phi_2 \text{ may be updated if a female politician is elected in period 1 and her valence is observed.}

\textsuperscript{1}We depart from the standard assumption of Bernhardt et al. (2011) by assuming that also ideology is observed.
**Signal.** When a candidate runs for the first time, voters observe a signal \( \sigma^k_t \in \{v, \bar{v}\} \) on his/her valence. The signal reveals whether the valence of the candidate is below (\(v\)) or above (\(\bar{v}\)) the median of its group. Hence the perceived expected valence given the signal \(\sigma\) differs for male and female candidates as follows:

\[
\mathbb{E}[v^k_t|\sigma^k_t, M] = \begin{cases} 
\frac{3}{4} & \text{if } \sigma^k_t = \bar{v} \\
\frac{1}{4} & \text{if } \sigma^k_t = v 
\end{cases} \quad \mathbb{E}[v^k_t|\sigma^k_t, F] = \begin{cases} 
\frac{3}{4} (1 - \phi_t (1 - V)) & \text{if } \sigma^k_t = \bar{v} \\
\frac{1}{4} (1 - \phi_t (1 - V)) & \text{if } \sigma^k_t = v 
\end{cases} .
\]

As a consequence the expected valence of a female candidate is lower than the one of a male candidate for any possible signal. Moreover \(\mathbb{E}[v^k_t|\sigma^k_t, F]\) is decreasing in the bias \(\phi_t\). The assumption \(V > \frac{1}{3}\), however, ensures that a woman with a high signal has a higher expected valence than a man with a low signal for any possible \(\phi_t\).

**Timing.** The sequence of events at any period \(t\) is:

- Two candidates (one per party) run for election. In period 1 they are two randomly drawn untried candidates, while in period 2 one of them may be the incumbent.
- Signals on candidates are observed.
- Given the information about candidates (party affiliation, gender and \(\sigma\) for the untried candidates; party affiliation, gender, \(x, v\) and past policy choice for the incumbent) citizens vote for their preferred candidate.
- The winning politician, \(P\), with ideology \(x^P_t\) and valence \(v^P_t\), implements the policy choice \(y_t = p(x^P_t, v^P_t)\).
- At the end of period 1 only, the incumbent optimally chooses whether to run for reelection or not.
3 Equilibrium analysis

An equilibrium of this political game is composed by policy choices and voting decisions that may involve incumbents, for which ideology \( x^I \) and valence \( v^I \) have already been observed, or untried candidates for which only party affiliation is known. Proposition 1 characterises a political equilibrium with weakly undominated voting strategies.

**Proposition 1** The following pure strategies constitute a political equilibrium in which the electoral outcome is decided by the median voter where \( x^m = 0 \).

**Policy choice.** In every period \( t = 1, 2 \) the elected politician, \( P \), implements \( y_t = x^P_t \).

**Voting on untried candidates.** When candidates are both untried the median voter votes according to the following ranking of gender-signal pairs

\[
(M, v) \succ (F, v) \succ (M, \bar{v}) \succ (F, \bar{v}),
\]

randomizing with equal probability when indifferent.

**Voting for the incumbent.** The median voter votes for the incumbent:

1. for any type of challenger when \( v^I \geq (x^I)^2 \);

2. if the challenger gender-signal is in \( \{(F, \bar{v}), (M, v) \} \) when \( v^I \geq (x^I)^2 - \frac{3}{4} \phi_2 (1 - V) \);

3. if the challenger gender-signal is in \( \{(M, \bar{v}), (F, v) \} \) when \( v^I \geq (x^I)^2 - \frac{1}{2} \);

4. if the challenger gender-signal is \( (F, v) \) when \( v^I \geq (x^I)^2 - \frac{1}{2} - \frac{1}{4} \phi_2 (1 - V) \)

**Re-candidacy decision.** The incumbent of \( t = 1 \) runs for re-election in \( t = 2 \) if one of the following two conditions holds:

1. \( v^I \geq (x^I)^2 - \frac{3}{4} \phi_2 (1 - V) \);

2. \( v^I \in \left[ \max \left\{ (x^I)^2 - \frac{1}{2}, \frac{3}{2} x - (x^I)^2 - \frac{1}{4} \right\}, (x^I)^2 - \frac{3}{4} \phi_2 (1 - V) \right] \).
In the last period of the game every elected politician implements his bliss point as policy choice since there are no incentives to do otherwise to gain voters for re-election. However, in this model we have assumed that both ideology and valence are observed when a politician is elected so the bliss point is the optimal policy choice even in period one as there’s no incentive to distort the implemented policy to mimic a different ideological position. Since politicians’ ideologies are uniformly distributed and parties are symmetric around the median voter’s position, the median voter’s expected utility from the policy component is the same for any untried candidate. Therefore, in elections involving an open seat (i.e., with two untried candidates) the median voter focuses on politician’s expected valence. Note that politicians are instead different in the valence dimension even ex-ante. As a matter of fact information on expected valence can be inferred from the gender-signal pairs. Therefore the median voter votes for the candidate with the higher expected valence as described by the ranking in Proposition 1. When an incumbent runs for re-election both his ideology and valence matter for the median voter’s decision as they are known. The incumbent is more likely to be re-elected for higher valences and lower ideological biases. Re-election is also more likely the lower the expected valence of the challenger. Finally the incumbent anticipates this electoral outcome and chooses to run for re-election only when his valence is sufficiently high.

