Contagion of Populist Extremism*

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Abstract
The objective of this study is to explore the propagation of populist extremism across countries.
To this end, we construct a multi-country model in which each country’s politician sequentially
implements a policy. Voters face information asymmetries about the incumbent politician’s type
(congruent type vs. non-congruent type) as well as the state of the world. We show that populist
extremism, wherein extreme policies are supported by voters, arises when the public opinion is
sufficiently radical. Moreover, populist extremism is contagious across countries, at least in the
short-run. Whether this effect stops in the long-run depends on the correlation of the state of the
world across countries. We show that extremism eventually disappears under the perfect correla-
tion, while either the convergence towards extremism or cycles of extremism hold when the state
of the world follows a Markov process without absorbing states. Our model is also applicable to
any agency problems not limited to political phenomena.

Keywords: Populism; Agency problems; Social learning; Yardstick competition; Signaling

JEL classification: D72; D83; H73

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Let us stop the domino effect right this week, this Wednesday.  
The domino effect of the wrong sort of populism winning in this world.  
Mark Rutte, the Dutch prime minister (March 13, 2017)

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1 Introduction

In representative democracy, voters delegate decisions to elected politicians. Ideally, the politicians choose appropriate policies for voters based on their expertise and information. However, in practice, such an ideal situation is hard to be expected because politicians are, at least to some extent, self-interested. In particular, to acquire high reputation for re-election or post-political life, politicians may pursue undesirable policies even if they know it is sub-optimal (Ashworth 2012). Hence, the reputation concerns may induce undesirable but popular extremism when it is strongly supported by voters for some reasons, such as to avoid electing corrupt leaders (e.g., Acemoglu, Egorov, and Sonin 2013). Such “populist extremism” arises as one of the symptoms of dysfunctional democracy.

The key implication we present in this paper is that such dysfunction of democracy may spread like falling dominoes across countries. In particular, populist extremism in one country may induce it in another country, leading to the “domino effect”. If this is the case, malfunctions of democracy in one country endanger democracy in other countries. This is a real concern given the massive concerns over the propagation of populist extremism in real-world politics and academic literature (Kaltwasser 2015). For instance, in Latin American politics, scholars often argue that there are waves of populism (e.g., neoliberal populism in the 1990s and left-wing populism in the 2000s). Furthermore, even in the current Europe, following the presidential election of the United States and the national referendum on Brexit, Dutch prime minister Mark Rutte expressed such concern, as our epigraph indicates. The central objective of the present study is to investigate whether the dysfunction of democracy — the emergence of populist extremism — spreads across countries or not, and if so, how—which will be analyzed by constructing a novel model that examines political agency problems and voters’ social learning. To our knowledge, this is the first attempt to analyze the diffusion process of malfunctioning democracy in the form of populist extremism.

Our analysis emphasizes the role of inter-country information propagation in explaining the domino effect. Traditional modes of spreading information were confined to local influence; it was extremely difficult for ordinary people to access to information about remote areas. In today’s world, however,
the world is interconnected more closely than any other period in the history. As a result, people in one area are now able to access rich information about other areas and utilize this knowledge in their political choices. As empirically demonstrated by Pachenco (2012) in the United States, the public opinion is influenced by neighboring states’ policies through voters’ social learning, which in turn affects policy outcomes. Even in the international context, Kayser and Peress (2012) show that, at least in developed countries, voters utilize information about foreign countries in evaluating the performance of their government. These arguments make it indispensable to analyze the interaction of voters’ social learning across borders and its political consequences.

At a first glance, the access to rich information seems to improve the democratic accountability. In particular, it has been pointed out that information externality mitigates political agency problems. For instance, the literature on yardstick competition (e.g., Besley and Case 1995) has shown that access to rich information from neighboring jurisdictions enables voters to partially resolve information asymmetries between themselves and politicians. This is because voters can use the behaviors of other jurisdictions’ politicians as a benchmark. Hence, populist extremism, a phenomenon induced by severe political agency problems, might be prevented by information propagation. However, this is not the case. In contrast to such an optimistic view, we show that voters’ benchmarking behavior caused by information propagation distorts voters’ learning and leads to the proliferation of populist extremism.

To analyze populist extremism and its patterns of spreading, we construct a political agency model that extends Acemoglu, Egorov, and Sonin’s (2013) model of populism. We begin our analysis with the single-country model. In this country, there are two types of politicians: the congruent type who shares the same policy preference with voters and the non-congruent type who has a biased policy preference. Voters do not know the incumbent politician’s type, and the incumbent has reputation concerns. In addition, there is information asymmetry about the state of the world, leading voters to be uncertain about the optimal policy. In this setting, we show that populist extremism could arise in the presence of high reputation concerns. In this equilibrium, the congruent type politician argues for a radical policy that the non-congruent type never chooses to signal that s/he is the good politician. Given this signaling role of the radical policy, voters support the politician arguing for such policy even if they know that the radical policy could be undesirable. In line with Acemoglu, Egorov, and Sonin (2013), we interpret this situation as populism because an undesirably extreme policy is strongly supported by voters. Interestingly, the emergence of such extremism depends on voters’ belief about the state of the world. In particular, we show that extremism arises if and only if the voters’ subjective probability of the optimal policy being radical exceeds a certain threshold. In other words, the more radical public opinion induces extremism more likely. Importantly, this threshold value could be less than a half, implying that extremism arises even when the radical policy is unlikely to be optimal.

Subsequent empirical studies include Hansen, Olsen, and Bech (2015) and Aytac (2018). Consistent with our idea, Kaltwasser (2015) also points out a possibility that the diffusion of populism could rely on learning by voters about neighborhood countries by referring to “demonstration effect.” At least conceptually, it could be the case that populism does not entail extremism. However, in reality, we often observe the strong connection between them and our model indeed shows such a connection (in particular, we show that extremism arises as a symptom of anti-elitism, which is the core of populism).
We then extend the model to a multi-country setting to analyze the domino effect of populist extremism. More specifically, we analyze the model in which the incumbent’s policy choice and the election are sequential across countries. This structure allows voters to learn the optimal policy through the policies previously implemented in other countries. Our goal is to illuminate the dynamic interaction between policies implemented by politicians and voters’ opinions, which triggers the proliferation of extremism. To see this point, suppose that in one country, extremism arises, leading the incumbent politician to argue for the radical policy. Since some fraction of the congruent type exhibits low reputation concerns and sincerely supports the optimal policy, the next country’s voters cannot rule out the possibility that the radical policy was implemented because it is indeed the optimal policy. Hence, voters upwardly update the probability of the radical policy being good. This radicalization of public opinion in turn induces extremism in the next country as seen in the single-country model. As such, policies and public opinions are distorted by each other through information propagation. Although the failure of social learning has been known in the literature of information cascade (e.g., Banerjee 1992; Bikhchandani, Hirshleifer, and Welch 1992), the novelty of our paper is to reveal that political agency problems create this failure. Given today’s rapid information spread, we believe that this story is of practical relevance and serves as an explanation for the contagion of populist extremism.

Given that populist extremism is contagious, a natural question is whether its spread comes to an end. Our analyses indicate that the result crucially depends on the correlation of the optimal policy across countries. We first consider the case in which the optimal policy remains the same across countries. In this case, we show that, in the long-run, the spread must come to an end. Although the spread of extremism is likely to be detrimental even in the short-run, this result indicates that the worst scenario—the permanent propagation—is rejected. We also demonstrate that the domino effect of extremism may suddenly end. In certain situations, a moderate policy in one country is always sufficient to stop the domino effect independently of past histories. This is surprising because, in the standard learning models, a single bounded signal is not necessarily sufficient to change belief significantly. These results imply that, at least in the long-run, populist extremism is not a severe problem when the state of the world is invariant across countries.

Unfortunately, this is not the case when each country’s optimal policy is only imperfectly correlated. In reality, elections are held only occasionally and the optimal policy may change between elections. Different countries also have different backgrounds, which again makes the optimal policy vary across countries. We introduce the imperfect correlation by assuming that the optimal policies follow a Markov process without absorbing states. In this case, unlike the case of perfect correlation, populist extremism never comes to an end. We show that, in the long-run, either the convergence to the extremism equilibrium or cycles of extremism always occur. Overall, it is more difficult to stop the domino effect of populist extremism under the conditions of imperfect correlation.

The intuitive explanation as to why populist extremism is prolonged in a changing world is as follows. Since the radical policy becomes the optimal policy with a positive probability, this policy could be implemented even without extremist sentiment. Furthermore, the implementation of the radical policy makes the public opinion of the next country more radical, leading to extremism in the next
country. Hence, even if extremism does not arise in one country, it could reemerge in subsequent countries, generating cycles of extremism. To make matters worse, deviation from extremism is difficult in a changing world. Suppose that countries cannot escape from extremism once captured unless the public opinion is highly moderate (i.e., suppose that extremism is likely to arise). To escape from extremism in this situation, the country’s voters must believe that the moderate policy is optimal with sufficiently high probability. However, even after observing the moderate policy in previous countries, voters in subsequent countries do not significantly update beliefs because the optimal policy might change from the previous one. This negatively impacts the learning process and countries can never escape from extremism. As such, the changing world situation drastically alters the long-term properties of populist extremism.

Although we frame our model in the context of populist extremism throughout this paper, the model is structured generally and can be applied to other settings that include agency problems. Beyond the analysis of extremism, our results indicate that the dynamic interactions between the principals’ opinions and agents’ actions catalyze the propagation of distorted policies. We briefly discuss other applications in Section 7.2.

The remainder of the paper is organized as follows. Section 2 discusses related literature and clarifies our contributions. Section 3 presents our model in a single-country setting and Section 4 analyzes it. In Section 5, the model is extended to a multi-country setting. Section 6 extends the multi-country model to the situation in which the optimal policies are imperfectly correlated. Section 7 provides several extensions and discusses other applications. Section 8 outlines our conclusions.

2 Related Literature

This study is related to several strands of literature in terms of both political economics and economics of social learning.

Political agency problems and reputation concerns: Politicians have an incentive to maintain high reputation. Although such reputation concerns may seem to improve voters’ welfare, it is not necessarily the case. Reputation concerns sometimes force congruent politicians to argue for inefficient policies: congruent politicians pander to public opinion and implement bad policies (e.g., Canes-Wrone, Herron and Shotts 2001; Maskin and Tirole 2004; Smart and Sturm 2013). Such bad reputation effect arises in various pricial-agent relationships including non-political issues (Ely and Välimäki 2003).

In more striking cases, to attract support, politicians might implement a sub-optimal policy even if it is perceived to be bad by voters. Acemoglu, Egorov, and Sonin (2013) indeed show this possibility: the congruent politician chooses an extreme policy that is known to be bad to signal that s/he is a good politician. Their mechanism provides a promising explanation for populism in which extremism arises in various pricial-agent relationships including non-political issues (Ely and Välimäki 2003).

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with strong support by voters arises.

Although Acemoglu, Egorov, and Sonin (2013)’s work provides a novel insight on populist extremism, their model has no uncertainty about the voter-optimal policy so that their is no connection between public opinion and pandering. By introducing uncertainty about the state of the world into the model based upon their work, we succeed in connecting the possibility of extremism and public opinion. This allows us to uncover the contagion of extremism through the dynamics of public opinion.

