Competition for influence in mixed oligopoly

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Abstract
The purpose of this paper is to analyze the effect of political lobbying on the degree of optimal privatization ratio in a closed and a two-country, mixed oligopoly model. We find that, in general, lobbying activity does not lead to a Pareto efficient allocation by optimal privatization policy in a closed and a two-country model. This is sharply contrast result in a common-agent and a multi-agent lobbying model.

JEL Classification: D43; D72; F12; F13; L33
Keywords: Privatization; Mixed oligopoly; Lobbying.
1. Introduction

The purpose of this paper is to analyze the effect of political lobbying on the optimal degree of privatization in a mixed oligopoly model in a closed and a two-country.

One of the salient propositions in public choice is the competitive political lobbying does not affect the original policy without political lobbying. This result, initially, is ascertained by using common agent framework (Dixit, Grossman and Helpman (1997) and, recently, Aidt and Hwang (2014) obtain the same result by using multi agent framework.

Dixit, Grossman and Helpman (1997) and Aidt and Hwang (2014) argue that when all special interest groups can lobby, the equilibrium policy with political lobbying is same as the equilibrium policy without lobbying, because a political pressure from each special interest group is fully canceled out as a result of political competition

Matsumura (1998) finds that neither full privatization (the government does not hold any shares) nor full nationalization (the government holds all shares) is optimal under moderate conditions, as the government can achieve the optimal allocation by choosing privatization ratio. Thus, if the government adjust the privatization ratio, this leads to the optimal privatization policy. However, we argue that if lobbying activity exists in a mixed oligopoly market, the privatization policy with political lobbying does not replicate the optimal policy, because the optimal decisions from a public firm involve not only their revenue as a firm but also the level of social welfare as a public interest or a public obligation.

Two types of a lobbying model considered here are a common-agent model and a multi-agent model. Firstly, using a common-agent model, we can analyze the case of a closed economy as in the case of a ban existing on foreign lobbying. Dixit, Grossman and Helpman (1997) analyze the effect of lobbying on a (common) government. They show that if all special interest groups (principals) lobby to a politician (common agent), the effect of lobbying is neutralized one another. Thus, Pareto optimal allocation can be replicated by this competitive lobbying activity as we explained above. Secondly, using a multiple-agent type model, we can address the case of no ban on foreign lobbying. Aidt and Hwang (2014) analyzes the effect of lobbying on foreign governments when there exist cross national externalities. They show that a foreign lobbying can maximize the world social welfare if all interest groups in all countries are organized in lobby groups and all governments are

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1 This result is ascertained in various environments (e.g. Lai (2010) and Winter (2017)).
equally receptive to campaign contributions. Thus, these results give an important and a powerful policy implication that lobbying activity can be justified in contrast to our results\(^2\).

The structure of this paper is organized as follows. In next section, we construct a general lobbying model in closed-economy. In Section 3, we extend the closed economy model to two-country model to analyze the difference between a common-agent and a multi-agent model. Section 4 discusses policy implications and generalizations for future research.

2. Closed economy

We describe a mixed oligopoly model with lobbying activity of a private firm, a public firm, and organized consumers. This section investigates the effect of lobbying activity on optimal privatization policy in a closed economy.

2.1 Basic setting in a closed economy

Consider a market in a closed economy served by a partially privatized firm (firm 0), which jointly owned by a government and private sector, and a pure private firm (firm 1)\(^3\). Both firms produce a homogenous goods, \(q_0\) and \(q_1\), representing the quantity of output of public firm and private firm, respectively, and face the inverse demand function \(p = 1 - 2Q\), where \(p\) and \(Q \equiv q_0 + q_1\) denote the price of goods and aggregate output in this market. Both firms have identical cost function, \(c_j = \frac{1}{2}q_j^2, j = 0, 1\), thus, the profit function of each firm becomes \(\pi_j = (1 - 2(q_0 + q_1))q_j - \frac{1}{2}q_j^2, j = 0, 1\). Assuming the demand function is linear, consumer surplus, \(cs\), is equal to \(Q^2\). Thus, social welfare is \(W = cs + \pi_0 + \pi_1\).

Following Matsumura (1998), government owns a share of \(1 - \theta \in [0,1]\) of the partially privatized firm. Here \(\theta\) can be seen as the degree of privatization; \(\theta = 1\) means perfectly privatized firm and \(\theta = 0\) means perfectly public firm. Thus, partially privatized firms’ objective function becomes the sum of the social welfare and producer surplus of its firm:

\(^2\) Grossman and Helpman (1994) proposed that special interest groups (principals) supply campaign contribution to a government (a common-agent) using the menu auction framework of Bernheim and Whinston (1986). Aidt and Hwang (2008, 2014) also proposed that special interest groups (principals) supply the campaign contribution to governments (a “multi”-agent) using the framework of Prat and Rustichini (2003).

\(^3\) This is the simplest case when we consider mixed oligopoly market. The number of firms does not affect our results.
\[ v = \theta \pi_0 + (1 - \theta)W \]

Maximizing the above objective function of public firm and the revenue function of private firm, we obtain the equilibrium output of partially privatized firm, \( q_0^* \), private firm, \( q_1^* \), and equilibrium price, \( p^* \) as:

\[
q_0^* = \frac{3}{11 + 10\theta}
\]

\[
q_1^* = \frac{1 + 2\theta}{11 + 10\theta}
\]

and

\[
p^* = \frac{3(1 + 2\theta)}{11 + 10\theta}
\]

where the effect of privatization on equilibrium outputs of both firms and equilibrium price are:

\[
\frac{dq_0^*}{d\theta} = -\frac{30}{(11+10\theta)^2} < 0, \quad \frac{dq_1^*}{d\theta} = \frac{12}{(11+10\theta)^2} > 0 \quad \text{and} \quad \frac{dp^*}{d\theta} = \frac{36}{(11+10\theta)^2} > 0.
\]

Using (1), producer surplus (or profits) of both firms, consumer surplus and objective function of privatized firm become

\[
\pi_0^* = \frac{9(1 + 4\theta)}{2(11 + 10\theta)^2}
\]

\[
\pi_1^* = \frac{5(1 + 2\theta)^2}{2(11 + 10\theta)^2}
\]

\[
cs^* = \frac{4(2 + \theta)^2}{(11 + 10\theta)^2}
\]

and

\[
v^* = \frac{46 + \theta(51 - 4\theta(6 + 7\theta))}{2(11 + 10\theta)^2},
\]

respectively.