3.1 Effects of the gender bias

Female probability of winning. The first effect of gender bias is a distortion in winning probability of women, as female candidates have a lower probability of winning for any given signal. Women are thought to be drawn from a distribution that gives higher probability to lower valences. This has two implications. First, when running for an open seat, they lose the electoral competition against males with the same signal. In that case the winning probabilities given $\sigma$ are as follows:

$$
\Pr_t\left[\text{win} | \sigma^k_t, M\right] = \begin{cases} 
\frac{7}{8} & \text{if } \sigma^k_t = v \\
\frac{3}{8} & \text{if } \sigma^k_t = v
\end{cases}; \quad \Pr_t\left[\text{win} | \sigma^k_t, F\right] = \begin{cases} 
\frac{5}{8} & \text{if } \sigma^k_t = v \\
\frac{1}{8} & \text{if } \sigma^k_t = v
\end{cases},
$$
so that the winning candidate is a woman with probability $\frac{3}{8}$ which is lower than the fraction of female candidates ($\frac{1}{2}$).

Second, the bias also affects the probability of winning when running against an incumbent. Specifically the difference is driven by challengers with high signals, as challengers with low signals always lose competitions against incumbents who optimally seek re-election. Male challengers with high signal win against incumbents with $v^I < (x^I)^2$, while female challengers with high signals win if the incumbent is characterised by $v^I < (x^I)^2 - \frac{3}{4} \phi_2 (1 - V)$. Hence incumbents with $v^I \in \left( (x^I)^2 - \frac{3}{4} \phi_2 (1 - V) , (x^I)^2 \right)$ defeat female candidates of the same (true) expected valence as the one of the male candidates who defeat them.

**Expected valence of elected politicians.** A second effect of the bias is in the link between the gender of the elected politician and his/her expected valence. Indeed the expected valence of elected politicians is higher for females. This is driven by electoral competitions for open seats. In this case, male elected politicians are above the median ($\sigma^P_t | M = \pi$) with probability of $\frac{7}{10}$ while female ones are above the median with probability $\frac{5}{6}$. As the true median is the same for both groups the result follows:

\[
\begin{align*}
\mathbb{E} \left[ v^P_t | M \right] &= \Pr (\sigma^P_t \geq \pi | M) \mathbb{E} \left[ v^P_t | \pi, M \right] + \Pr (\sigma^P_t < \pi | M) \mathbb{E} \left[ v^P_t | \pi, M \right] \\
&= \frac{7}{10} + \frac{3}{10} = \frac{3}{5} \\
\mathbb{E} \left[ v^P_t | F \right] &= \Pr (\sigma^P_t \geq \pi | F) \mathbb{E} \left[ v^P_t | \pi, F \right] + \Pr (\sigma^P_t < \pi | F) \mathbb{E} \left[ v^P_t | \pi, F \right] \\
&= \frac{5}{6} + \frac{1}{6} = \frac{2}{3}
\end{align*}
\]

**Voting for the incumbent.** In our model the gender bias has no effect on voters’ perception of the incumbent, since both valence and ideology are observed once in office. What instead affects the incumbent’s chances is the gender-signal pair of the challenger. Note that in an election between an incumbent $(x^I, v^I)$ and an untried challenger as the quality of the challenger as predicted by the gender-signal pair falls the set of incumbent’s characteristics $(x^I, v^I)$ that allows him/her to gain re-election.
grows larger.

**Running for re-election.** As discussed above voters’ evaluation of an incumbent does not depend on his/her gender. Therefore re-election incentives are the same for male and female politicians in office. Specifically, incumbents with high valences, *i.e.*, \( v^I \geq (x^I)^2 - \frac{3}{4} \phi_2(1 - V) \), always run for re-election and candidates with low valences, *i.e.*, \( v^I < (x^I)^2 - \frac{1}{2} \), never choose to do so, as running implies that the opposing party wins. An interesting effect can be observed for candidates with intermediate valences, *i.e.*, \( v^I \in \left[ (x^I)^2 - \frac{1}{2}, (x^I)^2 - \frac{3}{4} \phi_2(1 - V) \right) \). The condition for which incumbents in this parametric region find it optimal to run for re-election can be rewritten as:

\[
\begin{align*}
x^I & \leq \frac{3 - \sqrt{5 - 16v^I}}{4} \\
or \quad x^I & \geq \frac{3 + \sqrt{5 - 16v^I}}{4}.
\end{align*}
\]

Hence, we can first of all notice that if \( v^I \geq \frac{5}{16} \) the incumbent runs for re-election whatever his ideological position. For lower values of valence, instead, the incumbent runs only if he’s moderate or extreme. For a moderate, this is due to the fact that the event of losing is a rather small loss given that he/she is located quite close to the ideological positions of the other party. For an extremist, instead, the small chance of winning is still valuable given his/her distance from the challenger that belongs to the other party, even if the challenger is more likely to win.

These effects hold whatever the gender of the incumbent. There is however an indirect effect of gender on the re-candidacy decision. As a matter of fact, as discussed before, elected women have higher valences. Hence, elected women are more likely to be in those parametric regions in which re-candidacy is optimal, so that on average they run for re-election more often.
3.2 Dynamics of the gender bias

We have assumed that voters have the perception that women politicians are on average less competent than men, that is they are drawn from a distribution which gives a higher weight to lower valences. In standard economic models, the distribution of agents’ types is assumed to be common knowledge, and is not an object of inference. In particular, in our model, any observed female valence is consistent with the distorted distribution, as the supports of the true and the perceived distributions coincide. However, the main feature of our model is indeed the disalignment between the true distribution of female candidates’ valences and the perception that voters have of it. As a consequence, it is reasonable to presume that, when exposed to observations drawn from the true distribution (and therefore with frequencies of observation not consistent with voters’ misperception), voters may update their belief on the distribution of women’s valences. To model this, we adopt a bayesian statistics approach. In particular, we presume that voters have a prior on the parameter $\phi_1$, and that they update this prior if they observe an elected female politician (and her valence).