■ Policy diffusion and learning: The spread of populist extremism can be considered as policy diffusion. Furthermore, in our model, voters learn the state of the world based on other countries’ policies. Hence, our study is related to the literature of policy diffusion through learning (see Gilardi and Wasserfallen (2018) for the literature review). Notably, in this study, what is learned and by whom are different from most existing theoretical studies. In previous studies, the government learns the outcome of policies through other countries’ experiences (e.g., Volden, Ting, and Carpenter 2008; Callander and Harstad 2015). On the contrary, in our model, politicians know the state of the world, and instead, voters learn it. Furthermore, voters face multidimensional uncertainty: they learn two factors – the state of the world and the incumbent politician’s type – simultaneously.

The idea of the diffusion of political regimes has been widely known, as exemplified by President Dwight D. Eisenhower’s famous speech in 1954 mentioning the “domino theory” of communism. Citizens’ learning has not been theoretically investigated until recently, but Chen and Suen (2016) propose a model in which citizens learn the state through observing regime shifts in other countries and examine how this process leads to the spread of regime paradigm shifts. While their model is primarily concerned with collective action problems towards regime choice, we analyze the contagion of populist extremism across established democratic systems, focusing on political agency problems.

■ Yardstick competition: Voters’ learning from other countries has been partially investigated in the literature of yardstick competition (e.g., Besley and Case 1995; Belleflamme and Hindriks 2005). In yardstick competition models, voters observe the policies of other countries. However, governments decide policies only once, so there is no sequential learning and associated dynamics, which are essential to analyzing the domino effect.

The study by Hugh-Jones (2009) is also related. He analyzes yardstick competition in which the populism include Frisell (2009), Jennings (2011), Karakas and Mitra (2017), and Buisseret and Van Weelden (2018). None of them analyze the diffusion process of populism.

To analyze cheap talk messages in an election, Kartik and Van Weelden (2019) consider the model in which voters face uncertainty about the state of the world that follows a continuous distribution. However, as a result of the focus difference, voters’ learning about the state of the world plays no role and the uncertainty is introduced to create the imperfection of the signal on whether the incumbent implemented a good policy. Consequently, their model does not reveal a clear relationship between the possibility of extremism and voters’ beliefs about the state of the world.

In the literature of preliminary elections, voters’ learning and associated information cascades are often discussed (e.g., Callander 2007). However, in those models, politicians’ policy choice is exogenous; voters learn something through other voters’ previous voting strategies (i.e., the results of previous elections). Hence, the structure is different from ours.

The domino theory is not limited to communism. See Leeson and Dean (2009) and Leeson, Sobel, and Dean (2012) for its application to democracy and capitalism, respectively.
stage game—wherein each government simultaneously decides the policy—is repeated twice. Hence, his model incorporates dynamics of policies after voters’ social learning. However, due to the difference in the focus, his model does not include extremism or sequential political decisions so that it is not suitable to analyze the domino effect.

**Social learning:** From a broader perspective, our study makes three contributions to the literature of social learning. First, our model provides a new framework that captures an important aspect of observational learning under the principal-agent relationship. In our model, the agents knowing the state of the world choose policies and the principals learn the state of the world by observing the past policies. Hence, players who take actions perfectly know the state. Nonetheless, learning does not work well in the short term. This contrasts the existing studies in which learning does not work because in these models, players who take actions face uncertainty about the state of the world. Studies analyzing reputation concerns are also related (e.g., Scharfstein and Stein 1990; Ottaviani and Sørensen 2001), but even in these studies, players who take actions face uncertainty about the state of the world. As a result, the players have concerns about arguing for a policy that may be wrong and undermining their reputations, thus creating herding. Hence, the structure of these models is different from ours.

Second, we show that the extremism domino effect suddenly stops due to the discontinuous jump of voters’ beliefs. In the standard models, such paradigm shifts arise only when signals are unbounded (Smith and Sørensen 2000). In spite of bounded signals, our model shows that strategic interactions create paradigm shifts through the endogenous change in signal strength. This is another novelty of our study. In a one-person learning model, Ortoleva (2012) shows that paradigm shifts occur when people follow non-Bayesian updating, called “hypothesis testing”. Chen and Suen (2016) consider a social learning model in which each stage game is described as a global game. Then, they show that, when players face a model uncertainty, a paradigm shift occurs even if they follow Bayesian updating. In contrast to these studies, we implement neither non-Bayesian updating nor model uncertainty.

Lastly, we also analyze social learning when the state of the world is changing in Markovian manner. Moscarini, Ottaviani, and Smith (1998) and Nelson (2002) extend the canonical social learning model in this direction. Our results contrast those from previous studies. They show that the more likely the state of the world is to change, the shorter that the herding period is sustained. In contrast, we find the convergence to extremism in the Markovian environment, while it never occurs when the state of the world is constant. This result highlights the role of a changing world in social learning under a principal-agent framework.

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13 See Chamley (2004) for the background of social learning studies. Our model includes heterogeneous types of politicians. Hence, our study is also related to the literature of social learning under heterogeneous types of players. Smith and Sørensen (2000) analyze the case in which players have opposite preferences and Goeree, Palfrey, and Rogers (2006) explore the case in which a player’s payoff is partially dependent on a private shock. In addition, Wu (2015) analyzes heterogeneity in terms of signal accuracy.

14 By extending their models, Peck and Yang (2011) also analyze strategic delay and the associated information cascade.
3 Model

There are $N \in \mathbb{N}$ countries ($i = 1, ..., N$). For each country, there is an incumbent politician and a representative voter.\(^\text{15}\) Hereafter, we call the incumbent politician (the representative voter) in country $i$ politician $i$ (voter $i$). Our model is applicable to any principal-agent scenario. In other applications, the incumbent politician corresponds to the agent (or equivalently the expert) and the voter corresponds to the principal. We consider a model wherein each incumbent politician sequentially chooses a policy. At the beginning of period $i$, politician $i$ chooses policy $x_i$ from the set of available policies $X = \{0, 1, 2\}$ given the policies implemented before period $i$ by the other countries ($x_1, ..., x_{i-1}$) $\in X^{i-1}$. Then, voter $i$ evaluates politician $i$ given the policies implemented before in other countries and the policy implemented by politician $i$ ($x_1, ..., x_i$) $\in X^i$. This evaluation is denoted by $\pi_i$. The precise definition of strategies when $N \geq 2$ will be given in Section 5.

3.1 State of the World and Policies

Let $w_i$ be the state of the world in country $i$, which indicates the optimal policy for voters. It is denoted by either 1 or 2 i.e., $w_i \in \Omega \equiv \{1, 2\}$. Politician $i$ knows the value of $w_i$, while voter $i$ does not know its value. When $w_i = k$, the policy optimal for voter $i$ is $k$. Hence, policy 0 is never desirable for voters, whereas the other two policies can be appropriate. As seen in the next subsection, we consider single-peaked preferences so that policy 1 is close to policy 0, while policy 2 is the opposite of policy 0. Hence, we refer to policy 0 as the non-congruent policy, policy 1 as the moderate policy, and policy 2 as the radical policy. Note that some politicians and voters always have different tastes. Such a situation naturally arises when politicians come from special interest groups such as economic elites. For example, Acemoglu, Egorov, and Sonin (2013) consider a situation in which elites seek different policies than the average voter and elite groups might gain influence over a politician through bribery. We extend this situation so that the average voters’ ideal point may change depending on economic and social conditions, but voters always deem that what elites demand is undesirable.

As an illustration, suppose that taxation is the issue at the center of politics. The voter-optimal tax rate is low (i.e., policy 2) when the distortion of taxation is high (i.e., $w = 2$), whereas that is moderate (i.e., policy 1) when the distortion is low (i.e., $w = 1$). On the contrary, some politicians’ objective is to maximize the tax revenue for rent-seeking so that their optimal tax rate is quite high (i.e., policy 0) independently of the distortion of taxation. Our setting captures this situation. Note that by using an analogous setting, Besley and Smart (2007) analyze a fiscal policy under an agency problem.\(^\text{16}\)

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\(^{15}\) Voters’ heterogeneity is allowed for as long as the median voter theorem holds.

\(^{16}\) To be precise, they assume that the tax rate is a continuous variable. However, only three alternatives (the voter-optimal tax rate when the distortion is low, that when the distortion is high, and the revenue-maximizing tax rate) matter for the analysis so that the policy set essentially consists of three policies.
3.2 Politicians’ Types

There are two types of politicians: the congruent type and the non-congruent type. Voter $i$ does not know the type of politician $i$, meaning that information asymmetry about politicians’ types exists in addition to information asymmetry about the state of the world.

The payoff of the congruent type in country $i$ is given by

$$-L(|x_i - \omega_i|) + b_i V(p_i(x_i)),$$

where $L$ is the loss due to policy mismatch, $\pi_i(x_i)$ is the updated belief voter $i$ holds at the end of period $i$ about the probability of the incumbent $i$ being the congruent type given the implemented policy $x_i$, and $V : [0, 1] \mapsto [0, 1]$. The congruent type shares the same policy preference as the voter (i.e., the ideal policy is $\omega_i$). In this regard, this type of the politician is a good politician. Note that $L$ is assumed to be strictly increasing because we consider single-peaked preferences. As for normalization, $L(0) = 0$ and $L(1) = 1$. Furthermore, $L(2) = l > 1$, meaning that the loss due to policy mismatch is strictly convex.

On the contrary, the payoff of the non-congruent type in country $i$ is given by

$$-L(|x_i|) + b_i V(p_i(x_i)).$$

Hence, the ideal policy for the non-congruent type is policy 0 independently of the state of the world. Since policy 0 is never desirable for the voter, this politician is a bad type.

Politician $i$’s reputation is severely undermined if their policymaking makes voter $i$ believe that politician $i$ is likely to be the non-congruent type and prefer policies that are undesirable to voters. The low reputation might damage the quality of the post-political life or the incumbent’s soft legacy (Fong, Malhotra, and Margalit 2017). In addition, when the incumbent has a chance in the next term, their low reputation would prevent reelection. Such reputation concerns are added in the above payoff functions as the last term $b_i V(p_i(x_i))$. We impose that $V$ is strictly increasing, so that politicians always value higher reputation. We also assume $V(0) = 0$ and $V(1) = 1$.