The effect of privatization on producer surplus of both firms and consumer surplus can be also calculated as:
Political economic analysis of privatization

\[
\frac{d\pi_0^*}{d\theta} = \frac{36(3 - 5\theta)}{(11 + 10\theta)^3}
\]
\[
\frac{d\pi_1^*}{d\theta} = \frac{60(1 + 2\theta)}{(11 + 10\theta)^3} > 0,
\]
\[
\frac{dcs^*}{d\theta} = -\frac{72(2 + \theta)}{(11 + 10\theta)^3} < 0,
\]
and
\[
\frac{dv^*}{d\theta} = -\frac{359 + 2\theta(519 + 14\theta(33 + 10\theta))}{2(11 + 10\theta)^3} < 0,
\]
respectively.

From (2), equilibrium social welfare can be obtained as \(W^* = \frac{23 + 2\theta(22 + 7\theta)}{(11 + 10\theta)^2}\), and then we can obtain the optimal privatization ratio which satisfies the maximization condition \(\frac{dW^*}{d\theta} = 0\) as \(\theta^* = \frac{2}{11}\) in the closed economy.

**Lemma 1.** Optimal privatization ratio in closed economy (Han and Ogawa, 2008)

*In a closed economy, partial privatization is optimal policy and the privatization ratio is \(\theta^* = \frac{2}{11}\).*

### 2.2 Political economic model in a closed economy

In this sub-section, we analyze the effect of lobbying of a public firm, a private firm and organized consumers who are called by special interest group on optimal privatization ratio. From the equation (3), special interest group organized by the public firm, the private firm, and organized consumers has an incentive to influence the degree of privatization since an increase in \(\theta\) decreases the value of objective function of the public firm, increases the revenue of the private firm and decreases consumer surplus\(^4\). We assume that special interest group organized by the public firm,\(^4\)

\(^4\) It may seem that it is difficult to lobby by public firm since public firm is a part of government. However, a decision making by local public firm like a water supply, gus, and electricity industries is independent of a decision making by local government. Moreover, it may consider that it is strange to lobby for one of the factors of own objective function and what lobby can implement, because several shareholders and government have the stock of this partially privatized firm and thus objective function may differ. This problem is treated in Kamijo and Tomaru (2014) which consider how to determination of partially privatized firm’s objective function by the Nash bargaining approach. However, since we
the private firm and organized consumers can provide contributions, $Z^0$, $Z^1$ and $Z^{cs}$ to the policymaker in return for influencing the privatization ratio, $\theta$, respectively, and can offer the differentiable contribution schedule for privatization ratio, $Z^j(\theta)$ ($j = 0, 1$ and $cs$), to the policy makers. As a result, the payoffs for special interest group organized by the public firm, the private firm, and organized consumers are

$$V = v - Z^0(\theta), \Pi_1 = \pi_1 - Z^1(\theta), \text{ and } CS = cs - Z^{cs}(\theta)$$

As in Cai and Li (2014), the policymaker cares about the level of campaign contribution and social surplus because the number of voting depends not only on the size of campaign contribution but also the public endorsement. Thus, objective function of policymaker is the sum of consumer surplus $CS = cs - Z^{cs}(\theta)$, producer surplus of public firm $\Pi_0 = \pi_0 - Z^0(\theta)$, producer surplus of private firm $\Pi_1 = \pi_1 - Z^1(\theta)$ and political contribution $G(\theta) = CS + \pi_0 + \Pi_1 + \gamma_0 Z^0(\theta) + \gamma_1 Z^1(\theta) + \gamma^{cs} Z^{cs}(\theta)$

$$G(\theta) = CS + \pi_0 + \Pi_1 + \gamma_0 Z^0(\theta) + \gamma_1 Z^1(\theta) + \gamma^{cs} Z^{cs}(\theta)$$

where $\gamma_j (> 1)$ denotes the weight that policymakers for the political contributions of each special interest group.

Next, we consider the amount of political contribution from each special interest group. Following Grossman and Helpman (1994), we focus on the truthful contribution schedule: $Z^0(\theta) = \max\{0, v(\theta) - b^0\}, Z^1(\theta) = \max\{0, \pi_1(\theta) - b^1\}$ and $Z^{cs}(\theta) = \max\{0, cs(\theta) - b^{cs}\}$. The maximization condition of special interest group organized by the public firm, the private firm, and consumers have to satisfy:

$$\frac{\partial v}{\partial \theta} = \frac{\partial Z^0}{\partial \theta} \frac{\partial \pi_1}{\partial \theta} = \frac{\partial Z^1}{\partial \theta} \frac{\partial cs}{\partial \theta} = \frac{\partial Z^{cs}}{\partial \theta}$$

when this condition is satisfied, $Z^j > 0$.

We consider a three-stage game. In the first stage, each special interest group offers campaign contribution schedule to policymaker. In the second stage, policymaker determines the privatization level. In the third stage, a private firm and a public firm compete in the Cournot market. The game is solved by working backwards.

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want to focus on the effect of lobbying activity, we can assume that the objective of all stakeholders and government are the same as the simplest case.

5 Note that policy maker cares about not $CS + V + \Pi_1$ but social surplus, $CS + \pi_0 + \Pi_1$. 
2.3 Optimal privatization policy and the lobbying activity in closed economy

In the third stage, both firms choose their outputs to maximize their revenue as in (1) and (2). In the second stage, the policymaker chooses the optimal privatization policy. In the first stage, each special interest group determines the contribution schedule.