Specifically, the information that is relevant to voters is whether the observed female’s valence is higher or lower than $V$. To apply the standard tools of Bayesian statistics we perform the following change of variable. We call $q$ the probability of observing a “success”, that is, a valence higher than $V$ and $(1 - q)$ the probability of a “failure” that is a valence lower than $V$. The relation between $q$ and $\phi$ is as follows: $q = (1 - \phi) (1 - V)$, so that a lower $\phi$ implies a higher $q$, and $q < 1 - V$ indicates a distorted prior. We then consider the observation of a female’s valence as the realisation of a random variable in one Bernoulli trial with unknown probability of success $q$. This random variable follows $p(s) = q^s (1 - q)^{1-s}$ where $s$ is the event $v_t^P | F > V$. The conjugated prior is the Beta distribution

$$p(q) = \frac{q^{\alpha-1} (1 - q)^{\beta-1}}{B(\alpha, \beta)}.$$
The posterior distribution, after the observation of the random draw, is:

\[ p(q) = \frac{q^{s+\alpha-1} (1-q)^{1-s+\beta-1}}{B(s+\alpha, 1-s+\beta)}, \]

that is, a Beta distribution characterised by parameters \( \alpha' \) and \( \beta' \) where \( \alpha' = \alpha + 1 \) if a success is observed (and equal to \( \alpha \) otherwise) and \( \beta' = \beta + 1 \) is a failure is observed (and equal to \( \beta \) otherwise).

The expected value of the Beta distribution, which in our case translates into the expected probability of a success, is \( \frac{\alpha}{\alpha + \beta} \). Recall that a distorted prior implies \( q < 1 - V \). Hence the hyperparameters of the prior distribution must satisfy \( \alpha < \frac{1-V}{V} \beta \).

Note that if a valence greater than \( V \) is observed the posterior expected probability of a success is higher than the prior one, as \( \frac{\alpha'}{\alpha' + \beta'} = \frac{\alpha+1}{\alpha+1+\beta} > \frac{\alpha}{\alpha+\beta} \). If instead a valence lower than \( V \) is observed the posterior expected probability is lower than the prior one, as \( \beta \) increases while \( \alpha \) stays the same. As a consequence, observing a female politician with a valence above \( V \) decreases voters' bias while if the observed valence is below \( V \) the bias increases.

In our paper we take \( V \) as constant and exogenously given, and we model the intensity of the bias through the parameter \( \phi \). However, also \( V \) can be considered as a measure of the strength of gender bias. Specifically, if \( V \) is low, voters allow for the possibility that female politicians are characterised by valences that are much lower than the males' ones. We can therefore imagine \( V \) as being influenced by cultural country-specific characteristics. It is worth noting that a stronger bias, one that has a particularly low \( V \), will be reduced more easily than a weaker one (high \( V \)) because it will be contradicted by the observation of females valences above \( V \) with higher probability. On the contrary a weak bias will be more persistent.

**Possible effects of gender quotas.** Affirmative action policies, such as gender quotas, in our model can be interpreted as an exogenous variation in the frequency of observations coming from female politicians. In other words any policy that increases exogenously the number of candidate or elected women will allow voters to acquire information on a larger set of women. In our environment this implies a faster
reduction in the gender bias. As discussed above the process will be faster for biases with a low $V$ as it is more likely that the observed valences are above the threshold.

It’s worth noting that this effect would persist even in case these affirmative policies are then removed. This is consistent with the empirical evidence of De Paola et al. (2010).

4 Selection of candidates: the role of parties

In the baseline version of the model we worked with the assumption that candidates are randomly drawn from the population. This is not a realistic assumption as parties choose their candidates to maximise the probability of being in power. We now modify the model by adding an initial stage in which parties select their candidates from a pool that is smaller than the whole population (as in Le Barbanchon and Suvagnet 2018).

We extend the model by introducing parties $L$ and $R$ as separate agents whose aim is to maximise the total probability (across periods) of having one of its members elected. They do so by optimally selecting which politician is going to run for them.

We begin from the simplest possible assumption: each party, when selecting a candidate, has to choose between two politicians that are randomly drawn from the whole population.

Proposition 2 characterises the political equilibrium of the model with endogenous selection of candidates. Note that policy choices and voters’ behaviour are unchanged from the baseline model (see Proposition 1).

**Proposition 2** The following pure strategies, together with the policy choice and voting behaviour characterised in Proposition 1, constitute a political equilibrium in which the electoral outcome is decided by the median voter where $x^m = 0$.

**Selection of candidates.** In every period and type of election parties choose their candidate according to the following ranking of gender-signal pairs

$$(M, \overline{v}) \succ (F, \overline{v}) \succ (M, \overline{v}) \succ (F, \overline{v}),$$
randomizing with equal probability when indifferent.

**Re-candidacy decision.** The incumbent of $t = 1$ runs for re-election in $t = 2$ if one of the following two conditions holds:

1. $v^I \geq (x^I)^2$;

2. $v^I \in \left[ \max \left\{ (x^I)^2 - \frac{3}{4} \phi_2 (1 - V), \frac{7}{26} x^I - (x^I)^2 - \frac{1}{18} \right\}, (x^I)^2 \right]$;

3. $v^I \in \left[ \max \left\{ (x^I)^2 - \frac{1}{2}, \frac{9}{2} x^I - (x^I)^2 - \frac{1}{8} \right\}, (x^I)^2 - \frac{3}{4} \phi_2 (1 - V) \right]$.