Here, $b_i \geq 0$ represents the intensity of reputation concerns. The non-congruent type is assumed to have strong reputation concerns because they are self-interested. That is, $b_i = b > 0$ for the non-congruent type. On the contrary, some congruent politicians might only have low reputation concerns and always implement the policy that is optimal for voter $i$ (i.e., some politicians might be so-called statesmen). To capture this, we assume that the congruent type is divided into the congruent type $H$ and the congruent type $L$. The former type has high reputation concerns similar to the non-congruent type: $b_i = b$. In contrast, the latter type only has low reputation concerns: $b_i = b_L \in [0, b)$. The voter

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17 The function $L$ can be different across different types of politicians, though we assume the same function for simplicity.
18 As a whole, our setting allows the single-crossing condition to be satisfied under the binary state of the world. This is essential to generate extremism.
20 Even if some fraction of the non-congruent type have only low reputation concerns, the results do not change.
21 In the literature, there are two approaches to model the congruent type. The first one is to assume that they have...
does not know whether the congruent type is \(H\) or \(L\). In country \(i\), the incumbent is the congruent type \(H\) with probability \(q_H \in (0, 1)\), the congruent type \(L\) with probability \(q_L \in (0, 1)\), and non-congruent with probability \(1 - q_H - q_L\). We define \(q \equiv q_L + q_H\). \(q \in (0, 1)\) is also assumed. Politicians’ types are independently determined across countries. Denote this type space by \(T_i \equiv \{H, L, N\}\) in which \(H\) (\(L\)) represents that politician \(i\) is the congruent type \(H\) (\(L\)) and \(N\) represents that they are the non-congruent type.

In Section 7.1, we extend our model to a two-period election model and interpret reputation concerns as reelection concerns. That model is also useful to understand why some congruent-type politicians have low reputation concerns.

For simplicity, we assume henceforth that the reputation concern takes the linear form \(V(i) = \pi_i\). This assumption is standard in the literature (e.g., Maskin and Tirole, 2004). As discussed in Appendix B6, our conclusion does not depend on this assumption.

3.3 Timing of the Game and Equilibrium Concept

The timing of the game is summarized as follows.\(^{22}\) In period \(i\),

1. Politician \(i\) chooses a policy \(x_i\).

2. Voter \(i\) updates the belief on the incumbent’s type \(\pi_i(x_i)\).\(^{23}\)

3. Politician \(i\)’s payoff is realized.

The equilibrium concept is a (mixed strategy) perfect Bayesian equilibrium.

4 Equilibrium: Single-Country Model

We start with the case when \(N = 1\). Although this benchmark case cannot deal with the contagion of extremism, it provides a useful framework to consider the mechanism that induces the spread of extremism.

\(^{22}\)The evaluation by voter \(i\) does not depend on policies implemented after politician \(i\)’s policy choice, and thus politicians are not forward-looking. Such an assumption makes the analysis tractable especially when we consider the asymptotic properties. Indeed, it is often assumed in the literature of social learning including the seminal works by Banerjee (1992) and Bikhchandani, Hirshleifer, and Welch (1992). Furthermore, we believe that this situation is natural in many contexts because politicians are likely to care more about the reputation in the near future. It is particularly the case when the reputation concern is interpreted as the reelection concerns because the electoral result depends on the reputation at the time of the election. Moreover, policies just before the election are likely to be highly informative about their type and quite important, particularly given that the optimal policy as well as the salient issue vary over time. Thus, the reputation is likely to be heavily dependent on the policy implemented just before the election.

\(^{23}\)If one wants to explicitly model the voter’s action, we can consider the following situation: voter \(i\) chooses the conjecture about the probability of the incumbent being the congruent type \(y\) to minimize the quadratic loss \((1\{i \neq N\} - y)^2\). Then, \(y^* = \pi_i(x_i)\).
Suppose that $N = 1$. Let us denote the ex-ante probability that $\omega_1 = 1$ by $p_1 \in (0, 1)$. Voters know this probability, while they do not know the exact value of $\omega_1$. Without notational abuse, we omit the subscript “1” that represents country 1 since there is only one country in this section. Similarly, we denote $p_1$ by $p$.

To focus on meaningful cases in which extremism could arise, we suppose the following:

**Assumption 1.** $b_L \in [0, 1)$ and $b \in (2, l - 1)$.

That is, reputation concerns of the congruent type $L$ are sufficiently low, while those of the other types are relatively high.\(^{24}\) These assumptions are sufficient conditions to analyze meaningful cases in which extremism could arise (see Section B5). Henceforth, we focus on these values.

We allow players to mix actions. Let $\alpha^*(x; \omega)$ be the equilibrium probability that the congruent type $H$ chooses $x$ when the state is $\omega$. Similarly, let $\beta^*(x)$ be the equilibrium probability that the non-congruent type chooses $x$ and let $\gamma^*(x; \omega)$ be the equilibrium probability that the congruent type $L$ chooses $x$ when the state is $\omega$.\(^{25}\) Note that $\sum_{x=0}^{2} \alpha^*(x; \omega) = \sum_{x=0}^{2} \beta^*(x) = \sum_{x=0}^{2} \gamma^*(x) = 1$.

### 4.1 Equilibria

As a preliminary result, we show that the following types of equilibria never exist:

**Lemma 1.** (i) There is no equilibrium in which $\beta^*(0) = 1$. In addition, (ii) there is no equilibrium in which $\beta^*(2) > 0$, (iii) in any equilibrium, $\alpha^*(2; 2) = 1$, (iv) there is no equilibrium in which $\alpha^*(0; \omega) > 0$ for some $\omega$, and (v) in any equilibrium, $\gamma^*(\omega; \omega) = 1$.

**Proof.** See Appendix A.

The most important properties are (ii) and (v). The non-congruent type might have an incentive to choose a policy different from their own ideal policy (i.e., policy 0) to pretend to be the congruent type. (ii) argues that if the non-congruent type has the incentive, they implement policy 1, not policy 2. Since policy 2 is too different from policy 0, the loss due to implementing policy 2 is huge for the non-congruent type. In particular, this loss compared to that which occurs when implementing policy 1 is $l - 1$. Since the benefit of implementing policy 2 is at most $b$, the loss is always larger than the benefit as long as $b < l - 1$. Hence, the non-congruent type never chooses policy 2. This implies that the congruent type always can separate themselves from the non-congruent type by arguing for policy 2. That is, the radical policy works as a signal of the incumbent being the congruent type because it is too extreme for the non-congruent type. This property is the key creating populist extremism.

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\(^{24}\)The upper bound of $b$ is not restrictive when $l$ (i.e., the loss due to implementing policy 2 for the non-congruent type) is sufficiently high.

\(^{25}\)To be precise, we implicitly restrict our attention to equilibria in which each player’s equilibrium strategy depends on only payoff relevant information for him/herself. Hence, $\beta^*$ does not depend on $\omega$. This is consistent with the equilibrium concept we adopt in Section 5. Furthermore, Proposition 2, the key characterization of the single-country result, still holds even if we allow $\beta$ to depend on $\omega$. Alternatively, if we assume that the non-congruent type does not know the state of the world, all strategies always satisfy this property.
Given this, even the congruent type might have an incentive to distort policies to benefit their reputations. (v) argues that the congruent type $L$ always chooses $\omega$ as the policy because reputation concerns are sufficiently low (i.e., $b_L < 1$). Hence, we do not highlight the strategy of the congruent type $L$ in the following derivation of equilibria.

Based on this preliminary result, we obtain the following characterization of equilibria.

**Proposition 1.**  (a) There is an equilibrium in which $\alpha^*(2; \omega) = 1$ and $\beta^*(1) = 1$ if and only if \( \frac{1-q}{(b-1)q_L} \leq p \leq \frac{(b-1)(1-q)}{q_L} \). We refer to this as (E1) equilibrium.

(b) There is an equilibrium in which $\alpha^*(2; \omega) = 1$, $\beta^*(1) > 0$, and $\beta^*(0) > 0$ if and only if $p < \frac{1-q}{(b-1)q_L}$. Furthermore, in this equilibrium, $\beta^*(1) = \frac{(b-1)pq_L}{1-q}$; $\beta^*(0) = 1 - \beta^*(1)$. We refer to this as (E2) equilibrium.

(c) There is an equilibrium in which $\alpha^*(2; 2) = 1$, $\alpha^*(1; 1) > 0$, $\alpha^*(2; 1) = 1 - \alpha^*(1; 1)$, and $\beta^*(1) = 1$ if and only if \( \frac{(b-1)(1-q)}{q} \leq p \leq \frac{(b-1)(1-q)}{q_L} \). Furthermore, in this equilibrium, $\alpha^*(1; 1) = \frac{(b-1)(1-q)}{pq_L} - \frac{q_L}{q_H}$. We refer to this as (E3) equilibrium.

(d) There is an equilibrium in which $\alpha^*(\omega; \omega) = 1$ and $\beta^*(1) = 1$ if and only if $p \geq \frac{(b-1)(1-q)}{q}$. We refer to this as (NE) equilibrium.

(e) There is no other equilibrium.

**Proof.** See Appendix A.

Some of the equilibria in this proposition have an interesting feature that we call populist extremism or, interchangeably in this paper, extremism. To highlight this, let us define the extremism equilibrium. Let $X^*$ be the set of policies that can happen with a positive probability in an equilibrium i.e.,

$$X^*(\omega) \equiv \{x \in \{0, 1, 2\} : \alpha^*(x, ; \omega) + \gamma^*(x, \omega) > 0\}.$$

**Definition 1.** An equilibrium $(\alpha^*, \beta^*, \gamma^*, \pi^*)$ is called the (populist) extremism equilibrium if for some $\omega$, $X^*(\omega) \setminus \{\omega\} \neq \emptyset$ and $\pi^*(x) = 1$ for some $x \in X^*(\omega) \setminus \{\omega\}$.

If an equilibrium is not the extremism equilibrium, let us call it the non-extremism equilibrium. It is unsurprising that some politicians argue for extreme and undesirable policies because some politicians have a biased ideology. However, what we observe in the proliferation of populism is a more paradoxical situation in which the politician who chooses extreme policies obtains strong voter support. Our definition of populist extremism represents this paradoxical phenomenon. In the extremism equilibrium, some politicians choose a policy different from the voter-optimal policy. Nonetheless, their reputations are bolstered (i.e., $\pi^* = 1$), and they get re-elected or enjoy pleasant post-political life.\(^{26}\)

\(^{26}\)Our definition of extremism is close to the definition of populism given by Acemoglu, Egolov, and Sonin (2013), while they do not implement the formal definition. To be precise, not limited to the congruent type, they also regard the
(E1)-(E3) equilibria are indeed extremism equilibria. In the equilibria, the congruent type $H$ chooses policy 2 even if policy 1 is the voter-optimal policy. In addition, since only the congruent type chooses policy 2, politicians who implement policy 2 acquire high reputations.

The mechanism behind the emergence of populist extremism is outlined as follows. Due to reputation concerns, the non-congruent type has an incentive to pretend to be the congruent type. To this end, they choose policy 1, which could be chosen by the congruent type. This implies that the congruent type cannot separate themselves from the non-congruent type by choosing policy 1. On the contrary, the non-congruent type never chooses policy 2, as seen in Lemma 1 (ii). Hence, the congruent type $H$, who has high reputation concerns, implements the radical policy (i.e., policy 2) to signal that they are the congruent type. Furthermore, the politician who argues for the radical policy indeed acquires high reputation because it is a signal of the incumbent being the congruent type. As a result, extremism with a strong support by voters arises. This mechanism is analogous to that of Acemoglu, Egolov, and Sonin (2013), whereas they do not examine uncertainty about the state of the world. Notice that the mechanism does not depend on the existence of the congruent type $L$ at all. Its existence is crucial in the multi-country setting (see Appendix B4).