2.3.1 Lobbying from all principals

Using a truthful equilibrium as the solution concept to our common-agency game as in Bernheim and Whinston (1986), we only have to find a maximization condition of

\[ W(\theta) + (\gamma^0 - 1)v(\theta) + (\gamma^1 - 1)\pi_1(\theta) + (\gamma^c s - 1)c s(\theta), \]

that is, the policymaker determines the optimal privatization policy to maximize (4) subject to (5). Then we can get

\[ (\gamma^0 - 1)\frac{dv(\theta)}{d\theta} + (\gamma^1 - 1)\frac{d\pi_1(\theta)}{d\theta} + (\gamma^c s - 1)\frac{dcs(\theta)}{d\theta} + \frac{dW(\theta)}{d\theta} = 0. \]

We assume \( \gamma \equiv \gamma^0 = \gamma^1 = \gamma^c s \). The result brings the Pareto inefficient allocation except for \( \gamma = 1 \) as follows;

\[
\begin{align*}
(\gamma - 1) & \left\{ \frac{dv(\theta)}{d\theta} + \frac{d\pi_1(\theta)}{d\theta} + \frac{dcs(\theta)}{d\theta} \right\} + \frac{dW(\theta)}{d\theta} \\
= (\gamma - 1) & \left\{ \frac{dv_0(\theta)}{d\theta} + (1 - \theta) \frac{dW(\theta)}{d\theta} + \frac{d\pi_1(\theta)}{d\theta} + \frac{dcs(\theta)}{d\theta} \right\} + \frac{dW(\theta)}{d\theta} \\
= (\gamma - 1) & (\theta - 1) \frac{dv_0(\theta)}{d\theta} + (1 + (\gamma - 1)(2 - \theta)) \frac{dW(\theta)}{d\theta} = 0
\end{align*}
\]

From above first order condition, competitive lobbying does not lead to optimal privatization policy, \( \theta^{all} \).

**Proposition 1**

Even if special interest groups all use compensating contribution schedules, the competing bid for influence does not result in a choice that is Pareto efficient among the set of feasible policies in a closed-economy and common-agent model, and is lower than optimal privatization rate.

This proposition 1 is in contrast to a previous result summarized in Dixit, Grossman and Helpman (1997) who consider markets organized by private firms and consumers. In their model, since the sum of the benefits of each special interest group is equal to social welfare, the policymaker behaves as if (s)he were benevolent:
competition between special interest groups offset influence each other, and lobbying from all special interest groups do not affect resource allocation.

However, in our case, if the policymaker sets the privatization rate to optimal level, \( \theta^* = 2/11 \), as lemma 1, then \( \frac{dG}{d\theta}\big|_{\theta=\theta^*} = (y - 1)(\theta^* - 1) \frac{d\pi_0(\theta)}{d\theta} = (y - 1) \frac{3036}{103823} > 0 \). Thus, we find that privatization rate in the case of lobbying activity from all special interest groups is lower than optimal privatization rate. The reason why lobbying activity from all special interest groups cannot lead to a Pareto efficient allocation is that the sum of the objective function of each special interest groups, \( \nu(\theta) + \pi_1(\theta) + cs(\theta) \), is not equal to social welfare function.

In particular, public firm wants to maximize not only their revenue as firm, \( \pi_0 \), but also social welfare, \( W(\theta) \), based on the share of \( \theta \) which politician determines. Since the public firm’s behaviour differs from pure oligopolistic firm, that is, \( \frac{dv^*}{d\theta} < 0 \), manager supplies the campaign contribution in order to decrease the privatization rate and to maximize their objective function itself, \( \nu \), and thus the sum of benefit of each special interest group is not equal to social welfare, the policy is not the same as in the case of a benevolent policymaker and set lower privatization rate.

2.3.2 Asymmetric lobbying in closed economy

Now, we then investigate whether there are possibilities to achieve optimal privatization policy by asymmetric lobbying. We consider the following cases: (i) lobbying from private firm and consumers. (ii) lobbying from public firm and consumers. (iii) lobbying from public and private firms (iv) lobbying from a public firm. (v) lobbying from private firm and (vi) the case of lobbying from consumers.

The political equilibrium can be characterized as solutions to maximizing the following functions for all cases;

(i) \( \theta_{p,cs} = argmax(y - 1)(\pi_1(\theta) + cs(\theta)) + W(\theta) \)

(ii) \( \theta_{p,cs} = argmax(y - 1)(\nu(\theta) + cs(\theta)) + W(\theta) \)

(iii) \( \theta_{p,v} = argmax(y - 1)(\pi_1(\theta) + \nu(\theta)) + W(\theta) \)

(iv) \( \theta_{v} = argmax(y - 1)\nu(\theta) + W(\theta) \)

(v) \( \theta_{p} = argmax(y - 1)\pi_1(\theta) + W(\theta) \)

(vi) \( \theta_{cs} = argmax(y - 1)cs(\theta) + W(\theta) \)
From first-order conditions of them, we get the orderings of privatization rate.

\[ \theta^{v,cs} < \theta^{ali} < \theta^{cs} < \theta^{v} < \theta^{p,cs} < \theta^{p,v} < \theta^{*} < \theta^{p} \]  

(7)

The results in (7), show that, in closed economy, competitive lobbying and asymmetric lobbying cannot lead to a Pareto efficient allocation. That is, political externality from lobbying activity is one of the cause of preventing Pareto optimal allocation in a closed economy⁶.

**Lemma 2**

Asymmetric lobbying in a closed-economy never lead to a Pareto efficient allocation in any case. The ordering of privatization rate becomes \( \theta^{v,cs} < \theta^{ali} < \theta^{v} < \theta^{cs} < \theta^{p,cs} < \theta^{p,v} < \theta^{*} < \theta^{p} \).

**Proof**

From (i) to (vi), we ascertain that the policymaker does not choose optimal privatization rate \( \theta^{*} \). To determine the ordering of privatization rate in (7), we compare the size of lobbying pressure. Firstly, downward pressure due to consumer’s lobby is larger than public firm’s pressure as in 

\[ \frac{dc_{s}^{*}}{d\theta} - \frac{dv^{*}}{d\theta} = \frac{71+2\theta(447+14\theta(33+10\theta))}{2(11+10\theta)^3} \]

and that private firm gives upward pressure for politician from (3). Thus, we find that \( \theta^{v,cs} < \theta^{v} < \theta^{p,v} < \theta^{*} < \theta^{p} \).