**Selection of candidates.** In the second period parties maximise the probability that their current candidate is elected. Therefore when facing an open seat election, they choose their candidate according to the same ranking that describes the median voter’s preferences:

$$(M, \overline{v}) \succ (F, \overline{v}) \succ (M, \underline{v}) \succ (F, \underline{v}).$$

If instead the second period election involves an incumbent, indifferences may arise. For example, the party is indifferent among each type of politician if $v^I \geq (x^I)^2$, as every type of challenger is worse than the incumbent and loses the elections. Similarly, if $v^I \in \left[ (x^I)^2 - \frac{1}{2}, (x^I)^2 - \frac{3}{4} \phi_2 (1 - V) \right]$ the party is indifferent between a female and a male candidate with a high signal as they both win against the incumbent, and between a female and a male candidate with a low signal as they both lose. We assume that, even when indifferent, the party ranks politicians according to the same preference ordering as the median voter, i.e.,

$$(M, \overline{v}) \succ (F, \overline{v}) \succ (M, \underline{v}) \succ (F, \underline{v}).$$

We justify this assumption by noticing that this is strictly optimal if there exists the possibility of an unforeseen event that induces party $k$ to replace the incumbent with another candidate (e.g., the death of the incumbent).

Finally, parties choose according to the same ranking also in the first period as they do not know the true valences of the candidates but only their gender-signal
pair. This is true even if parties are not gender biased, as they anticipate the presence of a bias in the electorate.

**Female probability of candidacy and election.** Each party faces two politicians that are randomly drawn from the population, so that each politician is characterised by any specific gender-signal pair with equal probability. Given that parties select the candidates according to the ranking, the probability of having a candidate of a specific gender-signal pair is no longer the same across pairs. Such probability is:

$$
\Pr_2[(\sigma_2^k, M) | \text{cand.}] = \begin{cases} 
\frac{7}{16} & \text{if } \sigma_2^k = \nu \\
\frac{3}{16} & \text{if } \sigma_2^k = v
\end{cases} \quad \text{and} \quad
\Pr_2[(\sigma_2^k, F) | \text{cand.}] = \begin{cases} 
\frac{5}{16} & \text{if } \sigma_2^k = \nu \\
\frac{1}{16} & \text{if } \sigma_2^k = v
\end{cases}
$$

so that the candidate is a woman with probability $\frac{3}{8}$, which is lower than the fraction of female politicians (and therefore lower that the fraction of female candidates in the baseline model). The two selected candidates, one for each party, compete against each other in the election. As a consequence the probability of electing a politician with a specific gender-signal pair is:

$$
\Pr_2[(\sigma_2^k, M) | \text{win}] = \begin{cases} 
\frac{175}{256} & \text{if } \sigma_2^k = \nu \\
\frac{15}{256} & \text{if } \sigma_2^k = v
\end{cases} \quad \text{and} \quad
\Pr_2[(\sigma_2^k, F) | \text{win}] = \begin{cases} 
\frac{65}{256} & \text{if } \sigma_2^k = \nu \\
\frac{1}{256} & \text{if } \sigma_2^k = v
\end{cases}
$$

Note that females account only for $\frac{11}{32}$ (roughly 25.7% ) of elected politicians, which is less than what happens in the baseline model ($\frac{6}{16}$, around 37.5%). This is due to the gender bias distorting the selection twice: first with the party candidacy decision and then with the voters’ choice in the election.

**Expected valence of female candidates and elected politicians.** Female candidates, having been selected, are of higher expected valence than before. As a matter of fact, the expected valence of a female candidate in the baseline model is $\frac{1}{2}$ (as they are randomly drawn from the population) while after party selection it is $\frac{2}{3}$. Also, the expected valence of an elected female politician is now $\frac{49}{56}$, which is higher.
than her expected valence when candidacy is exogenous, $\frac{2}{3}$.

Endogenous selection of candidates highlights the effect of the gender bias since it operates twice. First of all, as discussed above, it reduces the overall probability of observing female politicians at every level at which it is at play (here both candidates and elected politicians). Second this reduction in probability is greater for women with low types, so that the expected valence of female politicians active in the political arena increases as the bias operates at more levels.

**The incumbent’s candidacy decision.** When parties select candidates, Incumbents run for re-election less often than in the baseline model. This is for two reasons, both due to the increased quality of candidates. First the expected valence of challengers is higher, thus incumbents have a higher expected utility from not running. Second, it is more likely to face a high quality challenger so that the incumbent looses with higher probability

**Female conditional probability of winning** Women’s probability of winning conditional on being a candidate drops to $\frac{11}{32}$, which is lower than the equivalent in the baseline model, $\frac{3}{8}$. This is contrast with empirical evidence that shows that when women run they win at the same rate as men (Burrell 1994, Darcy and Schramm 1977, Seltzer 1997). It is also in contrast with the intuition that observing less but more qualified women candidates should lead to probability of winning equal to the male counterpart. In our model, this is due to the discrete nature of the signal which does not allow parties to select women with such an high signal that lead them to be perceived as equally qualified as men.

# Concluding remarks

We have shown how modelling gender bias as an informational bias of voters when evaluating candidates allows us to obtain results that are consistent with the empirical evidence gathered in trying to explain female under-representation in elected
bodies. We show that women win less often than men but those that are elected are on average more competent than elected male politicians.

The fact that this bias evolves with each election and that being exposed to qualified women politicians contributes to its reduction should be considered a theoretical ground for those policies of affirmative action that favour women political participation. Our model predicts that they may have a positive effect even when temporary, but there’s a caveat: women that benefit from these policies have to be really qualified because the updating of the bias may work backward, thus favouring a confirmation of the distortion used by voters to evaluate women.

We also extended the model to consider the strategic candidacy choice by parties. When we allow them to choose their candidates strategically we observe that parties, even if they are not gender biased, choose women candidates only when they are sufficiently strong, because they anticipate the gender bias of the electorate. Therefore the gender bias operates twice both in the selection and election stages. As a consequence the probability of electing a woman falls even further, on the other hand though the expected valence of those that make it to office is greater.

We have highlighted the main effect of having a selection process in spite of the simple structure we gave to the informative set and choice of the parties. A deeper analysis may focus on parties with full information on candidates’ types or having access to a larger pool of candidates from which they can select. Future research will address these extensions.