Notably, (NE) is the non-extremism equilibrium. In this equilibrium, the non-congruent type chooses the non-optimal policy when $\omega = 2$. In this sense, an extreme policy could be implemented. However, the politician who chooses it does not bolster their reputation. Furthermore, the congruent type always chooses the voter-optimal policy. Hence, nothing is paradoxical and this equilibrium is the non-extremism equilibrium. Distinguishing from extremism, we refer to the “bad behavior” by the non-congruent type as tyranny based on the analogy that citizens do not wish to elect a tyrant but they may fail to distinguish a bad politician whose support will lead them to suffer from tyranny. Using this terminology, we can say that, in the equilibrium, extremism does not occur while tyranny exists.

In summary, we have the following properties.

**Fact 1.** All equilibria except for (NE) equilibrium are extremism equilibria.

We have one remark on two key assumptions that induce extremism. We believe that those assumptions reflect real aspects of populist extremism. The first key assumption is the existence of the non-congruent type (i.e., corrupt politicians), which makes the voter think that the incumbent might be bad. Without such politicians, the congruent type has no incentive to distort policies. In the literature on populism, it has been pointed out that populism has the characteristic of anti-elitism.\(^{27}\) Since voters’ beliefs that the incumbent might be the non-congruent type can be regarded as distrust towards politicians, our model reflects the fact that anti-elitism induces populist extremism.

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\(^{27}\)For instance, Mudde (2004: 543) defines populism as “an ideology that considers society to be ultimately separated into two homogeneous and antagonistic groups, ‘the pure people’ versus ‘the corrupt elite’, and which argues that politics should be an expression of the volonté générale (general will) of the people.”
The second assumption is that even the congruent type has reputation concerns. Without high reputation concerns, the congruent type never distorts policies, meaning that extremism does not arise. This implies that populists in our model choose extreme policies for opportunistic rather than ideological reasons. While, at first glance, populists’ motivations seem to be ideological, this is not necessarily the case. Indeed, some scholars argue that populists are primarily opportunistic. For instance, Weyland (2017: 62) points out “populism tailors its appeals in opportunistic ways to maximize the leader’s chances of capturing the government”. That is, populists’ policies are chosen in terms of how to attract voters. In these aspects, the underlying mechanism of populism in our model reflects the reality.

4.2 Extremism and Public Opinion

As seen in the previous proposition, the existence of each equilibrium depends on the public opinion \( p \). The result can be summarized as follows by focusing on the relationship with \( p \).

**Theorem 1.** Suppose first that \( \frac{1-q}{(b-1)q_L} < \frac{(b-1)(1-q)}{q} \). Then,

(i) When \( p < \frac{1-q}{(b-1)q_L} \), there is a unique class of equilibria: (E2) equilibrium.

(ii) When \( \frac{1-q}{(b-1)q_L} \leq p < \frac{(b-1)(1-q)}{q} \), there is a unique class of equilibria: (E1) equilibrium.

(iii) When \( \frac{(b-1)(1-q)}{q} \leq p \leq \frac{(b-1)(1-q)}{q_L} \), there exist three classes of equilibria: (E1), (E3), and (NE).

(iv) When \( p > \frac{(b-1)(1-q)}{q_L} \), there is a unique class of equilibria: (NE) equilibrium.

Next, suppose that \( \frac{1-q}{(b-1)q_L} > \frac{(b-1)(1-q)}{q} \). Then,

(i) When \( p < \frac{(b-1)(1-q)}{q} \), there is a unique class of equilibria: (E2) equilibrium.

(ii) When \( \frac{(b-1)(1-q)}{q} \leq p < \frac{1-q}{(b-1)q_L} \), there are three classes of equilibria: (E2), (E3), and (NE).

(iii) When \( \frac{1-q}{(b-1)q_L} \leq p \leq \frac{(b-1)(1-q)}{q_L} \), there are three classes of equilibria: (E1), (E3), and (NE).

(iv) When \( p > \frac{(b-1)(1-q)}{q_L} \), there is a unique class of equilibria: (NE) equilibrium.

**Proof.** From Proposition 1, we directly obtain the theorem.

While this characterization might seem complicated, it can be interpreted in a simple way. By using the definition of extremism equilibria, we obtain the following proposition.

**Proposition 2.** The non-extremism equilibrium exists if and only if \( p \geq \bar{p} \equiv \frac{(b-1)(1-q)}{q} \).

Hence, the emergence of extremism is highly related to the prior belief \( p \). As \( p \) becomes lower, extremism is more likely to arise. The intuition can be understood as follows. Suppose that sufficiently high reputations are maintained under policy 1. Then, even if policy 2 signals that the incumbent is the congruent type, the congruent type \( H \) might choose policy 1 when \( \omega = 1 \). Hence, whether the
non-extremism equilibrium exists depends on whether high reputations are maintained under policy 1. Interestingly, whether high reputations are maintained under policy 1 is in turn dependent upon the voters’ beliefs about the state of the world. To see this, assume first the non-extremism equilibrium. In this equilibrium, policy 1 is chosen either when the incumbent is the congruent type and \( \omega = 1 \) or when the incumbent is the non-congruent type. Thus, the updated belief on the probability of the incumbent being the congruent type given policy 1 is

\[
\pi(1) = \frac{pq}{pq + 1 - q}.
\]

Since the congruent type chooses policy 1 only when it is indeed the voter-optimal policy, this updated belief is increasing in \( p \). Hence, when \( p \) is relatively low—so that voters think \( \omega = 2 \) is likely—choosing policy 2 is beneficial for improving reputation even when \( \omega \) is actually 1, leading to an extremism equilibrium. However, note that \( p \) does not have to be less than 1/2. Thus, even when voters believe that \( \omega = 1 \) is more likely, politicians may choose policy 2. This is a key difference from “pandering” discussed in Maskin and Tirole (2004). In summary, the more strongly voters believe that the radical policy is the good policy, the more likely populist extremism is to arise. This property is key in catalyzing the spread of extremism in the next section.

In the previous subsection, we argued that the existence of the non-congruent type is essential in creating populist extremism. The comparative statics about the threshold value \( \bar{p} \) confirms this argument. Since \( \bar{p} \) is decreasing in \( q \), a higher \( q \) prevents extremism i.e., the less likely the incumbent politician is corrupt, the less likely extremism is to arise. This is indeed consistent with the feature of populism as anti-elitism.\(^{28}\)

We note two remarks. The first one is about welfare properties.\(^{29}\) When \( p \) is notably small, the equilibrium is (E2). In this equilibrium, the non-congruent type chooses policy 0, which is never optimal for the voter, with a certain probability. That is, the voter suffers from severe tyranny as well as extremism. This is the worst equilibrium for the voter, and so democracy performs worst. On the contrary, in the non-extremism equilibrium, the congruent type always chooses the optimal policy and the non-congruent type is also well-disciplined in the sense that they choose policy 1, which is the best equilibrium for the voter. These conclusions are summarized in the following fact.

**Fact 2.** For any \( \omega \), the equilibrium that gives the voter the highest payoff is (NE) equilibrium while the equilibrium that gives the voter the lowest payoff is (E2) equilibrium. Hence, when \( p > \bar{p} \), the voter-optimal equilibrium in the single-country model is (NE) equilibrium.

\(^{28}\)Acemoglu, Egorov, and Sonin (2013) also find a similar comparative statics result.

\(^{29}\)The presented welfare ranking implicitly assumes that the incumbent does not serve again because we only consider welfare from the current term. Such a situation is natural when the incumbent faces the term limit and is concerned about their reputation regarding the establishment of their legacy or improving their post-political life. When the incumbent is able to serve another term and reputation concerns are primarily reelection concerns (see the model presented in Section 7.1), on the other hand, the extremism equilibrium might be beneficial because it facilitates detecting bad incumbent politicians i.e., exhibiting a positive selection effect (Besley 2006). Even in this setting, we can show that the extremism equilibrium must be detrimental as long as the policy issue in the second term is not similarly relevant (\( k \) in the model of Section 7.1 is low) and the election result is sufficiently volatile (\( \varepsilon \) is large). The formal proof is available upon request.
The second remark is on the equilibrium when \( p = 0 \). So far, we have explored the case where \( p \in (0, 1) \). However, as seen later, \( p = 0 \) could be the case in the dynamic model. In the case that \( p = 0 \), we obtain the following result.

**Lemma 2.** When \( p = 0 \) and \( \omega = 2 \), \( \alpha^*(2; 2) = 1 \) and \( \beta^*(0) = 1 \) in any equilibria.

**Proof.** See Appendix A.

## 5 Equilibrium: Multi-Country Model

The results in the previous section demonstrate that voters’ beliefs regarding the state of the world affect whether extremism arises. However, this is not the entire process. The equilibrium behaviors of politicians should affect voters’ beliefs. Hence, it is expected that there is an interaction between public opinion dynamics and policy dynamics. This section is devoted to analyzing these interactions. To this end, we consider the case in which \( N \geq 2 \).\(^{30}\) In addition, we suppose that the state of the world is the same across countries—that is, the case in which \( \omega_i = \omega \) for all \( i \). For the case in which \( \omega_i \) varies across countries, see Section 6.

### 5.1 Equilibrium Concept

First, let us formally define the equilibrium concept. Define public history at the beginning of period \( i \) by \( h^i \equiv (x_1, ..., x_{i-1}) \in H^i \equiv X^{i-1} \). Politician \( i \)'s strategy is given by \( s_i : H^i \times \Omega \times T_i \rightarrow \Delta(X) \). \((p_i, 1 - p_i) \in \Delta(\Omega)\) denotes the belief that voter \( i \) attaches to state 1 and 2.

Our equilibrium concept is the voter-optimal Markov perfect Bayesian equilibrium:

**Definition 2.** \((s_i^*, \pi_i^*, p_i^*) \in \{1, ..., N\} \) constitutes an equilibrium if

(i) They constitute a perfect Bayesian equilibrium;

(ii) For the congruent type, \( s_i^* : \Delta(\Omega) \times \Omega \times T_i \rightarrow \Delta(X) \) and for the non-congruent type, \( s_i^* : \Delta(\Omega) \rightarrow \Delta(X) \) i.e., the equilibrium strategy of politician \( i \) depends only on their type and voter \( i \)'s belief \( p_i \), and in the case of the congruent type, it also depends on the state of the world; and

(iii) given \( p_i^* \in [0, 1] \), \((s_i^*, \pi_i^*) \) is the voter-optimal equilibrium in the static model.

(i) and (ii) together imply that we use the Markov perfect Bayesian equilibrium, which is a standard concept in the literature of social learning.\(^{31}\) Furthermore, (iii) implies that the equilibrium in each stage game is the voter-optimal one. When \( p_i \geq \bar{p} \), there are multiple equilibria; (iii) is concerned with

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\(^{30}\)Though we mainly consider the situation in which each \( i \) is a different country, our model can be regarded as the model that analyzes the domino effect of populist extremism overtime within one country.