Secondly, the size of negative pressure from consumer and that from public firm are larger than the size of positive pressure from private firm as in 

\[ \frac{dc_{s}^{*}}{d\theta} + \frac{d\pi_{1}^{*}}{d\theta} = -\frac{12(7-4\theta)}{(11+10\theta)^2} < 0 \]  

and 

\[ \frac{dv^{*}}{d\theta} + \frac{d\pi_{1}^{*}}{d\theta} = -\frac{239+14\theta(57+66\theta+200\theta^2)}{2(11+10\theta)^3} < 0 \]  

respectively. This means \( \theta^{p,v} < \theta^{p,cs} \). Moreover, since 

\[ \frac{dc_{s}^{*}}{d\theta} + \frac{d\pi_{1}^{*}}{d\theta} - \frac{dv^{*}}{d\theta} = \frac{191+14\theta(91+66\theta+200\theta^2)}{2(11+10\theta)^3} \]

\( 0, \theta^{v} < \theta^{p,cs} \).

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⁶ If we consider the case of different political weight for campaign contribution, \( \gamma^{s} \neq \gamma^{0} \neq \gamma^{1} \), we can show a case where the policymaker chooses an optimal level of privatization by multiple lobbying as in Appendix A. This would be the case when different groups have different ability to access the policymaker, with the weights acting as shadow prices of lobbying (see Mazza and van Winden, 2008). However, the policymaker would have to adjust such weights optimally (by regulating access by lobbyists). It is then reasonable to exclude the possibility of different political weight for campaign contribution as in Grossman and Helpman (1994).
Finally, since \( \frac{dW^s}{d\theta} - \frac{dv}{d\theta} = \frac{dc_{i}^s}{d\theta} + \frac{d\pi_{i}^s}{d\theta} = -\frac{12(7-4\theta)}{(11+10\theta)^2} < 0 \) and \( \frac{dW}{d\theta} - \left( \frac{dc_{i}^s}{d\theta} + \frac{d\pi_{i}^s}{d\theta} \right) = \frac{d\pi_{i}^s}{d\theta} = \frac{6\theta(1+2\theta)}{(11+10\theta)^2} > 0 \), \( \theta_{cs}^v < \theta_{ci} < \theta_{cs} \).

From these results, we get the result of (7). ■

Lemma 2 shows the result that political pressure due to campaign contribution gives positive/negative effect on the privatization rate and these effects never cancel out each other in any case because these sizes of effects differ, \( \frac{dc_{i}^s}{d\theta} < \frac{d\pi_{i}^s}{d\theta} \). This means political lobbying are not justified in a closed-economy and mixed oligopoly model.

3. Multi principal and multi agent model

In this section, we ascertain whether the result of common agent model can be applicable in multi-principal and multi-agent model built by Aidt and Hwang (2008), or not. Firstly, we extend the closed-economy model to two-country model as in Han and Ogawa (2008) by using common agent model and show lobbying activity in each country cannot lead to optimal policy as in last section.

As in last section, we show that there are the possibilities of over or underprivatization due to lobbying activity. Extending closed economy to two-country model, we will check whether the effect of lobbying can offset or not.

3.1. Basic settings of two-country

Basic model is same as previous section except that there are two countries of two symmetric economies, country \( d \) and \( f \). In each country, there is a single public firm and a single private firm. Firms in each country produce homogenous goods and compete in a Cournot fashion in a single integrated market. The inverse demand function for the integrated market is given by \( p = 1 - 2(q_0^d + q_1^d + q_0^f + q_1^f) \), where \( p \) and \( q_j^i \) represents market price and the amount of goods sold by firm \( j \) in country \( i \), \( i = d, f \) and \( j = 0, 1 \).

As the same with the closed economy case, we assume that both firms in each country has identical cost function, \( c_j^i = \frac{1}{2}(q_j^i)^2 \), \( i = d, f \) and \( j = 0, 1 \), then revenue function of both firms in each country becomes \( \pi_j^i = \frac{1}{2} \left( 1 - 2(q_0^d + q_1^d + q_0^f + q_1^f) \right) q_j^i \), \( i = d, f \) and \( j = 0, 1 \). Note that the consumer surplus, \( CS_i \), becomes \( CS_i = (Q_i)^2 = 0.25Q^2 \) since we assume the identical two countries, and have \( Q^d = Q^f \). Thus, social welfare in country \( i \) is given by \( W_i = CS_i + \pi_0^i + \pi_1^i \).
Objective function of manager of the public firm in each country becomes the sum of the social welfare and producer surplus of the firms of its country: $V_i = \theta_i \pi_1^d + (1 - \theta_i) \pi_1^f$, where $\theta_i$ represents privatization ratio in country $i$. From revenue maximization of public and private firms, the outcomes of each firm in Nash equilibrium are

$$q_0^d = \frac{4 + \theta f - \theta d}{4(4 + \theta d + \theta f)}$$

$$q_0^f = \frac{4 + \theta d - \theta f}{4(4 + \theta d + \theta f)}$$

and

$$q_1^d = q_1^f = \frac{2 + \theta d + \theta f}{4(4 + \theta d + \theta f)}$$

Substituting (8) to the inverse demand function, the world price becomes

$$p_w^w = \frac{1}{2} - \frac{1}{4 + \theta d + \theta f}$$

Using (8) and (9), we obtain the equilibrium producer surplus (or revenue) of firms and consumer surplus in each country are

$$\pi_1^d = \pi_1^f = \frac{3(2 + \theta d + \theta f)^2}{32(4 + \theta d + \theta f)^2}$$

$$\pi_0^d = \frac{(4 + 3\theta f + 5\theta d)(4 + \theta f - \theta d)}{32(4 + \theta d + \theta f)^2}$$

$$\pi_0^f = \frac{(4 + 3\theta d + 5\theta f)(4 + \theta d - \theta f)}{32(4 + \theta d + \theta f)^2}$$

and

$$CS_d = CS_f = \frac{(6 + \theta d + \theta f)^2}{16(4 + \theta d + \theta f)^2}$$