References


A Proof of Proposition 1

Policy choice. In the last period of the game the policy choice of the elected politician affects only his/her second period utility, as the game ends afterwards. Therefore he/she implements his/her most preferred policy which maximises his/her utility. In the first period of the game, the elected politician \( P \) in period \( t \) knows that his ideology \( x^P_t \) and his valence \( v^P_t \) are both observed by voters when he/she is in office. Moreover, voters know that in the last period every elected politician implements a policy equal to his/her ideology. As a consequence, the policy choice of the politician who is in office in period \( t = 1 \) does not affect his/her probability of winning the election in \( t = 2 \) (nor his/her re-candidacy choice). Therefore, also in period \( y_1 = x^P_1 \).

Voting on untried candidates. Voters anticipate the politicians’ behaviour. At the time of elections the median voter cannot distinguish two untried candidates according to their ideological position, as candidates are ex-ante symmetric in this dimension. Therefore the election is decided on the basis of the information available on candidates’ valence. The median voter’s utility is linear in \( v^P_t \) therefore he will prefer the candidate with the highest expected valence. The expected valence given the candidate’s gender and his/her \( \sigma^k_t \) is:

\[
\mathbb{E}[v^k_t | \sigma^k_t, M] = \begin{cases} 
\frac{3}{4} & \text{if } \sigma^k_t = \overline{v} \\
\frac{1}{4} & \text{if } \sigma^k_t = \underline{v}
\end{cases};
\mathbb{E}[v^k_t | \sigma^k_t, F] = \begin{cases} 
\frac{3}{4} (1 - \phi_t (1 - V)) & \text{if } \sigma^k_t = \overline{v} \\
\frac{1}{4} (1 - \phi_t (1 - V)) & \text{if } \sigma^k_t = \underline{v}
\end{cases}.
\]
therefore the median voter ranks the candidates according to their gender-signal pair as follows $(M, \overline{v}) \succ (F, \overline{v}) \succ (M, v) \succ (F, v)$.

Voting for the incumbent. Consider now a period 2 election in which the incumbent from period 1 runs for re-election. In period 2 voters know both valence $v^I$ and ideology $x^I$ of the incumbent. Given that valence is observed, the incumbent’s gender is does not affect his/her probability of re-election. Moreover, voters know that any politician winning in period 2 will implement a policy equal to his/her bliss point.

Consider therefore the voting incentives of the median voter when comparing an incumbent $(x^I, v^I)$ with an untried challenger characterised by his/her gender-signal pair. As the median voter is located at $x^m = 0$ it is indifferent whether the incumbent is from party $R$ and the challenger from party $L$ or viceversa.

The median voter’s expected policy disutility in period 2 from an untried challenger from party $k$, given the equilibrium policy choice, is

\[-\mathbb{E}(0 - y_2)^2 = -\mathbb{E}(x^k_2)^2 = -V[x^k_2] - (\mathbb{E}[x^k_2])^2
\]
\[= \frac{1}{12} \frac{9}{4} - \frac{9}{16} = -\frac{3}{4}.\]

Therefore, the median voter’s expected utility in period 2 when facing a challenger, depending on his/her gender-signal pair, is

- $\mathbb{E}u^m_2 (M, \overline{v}) = \mathbb{E} (0 - y_2)^2 + \mathbb{E} \left[ v^k_2 | \overline{v}, M \right] = -\frac{3}{4} + \frac{3}{4} = 0$;
- $\mathbb{E}U^m_2 (F, \overline{v}) = -\frac{3}{4} + \frac{3}{4} (1 - \phi_2 (1 - V)) = -\frac{3}{4} \phi_2 (1 - V)$;
- $\mathbb{E}u^m_2 (M, v) = -\frac{3}{4} + \frac{1}{4} = -\frac{1}{2}$;
- $\mathbb{E}U^m_2 (F, v) = -\frac{3}{4} + \frac{1}{4} (1 - \phi_2 (1 - V)) = -\frac{1}{2} - \frac{1}{2} \phi_2 (1 - V)$.

The median voter’s expected utility in period 2 from an incumbent characterised by $(x^I, v^I)$ is instead: $\mathbb{E}u^m_2 (x^I, v^I) = -(x^I)^2 + v^I$. Hence, the median voter prefers the incumbent to he challenger, given his/her gender-signal pair, in the following parametric regions:
• if the challenger’s gender-signal pair is \((M, v)\), when \(v \geq (x')^2\), that is, when \(x' \in [-\sqrt{v}, \sqrt{v}]\);

• if the challenger’s gender-signal pair is \((F, v)\), when \(v \geq (x')^2 - \frac{3}{4} \phi_2 (1 - V)\), that is, when \(x' \in [-\sqrt{v} + \frac{3}{4} \phi_2 (1 - V), \sqrt{v} + \frac{3}{4} \phi_2 (1 - V)]\);

• if the challenger’s gender-signal pair is \((M, v)\), when \(v \geq (x')^2 - \frac{1}{2}\), that is, when if \(x' \in [-\sqrt{v} + \frac{1}{2}, \sqrt{v} + \frac{1}{2}]\);

• if the challenger’s gender-signal pair is \((F, v)\), when \(v \geq (x')^2 - \frac{1}{2} - \frac{1}{4} \phi_2 (1 - V)\), that is, when \(x' \in [-\sqrt{v} + \frac{1}{2} + \frac{1}{4} \phi_2 (1 - V), \sqrt{v} + \frac{1}{2} + \frac{1}{4} \phi_2 (1 - V)]\).

Remark 3 Note that, in an election between an incumbent \((x', v')\) and an untried challenger as the quality of the challenger falls the set of \(x'\) by the incumbent that allows him/her to gain re-election grows larger, and also the threshold for \(v'\) decreases.