\(^{31}\)The non-congruent type’s strategy does not depend on \( \omega \) because that is payoff irrelevant for this type politician.
This selection is reasonable from the following two perspectives. First, we typically focus on the principal-optimal equilibrium in the analysis of agency problems, and the voter in our model corresponds to the principal. Secondly, from Fact 2, when there are multiple equilibria, the voter-optimal equilibrium is the non-extremism equilibrium. Hence, our selection is equivalent to the selection of the non-extremism equilibrium, if it exists. This is a conservative analysis of how likely extremism is to proliferate because we focus on the case in which extremism is least likely to arise.

Lastly, we assert the following assumption.

**Assumption 2.** \( \bar{p} < 1 \).

Combined with (iii) in Definition 2, this assumption guarantees that \( p \in [0, 1) \) exists such that the non-extremism equilibrium arises. Without this condition, extremism by definition always arises because all equilibria are extremism equilibria. We utilize this to focus on meaningful situations.

### 5.2 Updated Beliefs

Our starting point is to analyze voters’ learning processes regarding the state of the world. From now on, we assume that \( p_1 \in (0, 1) \). When \( p_{i-1} \in (0, 1) \), \( p_i \ (i \geq 2) \) is given recursively as follows based on the Bayes rule:

\[
\begin{align*}
    p_i(x_1, \ldots, x_{i-2}, 1) &= \begin{cases} 
        \frac{1+(b-1)p_{i-1}}{b} ( (E2) \ equilibrium) 
        & \text{if } x_{i-1} = 1 \\
        \frac{p_{i-1}(q_1+(1-q_1))}{p_{i-1}q_L+(1-q_1)} ( (E1) \ equilibrium) 
        & \text{if } x_{i-1} = 0 \\
        \frac{p_{i-1}}{p_{i-1}q_H+(1-q)} ( (NE) \ equilibrium) 
        & \text{if } x_{i-1} = 2
    \end{cases} \\
    p_i(x_1, \ldots, x_{i-2}, 2) &= \begin{cases} 
        1 - \frac{(1-p_{i-1})q}{(1-p_{i-1})q_L+q_H} ( (E1) \ and \ (E2) \ equilibria) 
        & \text{if } x_{i-1} = 1 \\
        0 ( (NE) \ equilibrium) 
        & \text{if } x_{i-1} = 0 \\
        p_{i-1} ( (E2) \ equilibrium). 
        & \text{if } x_{i-1} = 2
    \end{cases}
\end{align*}
\]

---

32 We cannot conduct the equilibrium selection using criteria such as the intuitive criterion. This is because the multiplicity of equilibria does not occur based on off-path belief formation.

33 Recent examples that focus on the voter-optimal equilibrium in the analysis of political agency problems include, for instance, Forand (2015).

34 Although we focus on the voter-optimal equilibrium for simplicity, the key for our following results is that there exists a threshold value of \( p \) such that the equilibrium is extremism if and only if \( p \) is less than that value. Hence, the wider class of equilibria indeed give us the almost same results. To illustrate, denote the equilibrium probability of policy 2 being implemented in country \( i \) given \( p_i \) by \( R^n(p_i) \). We then define the monotonic Markov perfect Bayesian equilibrium by the equilibrium in which (i) and (ii) are satisfied, and (iii') \( R^n(p) \) is weakly decreasing in \( p \) (i.e., monotonicity holds). So long as we consider the monotonic Markov perfect Bayesian equilibrium, we obtain similar results.

35 Here, we do not derive the updated beliefs for (E3) equilibrium because that equilibrium does not arise according to our equilibrium concept.
5.3 Spread of Extremism

The radical policy induced by populist extremism is problematic when the voter-optimal policy is the moderate policy. Hence, we assume that $\omega = 1$ in this section.

5.3.1 Populist Extremism Induces Opinion Radicalization

As seen in the updated belief derived in the previous subsection, the voter’s belief about the optimal policy changes depending on the policy. To understand these opinion dynamics more clearly, we define opinion radicalization as follows.

**Definition 3.** Fix $h^i$ and $\omega = 1$. An equilibrium in country $i$ $(\alpha^*, \beta^*, \gamma^*, \pi^*)$ induces more severe opinion radicalization if for some $x \in X^*(1)$, $p_{i+1}(h^i, x) < p_i(h^i)$ holds.

This definition represents that the equilibrium policy in country $i$ incorrectly changes the opinion voter $i + 1$ holds in the direction toward the radical policy.

From this definition and the updated belief derived in the above, we obtain the following fact.

**Fact 3.** Suppose $p_i \in (0, 1)$ and $\omega = 1$. Extremism equilibria induce more severe opinion radicalization, whereas non-extremism equilibria never induce more severe opinion radicalization.

Hence, when the moderate policy is the voter-optimal one, extremism equilibria induce more severe opinion radicalization. Let us illustrate this situation through an example of immigration policy. Suppose that the voter in country 2 is unsure about how much accepting immigrants adversely affects their social and economic situations. Now, the voter in country 2 learns that country 1 enacted quite stringent immigration policies toward immigrants (i.e., the radical policy). Although such policies may have been implemented by a biased populist (the congruent type $H$), they cannot rule out the possibility that the politician in country 1 did so because it was actually good (i.e., it was implemented by the congruent type $L$). Thus, observing country 1’s strict policy toward immigrants makes voters in country 2 believe that a strict immigration policy may be desirable even if the policy in country 1 is implemented by a populist. As such, extremism in country 1 induces opinion radicalization in country 2.

5.3.2 Spread of Extremism

We have seen that extremism may induce opinion radicalization. Further, opinion radicalization in turn induces extremism because of Proposition 2. Consequently, we have an interaction between opinion radicalization and extremism. The result of such an interaction can be seen in the following lemma.

**Lemma 3.** Suppose $\omega = 1$. For each $\tilde{p} \in (0, 1]$, $\Pr(p_{i+1} < \tilde{p}|p_i)$ is weakly decreasing in $p_i$.\footnote{We consider the following situation. No regulation on immigration is obviously too loose for voters (or at least voters think so) but some non-congruent politicians seek such policies. Then, policy 0 represents no regulation. Policy 2, for example, represents the very strict regulation, whereas policy 1 represents the moderate regulation.}

\footnote{In addition, we can easily observe that $\Pr(p_{i+1} < \tilde{p}|p_i)$ is not equal for all $p_i$.}
Proof. See Appendix A.

This result directly implies that the probability that the extremism equilibrium arises in country \( i + 1 \) is weakly decreasing in \( p_i \) i.e., opinion radicalization in country \( i \) induces extremism in country \( i + 1 \). When opinion radicalization is quite severe in country \( i \), the equilibrium entails extremism so that the radical policy may be implemented. After the implementation of the radical policy, opinion of country \( i + 1 \) becomes more radical so that country \( i + 1 \) is also captured by extremism. Furthermore, when country \( i \)'s opinion is too radical, the moderate policy in country \( i \) might not be enough to prevent extremism in country \( i + 1 \). To make matters worse, low \( p_i \) further limits the opportunities to learn the state of the world because the probability that the non-congruent type will implement policy 0 increases as \( p_i \) decreases. Since policy 0 is uninformative about the optimal policy, it further delays the learning process. As such, country \( i \)'s opinion radicalization induces country \( i + 1 \)'s extremism.

The following proposition implies that the iteration of this mechanism yields the domino effect of extremism:

**Proposition 3.** Suppose \( \omega = 1 \).

(a) Fix \( k \in \{1, ..., N - 1\} \). For \( i \in \{k + 1, ..., N\} \), \( \Pr(p_i \geq \bar{p}) \) is weakly increasing in \( p_k \).

(b) Suppose that \( p_1 < \bar{p} \). For \( x_1 \in \{0, 1\} \) that is on-path and \( i \in \{2, ..., N\} \), \( \Pr(p_i \geq \bar{p}|x_1 = 2) \leq \Pr(p_i \geq \bar{p}|x_1 = x) \). In addition, there exists \( p_1 \in (0, \bar{p}) \) such that for \( x_1 \in \{0, 1\} \) that is on-path and \( i \in \{3, ..., N\} \), \( \Pr(p_i \geq \bar{p}|x_1 = 2) < \Pr(p_i \geq \bar{p}|x_1 = x) \).

Proof. See Appendix A.

(a) is the straightforward extension of Lemma 3. The depth of opinion radicalization indicates the depth of extremism; thus, extremism arises in subsequent countries with a lower probability as the opinion radicalization in country \( k \) exacerbates. Since the implementation of the radical policy in country 1 induces opinion radicalization in country 2, (a) immediately implies (b), which shows the domino effect of extremism. (b) indicates that given that country 1 is in the extremism equilibrium, whether the radical policy is implemented in country 1 affects the probability of each subsequent country being in the extremism equilibrium. In particular, the implementation of the radical policy in country 1 induces extremism in subsequent countries. Since the implementation of the radical policy represents that the populist politician (the congruent type \( H \)) argues for the extreme policy to attract the support, this result indicates the domino effect of extremism.

To see this contagion process in more detail, we analyze a numerical example. In Figure 1, we describe a sample path of beliefs. We suppose that the radical policy is indeed implemented in country 1. This shock induces extremism in other countries. In Figure 1, even when \( N = 30 \), the domino effect of extremism has not stopped yet, implying that that the domino effect may indeed be a real concern.

Figure 2 also confirms that the policy implemented in country 1 induces the spread of extremism. In Figure 2, we present the respective average path of beliefs \( p_i \) when country 1 implements policy 1 (blue) and 2 (red). It also shows the green path that denotes the scenario in which county 1 takes policy
Figure 1: Contagion of populist extremism.

Notes: The parameter values are $l = 4$, $p_1 = 0.2$, $q_H = 0.5$, $q_L = 0.1$, $\omega = 1$, and $b = 2.1$. The left box plots the implemented policies and the right box represents the dynamic of voters’ beliefs. The orange dotted line in the right box is $\bar{p}$.

2 but the non-extremism equilibrium is hypothetically assumed to be realized in all of the subsequent countries.$^{38}$ Comparison of the blue and red lines reveals that the policy of country 1 crucially affects the significance of the contagion of extremism. If the country does not implement the radical policy, the extremism equilibrium ends relatively soon. On the contrary, the implementation of the radical policy propagates throughout many countries and induces extremism. The comparison between the red and green lines shows that the effect of the radical policy in country 1 is not limited to the change in country 2’s public opinion. Recall that the green line represents the hypothetical scenario in which the non-extremism equilibrium is always taken, provided that the belief at the beginning of period 2 is equal to that of the red line. In this case, the belief rapidly converges to one, indicating that the spread of populism immediately ends. However, this is not what happens in the actual equilibrium. When the radical policy is chosen in country 1, country 2 is also captured in the extremism equilibrium, which in turn induces the distorted learning in the subsequent countries. Hence, the red line is far different from the green line.

The severeness of the contagion of populist extremism might be more pronounced once we recognize the possibility of very long-lasting domino effect. Figure 3 shows the country at which the extremism stops for the first time in the case of the red line in Figure 2. When the extremism occurs in all countries $i \leq 20$ and the extremism occurs for country $i = 21$, the value is shown as 21.$^{39}$

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$^{38}$For each case, we simulate the equilibrium for 100,000 iterations and obtain the path of $p_i$. We then calculate the average for each $N$ and obtain the average path of $p_i$. This approach is also taken by Chen and Suen (2016).