Differentiating (8), (9), and (10) with respect to privatization ratio, we find the effect of privatization level on output of all firm, world price, producer surplus of all firm and consumer surplus in both countries are as follows,
\[
\frac{\partial q_0^i}{\partial \theta^i} = -\frac{4 + \theta^{-i}}{(4 + \theta^i + \theta^{-i})^2} < 0, \\
\frac{\partial q_0^{-i}}{\partial \theta^i} = \frac{\theta^{-i}}{2(4 + \theta^i + \theta^{-i})^2} > 0, \\
\frac{\partial q_1^i}{\partial \theta^i} = \frac{1}{2(4 + \theta^i + \theta^{-i})^2} > 0, \\
\frac{\partial p^w}{\partial \theta^i} = \frac{1}{(4 + \theta^i + \theta^{-i})^2} > 0, \\
\frac{\partial CS^i}{\partial \theta^i} = \frac{\partial CS^{-i}}{\partial \theta^i} = -\frac{6 + \theta^i + \theta^{-i}}{4(4 + \theta^i + \theta^{-i})^2} < 0, \\
\frac{\partial \pi_1^i}{\partial \theta^i} = \frac{8 - (\theta^{-i})^2 - 14\theta^i - \theta^{-i}(2 + 3\theta^i)}{8(4 + \theta^i + \theta^{-i})^3}, \\
\frac{\partial \pi_0^i}{\partial \theta^i} = \frac{3(\theta^{-i})^2 - (2 - \theta^i)\theta^{-i} + 2(4 + \theta^i)}{8(4 + \theta^i + \theta^{-i})^3} > 0, \\
\text{and} \\
\frac{\partial \pi_1^i}{\partial \theta^i} = \frac{\partial \pi_1^{-i}}{\partial \theta^i} = \frac{3(2 + \theta^i + \theta^{-i})}{8(4 + \theta^i + \theta^{-i})^3} > 0.
\]

where superscript \(-i\) means the country’s variables except country \(i\). From (11), the increase of the degree of domestic privatization decreases consumer surplus in each countries and increases foreign firms’ producer surplus.

By using (10), social welfare in each country becomes

\[
W^i = \frac{25 + \theta^i + \theta^{-i}(13 + 3\theta^i + 2\theta^{-i})}{8(4 + \theta^i + \theta^{-i})^2}
\]

The each benevolent policymaker maximize (12) by choosing \(\theta^i\) given by \(\theta^{-i}\), we can get the first-order condition

\[
\frac{dW^i}{d\theta^i} = \frac{13\theta^i + 3\theta^i\theta^{-i} + (\theta^{-i})^2 + \theta^{-i} - 2}{4 + \theta^i + \theta^{-i}} = 0.
\]
Solving this equation for $\theta_i$, the optimal reaction function about privatization policy of country $i$ becomes $\theta_i = \frac{1 - \theta^* - i(2 + \theta^*)}{13 + 3\theta^*}$. Using the symmetric assumption about two countries, we get the optimal privatization level in Nash equilibrium:

$$\theta_d = \theta_f = \frac{1}{4}(\sqrt{57} - 7).$$

**Lemma 3. Proposition 1 in Han and Ogawa (2008)**
The extent of privatization in the international mixed market with two countries is smaller than that in the mixed market with a single domestic market.

As mentioned above, privatization increases the producer surplus of the domestic public firm and foreign private firm and decreases the consumer surplus. Thus, domestic policymaker prevents to flow the domestic surplus to foreign producers. Actually, the coordinated problem gives the first-order condition:

$$\frac{dW^w}{d\theta^i} = \frac{dW^i}{d\theta^i} + \frac{dW^{-i}}{d\theta^{-i}} = \frac{(\theta^{-i})^2 - 5\theta^i - (1 + \theta^i)\theta^{-i} + 2}{4 + \theta^i + \theta^{-i}} = 0$$

By solving this, the optimal privatization levels in two-country model become

$$\theta_d = \theta_f = \theta^w = \frac{1}{3}.$$  

By comparing (13) and (14), we have the following Lemma 4.

**Lemma 4. Proposition 2 in Han and Ogawa (2008)**
When lobbying activity is prohibited, there is underprivatization in international mixed oligopoly with two countries.

### 3.2 Optimal privatization policy in two country, common agent model

In this section, we show the effect of domestic lobbying activity on privatization policy in a two-country setting of Grossman and Helpman (1994)’s common agent model. In order to be able to compare the result of a closed economy, basic setting is the same as in previous Section 2. This corresponds to the case with a ban of foreign lobbying.

As in section 2, the payoff of all principals in each country are,

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\(^7\) See (11).
\[ V^i = v^i - Z^{i,0}(\theta^i), \quad \Pi^i = \pi^i_1 - Z^{i,1}(\theta^i), \quad \text{and} \quad CS^i = cs^i - Z^{i,cs}(\theta^i) \]  

(15)

where \( Z^{i,j}(\theta^i) \) \((i = d, f, \text{and} \ j = 0, 1 \text{and} cs) \) represents the contribution schedule that private firms optimally choose, we put on the same assumption as in Sectior 2. The policymaker chooses the privatization level to maximize the weighted sum of social welfare, \( CS^i + \pi^i_1 + \Pi^i_1 \), and the amount of campaign contribution, \( Z^{i,0}(\theta^i) \), \( Z^{i,1}(\theta^i) \) and \( Z^{i,cs}(\theta^i) \);

\[
G^i(\theta^i) = CS^i(\theta^i) + \pi^i_1(\theta^i) + \Pi^i_1(\theta^i) + (y^i - 1) \sum_{j=0,1 \text{and} cs} Z^{i,j}(\theta^i)
\]