The incumbent’s candidacy decision. Consider an incumbent from party \(R\), characterised by \(x' \geq 0\) and \(v'\). If he/she does not run, voters have the same ex-ante probability of electing a politician from party \(L\) or \(R\). Therefore the incumbent, in period 2, obtains expected utility:

\[
\mathbb{E}u_2' = -\frac{1}{2} \mathbb{E}[(x' - x_2^L)^2] - \frac{1}{2} \mathbb{E}[(x' - x_2^R)^2] + \mathbb{E}(v_2^P)
\]

\[
= -(x')^2 - \frac{1}{2} \mathbb{E}[(x_2^L)^2] + x' \mathbb{E}[x_2^L] - \frac{1}{2} \mathbb{E}[(x_2^L)^2] + x' \mathbb{E}[x_2^L] + \mathbb{E}(v_2^P)
\]

\[
= -(x')^2 - \frac{3}{4} + \frac{5}{8}
\]

\[
= -(x')^2 - \frac{1}{8}.
\]

Note that in the model we assume that politicians are not biased and therefore their expected utility is not affected by the gender of the challenger. If we assumed instead that the incumbent suffered from the same gender bias as the voters, his/her utility from not running would have been lower, and therefore he/she would have run for re-election (sub-optimally) more often.

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Given the incumbent’s expected utility if he/she does not run in the second period, we discuss the optimal re-candidacy choice, which depends on the chances of re-election.

- If elected, the incumbent in period 2 implements his/her bliss point and his/her utility is equal to $v$. Therefore, an incumbent who wins against any type of challenger (i.e., such that $v' \geq (x^I)^2$) runs for re-election in period 2 for every value of $(x^I, v^I)$.

- An incumbent who wins against everybody but $(M, v)$, i.e., an incumbent such that $v \in \left((x^I)^2 - \frac{3}{4} \phi_2(1-V), (x^I)\right)$, has an expected utility from running for re-election which is the weighted average of the utility of winning (with probability $3/4$) and of the utility of losing against a challenger of the opposite party with a gender-signal pair equal to $(M, v)$, that is:

$$
\frac{3}{4} v^I + \frac{1}{4} \left(- (x^I)^2 - \frac{3}{4} x^I + \mathbb{E} (v^I_2|M, v) \right)
= \frac{3}{4} v^I + \frac{1}{4} \left(- (x^I)^2 - \frac{3}{4} x^I + \frac{3}{4} \right)
= \frac{3}{4} v^I - \frac{1}{4} (x^I)^2 - \frac{3}{8} x^I
$$

Note that the following inequality is always satisfied

$$
\frac{3}{4} v^I - \frac{1}{4} (x^I)^2 - \frac{3}{8} x^I \geq - (x^I)^2 - \frac{1}{8},
$$

therefore an incumbent with $(x^I, v^I)$ in this parametric region always runs for re-election.²

- An incumbent who wins only against challengers with low signals (of any gender), i.e., an incumbent such that $v \in \left((x^I)^2 - \frac{1}{2}, (x^I)^2 - \frac{3}{4} \phi_2(1-V)\right)$, has an expected utility from running which is the weighted average of the utility of winning (with weight $1/2$) and of the utility of losing against a challenger

²Observe that $v^I \geq \frac{1}{2} x^I - (x^I)^2 - \frac{1}{6}$ given that the rhs is negative.
of the opposite party with a high signal. Recall that politicians are not gender biased, so that the expected valence of a candidate with a high signal is $\frac{3}{4}$ regardless of his/her gender. Hence, in this parametric region, the incumbent’s utility from running in period 2 is:

$$\frac{1}{2} v^I + \frac{1}{2} \left( - \left( x^I \right)^2 - \frac{3}{4} - \frac{3}{2} x^I + \frac{3}{4} \right)$$

$$= \frac{1}{2} v^I - \frac{1}{2} \left( x^I \right)^2 - \frac{3}{4} x^I,$$

which is higher than the expected utility of the incumbent who chooses not to run in period 2 if:

$$v^I \geq - \left( x^I \right)^2 - \frac{1}{4} + \frac{3}{2} x^I.$$

• An incumbent that gains reelection only against $(F,v)$, i.e., an incumbent such that $v \in \left[ \left( x^I \right)^2 - \frac{1}{2} - \frac{1}{4} \phi_2(1 - V), \left( x^I \right)^2 - \frac{1}{2} \right]$, has an expected utility from running which is the weighted average of the utility of winning (with weight 1/4), of the utility of losing against a challenger of the opposite party with a high signal (with weight 1/2), and of the utility of losing against a challenger of the opposite party with a low signal. Hence, in this parametric region, the incumbent’s utility from running in period 2 is:

$$\frac{1}{4} v^I + \frac{1}{2} \left( - \left( x^I \right)^2 - \frac{3}{4} - \frac{3}{2} x^I + \frac{3}{4} \right) + \frac{1}{4} \left( - \left( x^I \right)^2 - \frac{3}{4} - \frac{3}{2} x^I + \frac{4}{4} \right)$$

$$= \frac{1}{4} v^I - \frac{3}{4} \left( x^I \right)^2 - \frac{9}{8} x^I - \frac{1}{8},$$

which is higher than the expected utility of the incumbent who chooses not to run in period 2 if:

$$v^I \geq \frac{9}{2} x^I - \left( x^I \right)^2.$$

However, $\frac{9}{2} x^I - \left( x^I \right)^2 > \left( x^I \right)^2 - \frac{1}{2}$. Hence, there is no pair $(x^I, v^I)$ in this parametric region such that the incumbent finds it optimal to run for re-election.