$^{39}$In this numerical example, after policy 2 implemented in country 1, country 3 must experience extremism even if country implements policy 1. On the other hand, if country 1 implements policy 1 (i.e., the blue scenario in Figure 2),
Figure 2: **Contagion of populist extremism.**

Notes: The parameter values are $l = 4$, $p_1 = 0.5$, $q_H = 0.5$, $q_L = 0.1$, $\omega = 1$, and $b = 2.1$. The blue (red) line describes the dynamics of beliefs when $x_1 = 1(2)$. The counter-factual green line describes the dynamics of beliefs when $x_1 = 2$ and the equilibrium is always the non-extremism equilibrium independently of $p$. For each case, we simulate the equilibrium dynamics of beliefs for 100,000 iterations and calculate the average. The orange dotted line is $\bar{p}$.

shows that the domino effect might be so strong that the extremism does not stop within 20 countries.

These arguments together suggest that the interaction between opinion radicalization and extremism causes the spread of extremism. When voters believe that the moderate policy is unlikely to be optimal, extremism is likely to occur, meaning that the radical policy is likely to arise. This, in turn, implies that voters more aggressively believe that the radical policy is optimal. This negative cycle induces the proliferation of extremism.

The resulting spread has a substantially negative impact on welfare. In Figure 4, we plot the path of welfare for the scenario of the red line in Figure 2. While the welfare loss is zero in the absence of extremism, it is significant in the presence of the spread of extremism.\(^4\) For instance, even after ten countries, the average welfare loss is around -0.6. Thus, even a single shock may propagate and have a detrimental effect in many countries. Note that in the extremism equilibrium, both populist extremism and tyranny take place, both of which are detrimental to welfare. In the extreme case of $p_i \simeq 0$, the congruent $H$ type always takes policy 2 while the non-congruent type almost always takes policy 0.

The contagion of populist extremism induces malfunctioning democracy in both respects.

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\(^3\)country 2 does not experience extremism.

\(^4\)We omit the welfare of country $N = 1$ since we exogenously impose the policy in this country. Figure 4 shows the welfare path from the second country.
Notes: This histogram shows the frequency that the first country out of the extremism is $i$. If the non-extremism equilibrium continues for all $i \leq 20$, it shows 21. It corresponds to the red line (i.e., policy 2 in country 1) and the parameter values are $l = 4$, $p_1 = 0.5$, $q_H = 0.5$, $q_L = 0.1$, $\omega = 1$, and $b = 2.1$. We simulate the equilibrium 100,000 times to obtain the frequency.

Figure 4: Contagion of populist extremism: Welfare comparisons.

Notes: The parameter values are $l = 4$, $p_1 = 0.5$, $q_H = 0.5$, $q_L = 0.1$, $\omega = 1$, and $b = 2.1$. The red line denotes the dynamics of welfare when $x_1 = 2$. The dotted line is the welfare under the blue and green scenario in Figure 2. We simulate the equilibrium dynamics of welfare for 100,000 iterations and calculate the average.
5.4 How to Stop the Domino Effect: Paradigm Shift

Next, we explore how the spread of extremism can suddenly stop. Since the radical policy induced by populist extremism is problematic only when $\omega = 1$, we again focus on the case where $\omega = 1$.

We assert the following proposition.

**Proposition 4.** Suppose $\omega = 1$. When $(b - 1)b \leq \frac{q}{1 - q}$, for any $i$ and any history $h^t$, which can happen on the equilibrium path such that $p_i(x_1, \ldots, x_i - 1) < \bar{p}$, $\Pr(p_{i+1} \geq \bar{p}|p_i) > 0$.

**Proof.** First of all, $p_i > 0$ since $\omega = 1$. Thus, when $p_i(x_1, \ldots, x_i - 1) < \bar{p}$, $p_{i+1}(x_1, \ldots, x_i - 1, 1) = \frac{1 + (b - 1)p_{i-1}}{b} > 1/b$. Here, when $(b - 1)b \leq q/(1 - q)$, $1/b > \bar{p}$. Hence, if the probability that $x_i = 1$ is positive for any $p_{i-1} \in (0, 1]$, the proposition holds.\footnote{The belief $p_{i+1}$ is also larger than $1/b$ when the equilibrium is (E1). To see this, note that the belief when $p_i = \frac{1 - q}{(b - 1)q L}$ is larger than $1/b$ and $p_{i+1}$ is increasing in $p_i$ in (E1) equilibrium.} Indeed, when $\omega = 1$, this probability is always positive because the congruent type $L$ chooses 1; thus, the probability of $x_i = 1$ is greater than or equal to $q_L > 0$.

Hence, under certain conditions, even if the public opinion is too radical (i.e., $p$ is small), the voter stops believing that the radical policy is optimal after observing the moderate policy (i.e., $p$ becomes large); thus, the politicians no longer choose the radical policy. As such, extremism stops. That is, a paradigm shift from extremism to non-extremism suddenly occurs, even if the degree of extremism is severe. We can see this in a numerical simulation presented in Figure 5. In this example, the condition $(b - 1)b \leq \frac{q}{1 - q}$ holds and the initial value of $p$ is quite small (0.01). Initially, politicians implement the radical policy so that extremism continues and the belief decreases lower than 0.01. Hence, we expect that, even if the moderate policy is chosen, $p$ does not increase significantly and the spread of extremism continues. However, this is not the case. Once either the congruent type $L$ or the non-congruent type is the incumbent in a country, the moderate policy is implemented and voters’ beliefs discontinuously change. The spread of extremism suddenly stops.

The paradigm shift is surprising. To see this, let us contrast our result to the following updating process. The state space is $\Omega = \{1, 2\}$, and the voter receives a signal about the state of the world: $s \in \{1, 2\}$, where $\Pr(s = \omega) = \alpha \in (1/2, 1)$. Then, the likelihood of the posterior $p'$ given $s = 1$ is

\[
\frac{p'}{1 - p'} = \frac{p}{1 - p} \frac{\alpha}{1 - \alpha},
\]

using the prior $p$. Hence, as $p \to 0$, $p' \to 0$, indicating that the result in Proposition 4 never holds. Our setting is similar with this process. The moderate policy is just an imperfect signal that $\omega = 1$; voters are Bayesian rational. Nonetheless, we have Proposition 4.

Strategic interactions play a key role in triggering the paradigm shift. When $p$ is sufficiently small, the non-congruent type mixes policies 1 and 0. In particular, the probability that this type chooses policy 1 is increasing in $p$.\footnote{Intuitively, smaller $p$ means that the reputation that the non-congruent type acquires by implementing policy 1 is lower. Hence, the non-congruent type implements policy 0 with a higher probability.} This implies that the smaller $p$ is, the higher the precision of policy 1 is as
A sample path when \( l = 4, p_1 = 0.01, q_H = 0.6, q_L = 0.1; b = 2.1, \omega = 1. \) The left box plots the implemented policies and the right box represents the dynamic of the voters’ belief. The orange dotted line in the right box is \( \bar{p} \).

5.5 Learning: Asymptotic Properties

As seen in Section 5.3, extremism is contagious—at least in the short-term. While this spread may occur long enough to have detrimental effects on many nations’ welfare, the result in Section 5.4 also suggests that such contagion can suddenly stop due to the paradigm shift. In this subsection, we investigate whether voters can learn the state of the world asymptotically.

Our starting point is showing that voters’ beliefs almost surely converge to a single point. By applying Martingale Convergence Theorem to our scenarios, we obtain the following result, which has been widely used in the literature of social learning under Bayesian updating.

**Lemma 4.** (Chamley 2004: Proposition 2.7). There exists \( p^* \in [0, 1] \) such that \( \Pr(\lim_{N \to \infty} p_N = p^*) = 1 \).

Hence, beliefs almost definitely converge to a point. In other words, there is no belief cycle. The convergence point is given by the following proposition.

**Proposition 5.** (a) Suppose \( \omega = 1 \). Then, \( \Pr(\lim_{N \to \infty} p_N = 1) = 1 \).
Notes: The parameter values are \( l = 4, p_1 = 0.2, q_H = 0.6, q_L = 0.005, \omega = 1, \) and \( b = 2.1. \) We simulate the equilibrium dynamics of beliefs for 100,000 iterations and calculate the average. The orange dotted line is \( \bar{p}. \)

(b) Suppose \( \omega = 2. \) Then, \( \Pr(\lim_{N \to \infty} p_N = 0) = 1. \)

Proof. See Appendix A.

Voters try to learn the state of the world only through politicians’ distorted policies. Nonetheless, this proposition argues that voters eventually learn the truth. Hence, at least in the long-term, politicians’ extremism does not influence voters to wrongly believe that the radical policy is good. Furthermore, since extremism does not arise when \( p \) is close to one, the spread of extremism eventually stops when the optimal policy is the moderate one. In other words, the contagion of extremism does not last forever when the radical policy is not good.

The key is the existence of the congruent type \( L, \) which can be arbitrarily small. Such politicians sincerely implement the voter-optimal policy, which allows information about the state of the world to be partially transmitted to voters. Consequently, voters can learn the truth asymptotically.\(^{43}\) That is, the existence of politicians who sincerely implement the voter-optimal policy prevents the domino effect of populism from continuing forever. Notably, any arbitrarily small fraction of the congruent type \( L \) is enough for the asymptotic result. This can be also seen from numerical simulations in Figure 6. In this example, the congruent type \( L, \) that always implements the optimal policy, is very rare. Even in this situation, the voters learn the state of the world in the long-term.

\(^{43}\) Using a different model, Goeree, Palfrey, and Rogers (2006) also find that social learning is successful in the long-term. They extend the standard social learning model à la Banerjee (1992) so that each player’s payoff consists of the common value, which depends on the state of the world as well as the individual value, which is independent of the state of the world. Then, they show that such heterogeneity of preferences creates players who choose the action consistent with the state of the world even during herding, and the herding stops.
Although whether the fraction of the congruent type $L$ is small does not matter for the asymptotic result, it certainly affects the stopping time of the domino effect. To illustrate, let us first consider the probability that the domino effect stops in country $i + 1$ given $p_i$. This is indeed decreasing in the fraction of the congruent type $L$, fixing the total fraction of the congruent type.

**Fact 4.** Suppose $\omega = 1$. Fix $q$. For any $p_i \in (0, \bar{p})$, $\Pr(p_{i+1} \geq \bar{p})$ is decreasing in $q_L$.

To see further, in Table 1, we present the frequency of long-lasting extremism (i.e., extremism continues at country $i = 21$) given the radical policy in country $1$. Again, we change the fraction of the congruent type $L$ keeping $q$ fixed. The results show that the frequency is highly sensitive to $q_L$ and long-lasting extremism occurs much more often when $q_L$ is small. These results together indicate that the domino effect is less likely to end with less congruent type $L$ politicians.

It should be emphasized that the asymptotic property does not mean that the spread of extremism is irrelevant. In reality, particularly in international contexts, the number of countries that share the same state of the world may not be notably large. The short-term effect is still important, as seen in Table 1.