(16)

where \( y^i(> 1) \) denotes the weight that policymakers in each country for the political contributions from a domestic public firm, a domestic private firm, and domestic organized consumers. As in Section 2.3, the special interest group organized consumer, public firm, and private firm can do no better than to select a contribution schedule of the following form: \( Z^{i,0}(\theta) = \max\{0, v^i(\theta^i) - b^{i,0}\} \), \( Z^{i,1}(\theta^i) = \max\{0, \pi^i_1(\theta^i) - b^{i,1}\} \) and \( Z^{i,cs}(\theta^i) = \max\{0, cs^i(\theta^i) - b^{i,cs}\} \). Thus, we can get maximization condition of domestic lobbying agent in country \( i \) as:

\[
\frac{\partial v^i}{\partial \theta^i} = \frac{\partial Z^{i,0}}{\partial \theta^i}, \quad \frac{\partial \pi^i_1}{\partial \theta^i} = \frac{\partial Z^{i,1}}{\partial \theta^i} \quad \text{and} \quad \frac{\partial cs^i}{\partial \theta^i} = \frac{\partial Z^{i,cs}}{\partial \theta^i} \quad \text{(17)}
\]

when \( Z^{i,j} > 0 \) is satisfied.

Policymaker in each country maximizes the objective function (16) by choosing the privatization level subject to (17):

\[
\frac{d G^i}{d \theta^i} = \frac{d W^i}{d \theta^i} + (y^i - 1) \left( \frac{\partial cs^i}{\partial \theta^i} + \frac{\partial v^i}{\partial \theta^i} + \frac{\partial \pi^i_1}{\partial \theta^i} \right) = 0
\]

\[
= \frac{1}{32} \left( 5 - 5y^i \right) + \frac{8(5 + \theta^{-i}) + y^i[2 + \theta^{-i}(7 + \theta^{-i})]}{(4 + T + \theta^i)^2} \left( 4 + \theta^{-i} + \theta^i \right) = 0
\]

By solving this first-order condition for \( \theta^i \), reaction function of country \( i \) becomes \( \theta^{i,all} = \theta^{i,all}(\theta^{-i,all}, y^i) \). The political equilibrium can be characterized as solutions of the following functions for all cases about asymmetric lobbying;
From first-order conditions of them, we get the orderings of privatization rate.

\[ \theta^{i,p,cs} = \arg\max (\gamma^i - 1)\{\pi^i_1(\theta^i) + cs^i_1(\theta^i)\} + W^i(\theta^i) \]

(i)

\[ \theta^{i,cs} = \arg\max (\gamma^i - 1)\{\pi^i_1(\theta^i) + cs^i_1(\theta^i)\} + W^i(\theta^i) \]

(ii)

\[ \theta^{i,p,v} = \arg\max (\gamma^i - 1)\{\pi^i_1(\theta^i) + v^i(\theta^i)\} + W^i(\theta^i) \]

(iii)

\[ \theta^{i,p} = \arg\max (\gamma^i - 1)v^i(\theta^i) + W^i(\theta^i) \]

(iv)

\[ \theta^{i,v} = \arg\max (\gamma^i - 1)v^i(\theta^i) + W^i(\theta^i) \]

(v)

\[ \theta^{i,cs} = \arg\max (\gamma^i - 1)cs^i(\theta^i) + W^i(\theta^i) \]

(vi)

From first-order conditions of them, we get the orderings of privatization rate.

\[ \theta^{i,v,cs} < \theta^{i,all} < \theta^{i,cs} < \theta^{i,p} < \theta^{i,p,cs} < \theta^{i,p,v} < \theta^{w*} = \theta^{i,p} \quad (18) \]

From (18), we can get the following proposition 2\textsuperscript{8}.

**Proposition 2**

Even if special interest groups all use compensating contribution schedules, the competing bid for influence generally does not result in a choice that is Pareto efficient among the set of feasible policies in two-country, common-agent, and mixed oligopoly model.

This proposition shows that, in a two-country model, the lobbying activity also cannot achieve a Pareto efficient allocation as in a closed economy because the leader of the game is competitive special interest group. The difference for the case of closed economy is whether there exist international fiscal externalities. The international fiscal externalities give downward pressure to privatization rate. Thus, privatization rate for all cases decreases and the ordering does not change. This means that \( \theta^{i,p} \) may correspond to social optimal privatization rate \( \theta^{w*} \).

The reason why there exist the possibility \( \theta^{i,p} = \theta^{w*} \) for privatization rate may are canceled out depending on the strength of \( \gamma^i \). Actually, when private firm lobbies for domestic politician, privatization level in a two-country model becomes

\[ \theta^{i,p} = \frac{1}{4}\left(\sqrt{3}\sqrt{28 - 12y^i + 3(y^i)^2} + 3y^i - 10\right) > \theta^{w*} \quad (19) \]

\textsuperscript{8} See Appendix B.
Thus, Nash equilibrium privatization level is affected by the weight of campaign contribution, $\gamma^i > 1$. When policymaker has a strong interest for campaign contribution compared with the flow of domestic social surplus, policymaker increases privatization level to acquire the campaign contribution.

**Lemma 5. Proposition and Corollary 1 in Shinozaki, Mazza and Kunizaki (2017)**

When private firms can lobby to policymaker and policymakers’ interest is stronger (weaker) than 25/18, there is under-(over-)privatization in international mixed oligopoly with two countries. Nash equilibrium privatization level in a two-country model corresponds to the social optimum level when the politicians’ interest is 25/18.

*Proof of Proposition 2*

By solving (20) to $\gamma^i > 0$, we have $\gamma^i = 25/18$. ■

### 3.3. Optimal privatization policy in two-country, multi-principal, multi-agent model

Last section shows that international fiscal externalities cannot modify the resource allocation although there is the possibility to alleviate lobbying effect. In this section, we check the effect of international lobbying for multiple agent, which means each agent can lobby not only own government but also foreign government, on the privatization rate as in Aidt and Hwang (2008, 2014).

Aidt and Hwang (2014) shows, when all special interest groups are organized and all politicians are equally corrupt, that is $\gamma \equiv \gamma^d = \gamma^f > 0$, the political externalities due to equilibrium policy with foreign lobbying internalizes all cross national externalities which correspond to international fiscal externalities, and is equal to the policy vector that maximizes global social welfare.