• Finally, an incumbent who wins against no type of challenger does never find
it optimal to seek re-election as by running he ensures that the challenger from
the opposite party (which can have a high or low signal with equal probability)
wins. His/her expected utility from running is:

\[
\frac{1}{2} \left( - \left( x^I \right)^2 - \frac{3}{4} - \frac{3}{2} x^I + \frac{3}{4} \right) + \frac{1}{2} \left( - \left( x^I \right)^2 - \frac{3}{4} - \frac{3}{2} x^I + \frac{1}{4} \right) \\
= - \left( x^I \right)^2 - \frac{3}{2} x^I - \frac{1}{4} < - \left( x^I \right)^2 - \frac{1}{8} .
\]

Results for an incumbent from party \( L \) are derived symmetrically. ■

B Proof of Proposition 2

The elected politician’s policy choice, and the voting behavior of voters are the ones
characterised in Proposition 1. Please refer to the proof of Proposition 1 for their
optimality.

Selection of candidates I: open seats elections in the second period. In
the second period parties maximise the probability that their current candidate is
elected. Therefore when facing an open seat election, they choose their candidate
according to the same ranking that describes the median voter’s preferences:

\((M, \overline{v}) \succ (F, \overline{v}) \succ (M, \underline{v}) \succ (F, \underline{v})\).

Each party faces two politicians that are randomly drawn from the population, so
that each politician is characterised by any specific gender-signal pair with equal
probability. As the two politicians are independently drawn the conditional candida-
dcy probabilities are as follow:

\[
\Pr_2[\text{cand.}|\sigma^k, M] = \begin{cases} 
\frac{7}{8} & \text{if } \sigma^k = \overline{v} \\
\frac{3}{8} & \text{if } \sigma^k = \underline{v} 
\end{cases} ; \quad \Pr_2[\text{cand.}|\sigma^k, F] = \begin{cases} 
\frac{7}{8} & \text{if } \sigma^k = \overline{v} \\
\frac{3}{8} & \text{if } \sigma^k = \underline{v} 
\end{cases} .
\]
so that the candidate is a woman with probability \(3/8\). The probability of having a candidate of a specific gender-signal pair is:

\[
Pr_2[(\sigma_2^k, M) \mid \text{cand.}] = \begin{cases} 
\frac{7}{16} & \text{if } \sigma_2^k = \overline{v} \\
\frac{3}{16} & \text{if } \sigma_2^k = \underline{v}
\end{cases} 
Pr_2[(\sigma_2^k, F) \mid \text{cand.}] = \begin{cases} 
\frac{5}{16} & \text{if } \sigma_2^k = \overline{v} \\
\frac{1}{16} & \text{if } \sigma_2^k = \underline{v}
\end{cases} .
\]

The two selected candidates, one for each party, compete against each other in the election. Their probability of winning, conditional on the gender-signal pair and on having been selected as candidates, is:

\[
Pr_2[\text{win} \mid (\sigma_2^k, M), \text{cand.}] = \begin{cases} 
\frac{25}{32} & \text{if } \sigma_2^k = \overline{v} \\
\frac{5}{32} & \text{if } \sigma_2^k = \underline{v}
\end{cases} 
Pr_2[\text{win} \mid (\sigma_2^k, F), \text{cand.}] = \begin{cases} 
\frac{13}{32} & \text{if } \sigma_2^k = \overline{v} \\
\frac{1}{32} & \text{if } \sigma_2^k = \underline{v}
\end{cases} .
\]

As a consequence the probability of electing a politician with a specific gender-signal pair is:

\[
Pr_2[(\sigma_2^k, M) \mid \text{win}] = \begin{cases} 
\frac{175}{256} & \text{if } \sigma_2^k = \overline{v} \\
\frac{15}{256} & \text{if } \sigma_2^k = \underline{v}
\end{cases} 
Pr_2[(\sigma_2^k, F) \mid \text{win}] = \begin{cases} 
\frac{65}{256} & \text{if } \sigma_2^k = \overline{v} \\
\frac{1}{256} & \text{if } \sigma_2^k = \underline{v}
\end{cases} .
\]

**Selection of candidates II: second period election with an incumbent.**

Consider a second period election in which an incumbent from party \(k\) is running for re-election. The opposing party knows \((x^I, v^I)\) and has to choose among two possible candidates. In this case, indifferences may arise. For example, the party is indifferent among each type of politician if \(v^I \geq (x^I)^2\), as every type of challenger is worse than the incumbent and loses the elections. Similarly, if \(v^I \in \left[ (x^I)^2 - \frac{1}{2}, (x^I)^2 - \frac{3}{4} \phi_2(1-V) \right] \) the party is indifferent between a female and a male candidate with a high signal as they both win against the incumbent, and between a female and a male candidate with a low signal as they both lose. We assume that, even when indifferent, the party ranks politicians according to the same preference ordering as the median voter, i.e.,

\[(M, \overline{v}) \succ (F, \overline{v}) \succ (M, \underline{v}) \succ (F, \underline{v}) .\]
We justify this assumption by noticing that this is strictly optimal if there exists the possibility of an unforeseen event that induces party \( k \) to replace the incumbent with another candidate (e.g., the death of the incumbent).

**Selection of candidates III: first period elections** Parties do not observe the true valences of politicians but only the public signal on them. Therefore they cannot anticipate who will have higher chances of running for re-election in the second period, when they compare two politicians with the same signal (but possibly different gender). Therefore they select candidates by maximising the probability that their candidate wins the first period election, and the problem is equivalent to the selection of candidates for an open seat election in the second period. Therefore, they choose the candidate according to the order

\[(M, v) \succ (F, v) \succ (M, v) \succ (F, v).\]

**The incumbent’s candidacy decision.** Consider an incumbent from party \( R \), characterised by \( x^I \geq 0 \) and \( v^I \). If he/she does not run, voters have the same ex-ante probability of electing a politician from party \( L \) or \( R \). Therefore the incumbent, in period 2, obtains expected utility:

\[
\mathbb{E} u^I_2 = \mathbb{E}[(x^L - x^R)^2] - \mathbb{E}[(x^L - x^R)^2] + \mathbb{E}(v^P_2) \\
= -(x^I)^2 - \frac{1}{2} \mathbb{E}(x^L)^2 - x^I \mathbb{E}(x^L) - \frac{1}{2} \mathbb{E}(x^L)^2 + x^I \mathbb{E}(x^L) + \mathbb{E}(v^P_2) \\
= -(x^I)^2 - \frac{3}{4} + \frac{23}{32} \\
= -(x^I)^2 - \frac{1}{32}.
\]

Recall that, as in the baseline model, politicians are not biased and therefore their expected utility is not affected by the gender of the challenger.