![Table 1](https://example.com/table.png)

**Table 1: Frequency of long-lasting extremism.**

*Notes:* The table shows the frequency that extremism takes place in country 21. Let $\nu \in (0, 1)$ be the parameter such that $q_H = \nu q$ and $q_L = (1 - \nu)q$. By changing $\nu$, we investigate the change in $q_L$, keeping $q$ fixed. The parameter values are $l = 4$, $q = 0.65$, $\omega = 1$, $b = 2.1$, and $p_1 = 0.4$. We suppose that policy 2 is implemented in country 1. We simulate the economy 100,000 times in calculating the frequency. All numbers are rounded up to three decimal places.

6 Markovian Environment

So far, we have assumed that $\omega$ is common across countries. This is a useful simplification in investigating the nature of the spread of extremism. However, in practice, the state of the world is not necessarily the same across countries. Social and economic conditions may be different across countries. National elections are held only occasionally, implying that there is some interval in the election of country $i$ and $i + 1$. In either case, the state is likely to be correlated only imperfectly.

In this section, we investigate the environment in which the state of the world follows a Markov process. That is, for all $i$, $\Pr(\omega_{i+1} = j | \omega_i = j) = \theta_j \in (1/2, 1)$ for each $j \in \{1, 2\}$. $\theta_1 (\theta_2)$ represents the stability of the state 1 (2). The values of $\theta_1$ and $\theta_2$ are known to voters as well as politicians. Other settings are exactly the same as those in the previous sections.

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44By fixing $q$, we can keep $\bar{p}$ fixed.

45This relationship does not depend on the parameter values. For various parameter values, we obtain the same relationship. The additional numerical examples are available upon request.
Then, the updated beliefs are given as follows:

$$p_{i+1}(x_1, \ldots, x_{i-1}) = \theta_1 p_i^* + (1 - \theta_2)(1 - p_i^*),$$  \hspace{1cm} (2)

where $p_i^*$ is defined by $p_{i+1}$ in Section 5.1.

In the analysis, we assume the following inequality to focus on meaningful cases:

**Assumption 3.** \(1 - \theta_2 < \bar{p} \leq \theta_1\) holds.

When $\theta$ is too low, the policy in the previous country is not informative. For instance, when $\theta_1 = \theta_2 = 1/2$, $p_{i+1} = 1/2$ independently of $x_i$. Assumption 3 argues that the informativeness of the previous policy should not be too low. When the previous policy $x_i$ is the perfect signal of the previous state of the world $\omega_i$, the previous policy should largely affect voters’ belief. In particular, the following property (*) should hold: when voter $i + 1$ knows that $\omega_i = 1$ (2), $p_{i+1}$ is large (small) so that $p_{i+1} > \bar{p}$ ($p_{i+1} < \bar{p}$). Otherwise, the informativeness of the previous state of the world is too low so that whether the equilibrium is extremism becomes invariant. In such an environment, it is meaningless to analyze the contagion of extremism. Hence, we impose (*), that is, Assumption 3.

### 6.1 Convergence towards Extremism

In the model in Section 5, voters’ beliefs converge towards the truth and thus at the limit, extremism never happens so long as $\omega = 1$. However, this no longer holds when $\omega_i$ is variant. The following proposition highlights this fact. Let

$$p_S \equiv \frac{1 - \theta_2}{2 - \theta_1 - \theta_2} \in (0, \theta_1),$$

which is equal to the steady state probability of $\omega$ being 1.\(^{46}\)

**Proposition 6.** (a) There exists $p_E \in (p_S, \theta_1)$ such that for any $i$, $p_i < \bar{p}$ implies that $p_j < \bar{p}$ holds for all $j \geq i + 1$ if and only if $\bar{p} \geq p_E$.

(b) When $\bar{p} \geq p_E$, $\lim_{N \to \infty} \Pr(p_N < \bar{p}) = 1$.

(c) $p_E$ is strictly increasing in $\theta_1$ and weakly increasing in $q_L$ (while $q$ kept fixed).

**Proof.** See Appendix A.

(a) and (b) argue that when $\bar{p} \geq p_E$, the equilibrium eventually shifts into the region in which extremism arises; after that, extremism continues forever. In that sense, the convergence to extremism occurs which contrasts the result obtained in Section 5. This result can be illustrated in the numerical example presented in Figure 7. In this example, the initial state is 1 and the implemented policy in

\(^{46}\)Since $\theta_1, \theta_2 \in (0,1)$, the Markov chain converges to the steady state distribution.
country 1 is also 1. However, eventually, voters’ beliefs decrease lower than $\bar{p}$, and the equilibrium never shifts outside of extremism equilibria.\footnote{Note that when the states are observable, the convergence to extremism does not occur in this case. Indeed, if $\omega_i = 1$, $p_{i+1} = 0.7 > \bar{p}$.}

Countries cannot escape from extremism once captured because of the following mechanism. Suppose that $\bar{p} > p_S$ because otherwise, the convergence result does not hold. In general, from (2), the following relationship holds in the Markovian environment: $p_{i+1} \leq p_{i+1}^* \Leftrightarrow p_{i+1}^* \geq p_S$. This property can be easily understood when $\theta_1 = \theta_2$. In this symmetric case, $p_S = 1/2$, indicating that the updated belief becomes less extreme due to the possibility that the state of the world might change. Suppose that country $i$ is in the extremism equilibrium, but the incumbent is the congruent type $L$ and implements the moderate policy. In this case, country $i + 1$ can escape from extremism only if $p_{i+1}^* > \bar{p} > p_S$. However, the updated belief becomes moderate so that $p_{i+1} < p_{i+1}^*$ because the possibility of the change in the state of the world is considered. Hence, $p_{i+1} < \bar{p}$ might hold, implying that country $i + 1$ might not be able to escape from extremism. This mechanism can be understood using the comparative static parameters provided in (c). $p_E$ is increasing in $\theta_1$ and $q_L$, meaning that the higher $\theta_1$ and $q_L$ are, the less likely countries are to be captured in extremism equilibria. When $\theta_1$ is large, the state of the world is highly stable, given that the previous state is 1. Hence, the above mechanism is unlikely to work. In addition, when $q_L$ is high, voters strongly believe that $\omega_i$ is likely to be 1; i.e., $p_{i+1}$ is high because the congruent type $L$ always takes the optimal policy. To see this, note first that policy 1 in the extremism equilibrium comes from either the congruent type $H$ or the non-congruent type. When $q_L$ is not large enough, policy 1 is likely to come from the non-congruent type under $\omega = 2$. As a result, the infor-

Figure 7: Convergence to extremism.

Notes. A sample path when $l = 4; \, p_1 = 1; \, \omega_1 = 1; \, \theta_1 = \theta_2 = 0.7; \, q_H = 0.4; \, q_L = 0.3; \, b = 2.5$. The orange dotted line is $\bar{p}$. 
mation value of policy 1 about the true state of the world is not sufficient to overturn the extremism equilibrium.

Lastly, extremism always occurs because of the following mechanism. Suppose that \( p_i \geq \bar{p} \). With a positive probability, the radical policy is observed since there could be the case that the state of the world is 2. When the radical policy is observed in country \( i \), \( p_{i+1} = 1 - \theta_2 < p_S < \bar{p} \). Hence, country \( i+1 \) is captured in extremism equilibria. Here, the key is the fact that the state of the world is changing. Because of this property, with a positive probability, the radical policy is observed, which influences the spread of extremism in subsequent countries.

### 6.2 Cycles of Extremism

Next, we consider the case in which the convergence does not hold (i.e., \( \bar{p} < p_E \)). In this case, cycles of extremism are exhibited as seen in the following proposition.

**Proposition 7.** Suppose \( \bar{p} < p_E \).

(a) For any integer \( M \geq 1 \), \( \lim_{N \to \infty} \Pr(\forall i \text{ s.t. } M \leq i \leq N : p_i \geq \bar{p}) = 0 \).

(b) For any integer \( M \geq 1 \), \( \lim_{N \to \infty} \Pr(\forall i \text{ s.t. } M \leq i \leq N : p_i < \bar{p}) = 0 \).

**Proof.** See Appendix A.

Proposition 7 shows that the probability of staying in either the non-extremism or extremism equilibrium forever is zero. Thus, in the long-term, we observe both the non-extremism and extremism equilibria.

Such cycles can be seen in the numerical example presented in Figure 8. In this example, extremism initially arises. Although it spreads to around 20 countries due to the contagion mechanism described in Section 5, its proliferation finally ceases. After a while, extremism then re-emerges because, in the case of \( \omega = 2 \), voters may observe policy 2 even when \( p > \bar{p} \). As such, cycles of extremism exist. The mechanism itself is straightforward. Since the state of the world changes across countries, voters’ beliefs fluctuate highly. Hence, we obtain cycles.

In Table 2, we report the results of several numerical simulations to illustrate the potential importance of the spread of extremism in the Markovian environment without the convergence toward extremism.\(^{48}\) In column 2, we report the fraction of the extremism equilibrium (\( p_i < \bar{p} \)) in the simulated equilibrium of 1,000,000 countries. In columns 3 and 4, we obtain all consecutive sequences of extremism and non-extremism equilibriums and calculate the average length of each.\(^{49}\)

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\(^{48}\)The action taken by the incumbent depends on the voter’s belief so that neither the transition matrix about the implemented policy nor that about the class of the static equilibrium is time-homogenous. Consequently, the dynamics of the belief cannot be described as a finite-state Markov process when \( N \to \infty \). Hence, we cannot obtain analytical results when \( N \to \infty \). Peck and Yang (2011) also face the same difficulty and resort to simulations. However, in Appendix B3, we provide analytical characterizations by reducing the effective number of states by imposing additional assumptions.

\(^{49}\)To illustrate, suppose that the extremism equilibrium is observed for countries 12-21, 26-40, and 71-100. Then, the length of each sequence is 9, 15, and 30, respectively. The average duration of extremism equilibrium is calculated as \( (9+15+30)/3=18 \).
Figure 8: Cycles of extremism.

Notes. A sample path when \( l = 4; \ p_1 = 1; \ \omega_1 = 1; \ \theta_1 = \theta_2 = 0.8; q_H = 0.5; q_L = 0.2; \ b = 2.3 \). The orange dotted line is \( \bar{p} \).

<table>
<thead>
<tr>
<th>Parameters ( {q_H, q_L, \theta_1, \theta_2} )</th>
<th>Frequency of extremism equilibrium</th>
<th>Duration of ( p_i &lt; \bar{p} )</th>
<th>Duration of ( p_i \geq \bar{p} )</th>
<th>Average welfare</th>
</tr>
</thead>
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<tr>
<td>{0.5, 0.2, 0.99, 0.99}</td>
<td>0.54</td>
<td>76.67</td>
<td>68.19</td>
<td>-0.60</td>
</tr>
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<td>10.69</td>
<td>7.76</td>
<td>-0.56</td>
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<tr>
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<td>0.61</td>
<td>5.86</td>
<td>3.79</td>
<td>-0.53</td>
</tr>
<tr>
<td>{0.05, 0.65, 0.9, 0.9}</td>
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<td>9.57</td>
<td>8.79</td>
<td>-0.39</td>
</tr>
<tr>
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<td>14.84</td>
<td>8.04</td>
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<tr>
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<td>0.92</td>
<td>94.75</td>
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</tr>
<tr>
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<td>5.56</td>
<td>6.45</td>
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</tr>
<tr>
<td>{0.75, 0.1, 0.9, 0.9}</td>
<td>0.38</td>
<td>2.67</td>
<td>4.18</td>
<td>-0.26</td>
</tr>
</tbody>
</table>

Table 2: Numerical examples: Spread of extremism under the cycles.