In preceding section, we showed that international lobbying in common agency model cannot internalize international fiscal externalities in general because each country’s government does not care about other country’s welfare. In this subsection, we check whether Aidt and Hwang (2014)’s result can adopt by using international lobbies from all special interest groups or not.

In order to compare the result of Aidt and Hwang (2008, 2014), we put on the same assumption of their paper;

**Assumption:**  *international lobbying in multi-principal and multi-agent model*

(i) Three trade unions, the international union of public firms, private firms, and consumer, are organized as an international lobbying group.

(ii) The unions can lobby not only their own country but also the foreign government, directly.
Following Aidt and Hwang (2008), we introduce the international organized trade union they can lobby. Letting \( \hat{\nu}(\theta^d, \theta^f) = \nu^d(\theta^d, \theta^f) + \nu^f(\theta^f, \theta^d) \), \( \hat{\pi}^i(\theta^d, \theta^f) = \pi^i(\theta^d, \theta^f) + \pi^i(\theta^f, \theta^d) \) and \( \hat{c}_s(\theta^d, \theta^f) = c_s^d(\theta^d, \theta^f) + c_s^f(\theta^f, \theta^d) \), the hat means the objective function of each international organized union. The payoff of international union of a public firm, a private firm, and consumers each principal are,

\[
V^w = \hat{\nu} - Z^d(\theta^d, \theta^f) - Z^f(\theta^f, \theta^d),
\]

\[
\Pi^w_i = \hat{\pi}^ i - Z^{d,i}(\theta^d, \theta^f) - Z^{f,i}(\theta^f, \theta^d),
\]

and

\[
C_S^W = \hat{c}_s - Z^{d,cs}(\theta^d, \theta^f) - Z^{f,cs}(\theta^f, \theta^d)
\]

Focusing on truthful equilibrium, the political equilibrium with international lobbying can be characterized as a solution to maximizing the following function:

\[
G^{\text{MI}}(\theta^i, \theta^{-i}) = W^i(\theta^i, \theta^{-i}) + W^{-i}(\theta^i, \theta^{-i}) + (\gamma - 1)\{\hat{c}_s(\theta^d, \theta^f) + \pi^{dW}(\theta^d, \theta^f) + \hat{\nu}(\theta^d, \theta^f)\}
\]

\[
= 1 + (\gamma - 1)(2 - \theta^i)\{W^i(\theta^i, \theta^{-i}) + W^{-i}(\theta^i, \theta^{-i})\}
\]

\[
+ (\theta^i - 1)(\gamma - 1)\{\pi^{dW}(\theta^i, \theta^{-i}) + \pi^{-1}(\theta^i, \theta^{-i})\}
\]

If \( \theta^i = 1 \), (21) becomes \( G^{\text{MI}}(\theta^i, \theta^{-i}) = \gamma(W^i + W^{-i}) \). However, fully privatization policy does not corresponds to optimal (coordinated) privatization policy \( \theta^w = \frac{1}{3} \) from (14). Thus, we find international lobbying activity in multi-agent model cannot internalize international externalities. This result also contrasts to Aidt and Hwang (2014).

**Proposition 3**

Even if international special interest groups all use compensating contribution schedules, the competing bid for influence does not result in a choice that is Pareto efficient among the set of feasible policies in two-country, multi-agent, and mixed oligopoly model.

The reason of Proposition 3 is just the same as the case of a closed-economy. That is, as a result of this game, the sum of objective function of each international special
interest group differs from the sum of objective function of a politician of two countries in contrast to Aidt and Hwang (2014), although political externalities can internalize by international fiscal externalities.

Now let’s turn to examine whether policymaker in each country can internalize by asymmetric international lobbying in Nash equilibrium. As in proceeding section, the political equilibrium can be characterized as solutions to maximizing the following functions for all cases;

\[(i) \quad \theta_{M,p,cs} = \text{argmax}_{\gamma - 1}{\pi^W_1(\theta^d, \theta^f) + \bar{c}_s(\theta^d, \theta^f)} + W^i(\theta^i, \theta^-)\]

\[(i) \quad \theta_{M,v,cs} = \text{argmax}_{\gamma - 1}{\bar{v}(\theta^d, \theta^f) + cs^i(\theta^i)} + W^i(\theta^i, \theta^-)\]

\[(i) \quad \theta_{M,p,v} = \text{argmax}_{\gamma - 1}{\pi^V_1(\theta^i) + \bar{v}(\theta^d, \theta^f)} + W^i(\theta^i, \theta^-)\]

\[(i) \quad \theta_{M,v} = \text{argmax}_{\gamma - 1}{\bar{v}(\theta^d, \theta^f) + W^i(\theta^i, \theta^-) + W^{-i}(\theta^- i, \theta^i)}\]

\[(i) \quad \theta_{M,p} = \text{argmax}_{\gamma - 1}{\pi^W_1(\theta^d, \theta^f) + W^i(\theta^i, \theta^-) + W^{-i}(\theta^- i, \theta^i)}\]

\[(i) \quad \theta_{M,c} = \text{argmax}_{\gamma - 1}{\bar{c}_s(\theta^d, \theta^f) + W^i(\theta^i, \theta^-) + W^{-i}(\theta^- i, \theta^i)}\]

From first-order conditions of them, we get the orderings of privatization rate.

\[
\theta_{M,v,cs} < \theta_{M,r,all} < \theta_{M,c} < \theta_{M,p,cs} < \theta_{M,p,v} < \theta_{W^i} < \theta_{M,p} \tag{22}
\]

From (22), we can show that, even though we consider international lobbying environment in multiple agent model, it can not achive the Pareto efficient allocation for \(\gamma > 1\) in mixed oligopoly model\(^9\).

**Lemma 6**

Asymmetric international lobbying cannot achieve a Pareto efficient allocation and the ordering of optimal privatization rate is just same as in a closed economy and a two-country, common agent model.

The reason why socially optimal privatization policy does not be replicated as in lemma 6 is that the formation of international trade union and lobbying to domestic and foreign politician does not modify political externalities although its can partly internalize international fiscal externalities.