Given the incumbent’s expected utility if he/she does not run in the second period, we discuss the optimal re-candidacy choice, which depends on the chances of re-election.
• If elected, the incumbent in period 2 implements his/her bliss point and his/her utility is equal to $v$. Therefore, an incumbent who wins against any type of challenger (i.e., such that $v^I \geq (x^I)^2$) runs for re-election in period 2 for every value of $(x^I, v^I)$.

• An incumbent who wins against everybody but $(M, v)$, i.e., an incumbent such that $v^I \in \left[\left((x^I)^2 - \frac{3}{4} \phi_2 (1 - V), (x^I)^2\right]\right]$, has an expected utility from running for re-election which is the weighted average of the utility of winning (with probability $9/16$) and of the utility of losing against a challenger of the opposite party with a gender-signal pair equal to $(M, v)$, that is:

\[
\frac{9}{16} v^I + \frac{7}{16} \left(- (x^I)^2 - \frac{3}{4} - \frac{3}{2} x^I + \mathbb{E} \left( v^L^I | M, v \right) \right)
\]

\[=
\frac{9}{16} v^I + \frac{7}{16} \left(- (x^I)^2 - \frac{3}{4} - \frac{3}{2} x^I + \frac{3}{4} \right)\]

\[=
\frac{9}{16} v^I - \frac{7}{16} (x^I)^2 - \frac{21}{32} x^I
\]

Note that the incumbent finds optimal to run for re-candidacy when

\[
\frac{9}{16} v^I - \frac{7}{16} (x^I)^2 - \frac{21}{32} x^I \geq - (x^I)^2 - \frac{1}{32},
\]

that is when $v^I \geq - (x^I)^2 + \frac{7}{6} x^I - \frac{1}{18}$.

• An incumbent who wins only against challengers with low signals (of any gender), i.e., an incumbent such that $v^I \in \left[\left((x^I)^2 - \frac{1}{2}, (x^I)^2 - \frac{3}{4} \phi_2 (1 - V)\right]\right]$, has an an expected utility from running which is the weighted average of the utility of winning (with weight $1/4$) and of the utility of losing against a challenger of the opposite party with a high signal. Recall that politicians are not gender biased, so that the expected valence of a candidate with a high signal is $\frac{3}{4}$ regardless of his/her gender. Hence, in this parametric region, the incumbent’s
utility from running in period 2 is:

\[
\frac{1}{4} v' + \frac{3}{4} \left( - (x')^2 - \frac{3}{4} - \frac{3}{2} x' + \frac{3}{4} \right)
\]

\[
= \frac{1}{4} v' - \frac{3}{4} (x')^2 - \frac{9}{8} x,
\]

which is higher than the expected utility of the incumbent who chooses not to run in period 2 if:

\[
v' \geq - (x')^2 - \frac{1}{8} + \frac{9}{2} x'.
\]

Note that, if \( \phi_2 (1 - V) \geq \frac{1}{6} \), the inequality never holds in this parametric region, as \(- (x')^2 - \frac{1}{8} + \frac{9}{2} x' \geq - \frac{3}{4} \phi_2 (1 - V)\), and the incumbent never chooses to run for a second election.

- An incumbent that gains re-election only against \((F, v)\), i.e., an incumbent such that \(v' \in \left( (x')^2 - \frac{1}{2} - \frac{1}{4} \phi_2 (1 - V), (x')^2 - \frac{1}{2} \right)\), has an expected utility from running which is the weighted average of the utility of winning (with weight \(\frac{1}{16}\)), of the utility of losing against a challenger of the opposite party with a high signal (with weight \(\frac{3}{4}\)), and of the utility of losing against a challenger of the opposite party with a low signal. Hence, in this parametric region, the incumbent’s utility from running in period 2 is:

\[
\frac{1}{16} v' + \frac{3}{4} \left( - (x')^2 - \frac{3}{4} - \frac{3}{2} x' + \frac{3}{4} \right) + \frac{3}{16} \left( - (x')^2 - \frac{3}{4} - \frac{3}{2} x' + \frac{1}{4} \right)
\]

\[
= \frac{1}{16} v' - \frac{15}{16} (x')^2 - \frac{45}{32} x' - \frac{3}{32},
\]

which is higher than the expected utility of the incumbent who chooses not to run in period 2 if:

\[
v' \geq \frac{45}{2} x' - (x')^2 + 1.
\]

Given that \(\frac{45}{2} x' - (x')^2 + 1 > - (x')^2 - \frac{1}{2}\) the above inequality is never satisfied, so that such an incumbent never choose to run for re-election.
Finally, an incumbent who wins against no type of challenger does never find it optimal to seek re-election as by running he ensures that the challenger from the opposite party (which can have a high or low signal with equal probability) wins. His/her expected utility from running is:

\[
\frac{3}{4} \left( - (x^I)^2 - \frac{3}{2} x^I + \frac{3}{4} \right) + \frac{1}{4} \left( - (x^I)^2 - \frac{3}{2} x^I + \frac{1}{4} \right) \\
= - (x^I)^2 - \frac{3}{2} x^I - \frac{1}{8} < - (x^I)^2 - \frac{1}{18}.
\]

Results for an incumbent from party \( L \) are derived symmetrically.