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\(^9\) See Appendix B.
4. Summary

We discussed about the effect of lobbying on the degree of optimal privatization ratio and on the social surplus in a closed-country, and a two-country mixed oligopoly model. We found that in general, lobbying activity does not lead to a Pareto efficient allocation by optimal privatization policy.

This result is sharply contrast to the previous result. Grossman and Helpman (2001, section 8.6) which is the one of the famous contribution of this region summarizes “if the SIGs all use compensating contribution schedules,” “then the competing bids for influence must result in a choice that is Pareto efficient among the set of feasible policies.”. However, our results shows, in a mixed oligopoly market, a government has to prohibit the domestic and foreign lobbying by a law.

Note that our analysis focuses on the normative aspects. In order to apply this result to economic policies, we have to consider the effect of voting, legislature, trade policies, and the endogenous number of private firm. Moreover, we can consider other type of asymmetric lobby. For example, although only special interest group of consumers can lobby in one country, all special interest groups may lobby in other country by the difference of each country’s law, institutions and culture.

Appendix A.

We show that if politician has different political weight for campaign contribution, there is a possibility to choose social optimal privatization rate. Actually, from first order condition , \((y^0 - 1) \frac{dv(\theta)}{d\theta} + (y^1 - 1) \frac{d\pi_1(\theta)}{d\theta} + (y^{cs} - 1) \frac{dcs(\theta)}{d\theta} + \frac{dW(\theta)}{d\theta} = 0\), if \((y^0 - 1) \frac{dv(\theta)}{d\theta} + (y^1 - 1) \frac{d\pi_1(\theta)}{d\theta} + (y^{cs} - 1) \frac{dcs(\theta)}{d\theta} = 0\) is satisfied, social welfare is maximized. Thus, if someone adjust the value of \(y^0, y^1, \) and \(y^{cs}\), Pareto optimal allocation can be achieved.

Appendix B.

Proof of (18)

From (i) to (vi), we understand politician does not choose optimal privatization rate \(\theta^W^*\) as in a two-country economy. We show the ordering of privatization rate is same as a closed economy by same procedure. First is that downward pressure due to consumer’s lobby is larger than public firm’s pressure as in \(\frac{d\pi_1(\theta)}{d\theta} > \frac{dv(\theta)}{d\theta}\)
\[ \frac{1}{32} \left(5 + \frac{\phi(1+i^{-1})}{(4+i^{-1})^{-1}} - \frac{\phi(2+i^{-1})(7+i^{-1})}{(4+i^{-1})^{-1}}\right) > 0, \] and that private firm gives upward for politician as in (11). Thus, we find that \( \theta^{i,cs} < \theta^{i,ts} < \theta^{i,v} < \theta^{W,v} < \theta^{i,p} \).

Second is that the size of negative pressure from consumer and of negative pressure from public firm are larger than the of size of positive pressure from private firm as in \( \frac{dcSi}{d\theta^i} + \frac{dcW^i}{d\theta^i} = \frac{\theta^i+\theta^i-6}{u(4+i^{-1})} < 0 \) and \( \frac{dcI}{d\theta^i} + \frac{dcL}{d\theta^i} = -\frac{1}{32} \left(5 - \frac{\phi(2+i^{-1})(7+i^{-1})}{(4+i^{-1})^{-1}}\right) < 0 \). This means \( \theta^{I,p,cs} < \theta^{I,p,v} \). Moreover, since \( \frac{dcSi}{d\theta^i} + \frac{dcW^i}{d\theta^i} = \frac{1}{32} \left(\phi(2+i^{-1})(7+i^{-1}) + 4(5+i^{-1})\right) > 0, \theta^{i,v} < \theta^{I,p,cs} \).

Finally, since \( \frac{dcW^i}{d\theta^i} = \frac{dcI}{d\theta^i} + \frac{dcL}{d\theta^i} = \frac{3(\theta^i+\theta^i-6)}{u(4+i^{-1})} > 0 \) \( \theta^{i,v,cs} < \theta^{I,a,ts} < \theta^{i,cs} \). Summarizing these results, we get the result of (18).

**Proof of (22)**

From (i) to (vi), we understand that optimally privatization rate \( \theta^{W,v} \) as in a closed and a two-country, multi-agent model.

First is that downward pressure due to consumer’s lobby is larger than public firm’s pressure as in \( \frac{dcS}{d\theta^i} + \frac{dcL}{d\theta^i} = \frac{\theta^i+\theta^i-6}{u(4+i^{-1})} > 0, \) and that private firm gives upward for politician, \( \frac{dcI}{d\theta^i} + \frac{dcL}{d\theta^i} = \frac{3(\theta^i+\theta^i-6)}{2(4+i^{-1})} > 0 \). Thus, we find that \( \theta^{M,v,cs} < \theta^{M,v,ts} < \theta^{W,v} < \theta^{M,p} \).

Second is that the size of negative pressure from consumer and of negative pressure from public firm are larger than the of size of positive pressure from private firm as in \( \frac{dcS}{d\theta^i} + \frac{dcL}{d\theta^i} = \frac{\theta^i+\theta^i-6}{u(4+i^{-1})} < 0 \) and \( \frac{dcI}{d\theta^i} + \frac{dcL}{d\theta^i} = \frac{3(\theta^i+\theta^i-6)}{2(4+i^{-1})} > 0 \). This means \( \theta^{M,p,cs} < \theta^{M,p,v} \). Moreover, since \( \frac{dcS}{d\theta^i} + \frac{dcL}{d\theta^i} = \frac{\theta^i+\theta^i-6}{u(4+i^{-1})} < 0 \) \( \theta^{M,v,cs} < \theta^{M,v,ts} < \theta^{M,v} \).

Finally, since \( \frac{dcW^i}{d\theta^i} = \frac{dcI}{d\theta^i} + \frac{dcL}{d\theta^i} = \frac{3(\theta^i+\theta^i-6)}{u(4+i^{-1})} > 0 \) \( \theta^{M,v,cs} < \theta^{M,a,ts} < \theta^{M,v} \).

From these results we get the result of (22).
References