Notes: All numbers are rounded up to two decimal places. In all cases, \( b = 2.3; \ p_1 = 1; \ \omega_1 = 1; \) and \( l = 4 \). The figures are calculated by simulating the model by 1,000,000 times and taking the average. In all cases, the frequency of extremism equilibrium when the previous \( \omega \) is observed is 0.5, since observing \( \omega_i = 1(2) \) makes \( p_i+1 \geq \bar{p}(p_i+1 < \bar{p}) \) and each state occurs by the same frequency in the long-term.
The results in Table 2 indicate that when the duration of extremism equilibrium ($p_i < \bar{p}$) is large relative to that of non-extremism equilibrium ($p_i \geq \bar{p}$), the extremism equilibrium is observed more frequently. For example, in rows 3, 5, and 6, the extremism equilibrium is more persistent and is indeed more frequently observed while in rows 7 and 8, the extremism equilibrium is less persistent and is less frequently observed. When the spread of extremism prevents voters from learning that the state is 1, it prolongs the duration of the extremism equilibrium and leads to more frequent observations of it. Furthermore, the welfare loss associated with the prolonged extremism is fairly large. For example, in rows 1, 2, and 3, the average welfare is less than $-0.5$, meaning that voters’ obtained payoff is less than the expected payoff when voters randomly choose the policy between policies 1 and 2 without knowing anything about the state of the world.

In rows 1-3, we change the stability of each state while keeping other parameters fixed. In this case, the frequency of extremism is increasing as the state becomes more unstable. This is important in linking the analyses of Sections 5 and 6; we discuss this in detail in the next subsection.

In rows 4-6, we change the relative magnitude of $q_H$ while keeping $q$ fixed. Note that the congruent $H$ type is the source of extremism in the sense that it takes the extreme policy; thus, its effect on the contagion is of interest. Here, the increase in $q_H$ significantly prolongs the duration of the extremism equilibrium, and its frequency also increases. Thus, as the fraction of this type increases, the radical policy is more likely to be taken in the moderate state which catalyzes the spread of extremism. This can be seen most clearly in row 6, which takes the very large value of $q_H$.

However, this does not always hold. In rows 7 and 8, the duration of the extremism equilibrium is decreasing in $q_H$. This occurs because the informativeness of the radical policy (i.e., policy 2) is important in determining the significance of contagion effect. The mechanism is as follows. As in Section 5, the congruent type $H$ implements the radical policy, and in this sense, their existence induces extremism. However, we also have the opposite effect. $p_{i+1}(2)$ increases in $q_H$ because the larger fraction of the congruent type $H$ indicates that the radical policy is less informative about the true state of the world. To see this point, note that the congruent type $H$ implements policy 2 even when the state is 1. In the extremism equilibrium, voters expect that an extreme policy might be chosen even when the state is moderate, which reduces the information value of the policy on the state of the world. As a result, the belief $p$, which we regard as the depth of extremism in Section 5, may not significantly decrease after the observation of policy 2.\footnote{When we analyze the contagion of extremism in Section 5, observing policy 2 always decreases the belief $p$. In Markovian environment, on the other hand, it may even increase after observing the radical policy because the belief tends to move toward the stationary state distribution of the states. Indeed, we can show that fixing $q$, $\Pr(p_{i+1} \geq \bar{p})$ is not necessarily decreasing in $q_H$. Moreover, in an extreme case, the extremism equilibrium cannot be realized in consecutive countries because $p_{i+1}(2) > \bar{p}$ always holds in the extremism equilibrium.} This effect mitigates the spread of extremism. The presence of these two counterforces implies that the significance of the contagion of extremism exhibits no simple pattern. Depending on which effect is dominant, the contagion becomes more moderate as $q_H$ increases.

It is important to keep in mind that in terms of the average welfare, the last column of Table 2 shows that the frequent extremism equilibrium may not necessarily translate into the larger welfare loss. For
example, in columns 1-3, they exhibit the negative relationship while the positive relationship appears in columns 4-6. The reason is that the welfare loss is caused not only by extremism but also by tyranny. In particular, the non-congrucent type is more likely to take policy 0 as \( p \) decreases. Since the loss of policy 0 under \( \omega = 2 \) is the largest, the welfare loss from such tyranny might be more important than that from extremism. (2) implies that the lower bound of \( p \) is determined by \( \theta_2 \) (i.e., the persistence of the extreme state). Roughly speaking, in such a case, voters do not think that the extremism is not very severe even when it does happen. It prevents the most detrimental form of tyranny and enhances the average welfare. Whether the more frequent extremism equilibrium implies the larger welfare loss depends on the source of extremism as well as the way to evaluate welfare.

Overall, we conclude that even if the convergence to extremism does not occur, the extremism equilibrium could be frequently observed and persistent in the long-run when the spread of populism identified in Section 5 is serious.

We have experimented various numerical examples and confirmed the robustness of the arguments in this subsection. Moreover, several available analytical characterizations also support the arguments. In the next section, we analytically argue the comparative statics regarding the stability of the state of the world \((\theta_1 \text{ and } \theta_2)\). In Appendix B3, we analytically characterize the long-run behavior of the model by restricting parameter values, and derive the analogous comparative static results.

### 6.3 Effect of Imperfect Correlation

So far, we have observed that the results in the Markovian environment are quite different from those in Section 5. When the state of the world is the same across countries, extremism eventually disappears. In contrast, when the state of the world is fluctuated, the spread of extremism may be serious, even in the long-term. Our analysis in this subsection bridges the two findings in Sections 5 and 6.

We start with obtaining the following proposition.

**Proposition 8.** Suppose \( \theta_1 = 1 \). Then, for any \( \epsilon > 0 \), \( \lim_{N \to \infty} \Pr(|p_N - 1| > \epsilon) = 0 \).

**Proof.** See Appendix A.

This proposition considers the case in which \( \omega = 1 \) is the absorbing state. In such a case, the belief probabilistically converges to one, meaning that, in the long-term, extremism does not matter, and social learning works well. This suggests that what is key for the optimistic result is that the moderate policy is stably optimal (i.e., \( \omega = 1 \) is stable). What prevents this from stopping the domino effect is that the optimal policy might change from the moderate one to the radical one over time.

This property can be extended to the more general case in which \( \theta_1 < 1 \). Remember that \( p_E \) is the threshold value at which the convergence to extremism occurs and \( p_E \) is increasing in \( \theta_1 \). From this observation, we first directly obtain the following fact.

**Fact 5.** There exists \( \bar{\theta} \in (1/2, 1) \) such that \( \bar{p} \geq p_E \) if and only if \( \theta_1 \leq \bar{\theta} \).
This fact argues that there is a threshold value \( \tilde{\theta} \) such that if \( \theta_1 \leq \tilde{\theta} \), the convergence to extremism equilibria occurs. Otherwise, cycles of extremism arise. This implies that the instability of the moderate state (i.e., state 1) leads to the convergence to extremism.

Furthermore, the expected time to remain in the non-extremism equilibrium also depends on \( \theta_1 \). The expected duration of the non-extremism equilibrium conditional on \( p_i \) is given by

\[
D(p_i) \equiv E \left[ \{ j \in \{i,\ldots,N\} : p_k \geq \tilde{p} \ \forall k \in \{i,\ldots,j\} \} \mid p_i \right].
\]

Given this definition, we obtain the following proposition indicating that the more stable the moderate state is, the longer the duration of the non-extremism equilibrium is.

**Proposition 9.** \( D(p_i) \) is weakly increasing in \( \theta_1 \).

**Proof.** See Appendix A.

These arguments highlight that the parameter \( \theta_1 \) —the stability of the moderate state—is the key in determining the long-run properties of extremism. So long as the moderate state is perfectly stable, the domino effect of extremism definitively stops from the results in Section 5 and Proposition 8. However, if the state is very unstable, we observe the convergence to extremism. In intermediate cases, the cycle of extremism emerges. Proposition 9 further validates this interpretation since it shows that the non-extremism equilibrium is unstable when the moderate state is also unstable.

To be precise, an increase in \( \theta_1 \) implies an increase in the steady state probability of the state being moderate and the higher stability of the moderate state. Hence, one might think that the aforementioned result simply indicates that the higher steady state probability of the moderate state is the one that prevents extremism. To see that this is not the case, suppose that the steady state probability distribution is fixed: \((1 - \theta_2)/(1 - \theta_1) = s^*\), where \( s^* > 0 \) is constant.\(^{51}\) For such \( \theta_1 \) and \( \theta_2 \), \( \theta_1 = 1 - \rho \) and \( \theta_2 = 1 - s^* \rho \) hold, where \( \rho \in (0, 1/2 \min\{1, 1/s^*\}) \). A decrease in \( \rho \) implies a higher \( \theta_1 \), keeping the steady state probability distribution. Hence, if a decrease in \( \rho \) makes extremism more easily sustained, a change in the stability of the state of the world, rather than that in the steady state probability, is the key mechanism. We obtain such a result:

**Fact 6.** \( p_E \) is strictly decreasing in \( \rho \).

**Proof.** See Appendix A.

Fact 6 implies that, given other parameters, increasing \( \rho \) may trigger convergence to extremism, indicating that it is more likely to occur as the stability of each state increases. To confirm this result in our numerical examples reported in the first three rows of Table 2, we change only \( \theta \) while fixing other parameters.\(^{52}\) In this case, the convergence to extremism occurs as \( \theta \) decreases, which is consistent with Fact 6 that \( p_E \) is increasing in \( \theta \). Moreover, rows 1-3 indicate that the fraction of the extremism

\(^{51}\)The steady state probability of \( \omega \) being 1 (say, \( p_S \)) is given by \( p_S/(1 - p_S) = (1 - \theta_2)/(1 - \theta_1) \).

\(^{52}\)In these examples, \( \theta_1 = \theta_2 \) i.e., \( s^* = 1 \).
equilibrium is decreasing in $\theta$ even when perfect convergence does not occur. These results imply that the spread of extremism may be more serious in an unstable world.\textsuperscript{53}

This leads to an interesting implication about the effect of the length of the interval between each country’s election. In reality, there is some time interval between the election in one country and that in another country. The longer such interval is, the more unstable the state of the world would be. Hence, the longer interval might prevent us from stopping extremism. This suggests that a year in which elections will be held in various countries is the chance to stop the domino effect of populist extremism because the interval between elections is short in the year.

\section{Discussions}

\subsection{Dynamic Election Model}

Consider the following extension of our basic model. There are two periods ($t=1,2$). In period 1, there is an incumbent politician. In each period, there is one policy issue. The policy issue in period 1 is the same as that of our basic model. In period 2, there is another policy issue $y$. The policy regarding this issue is chosen from $\{0,1,2\}$. Let the policy chosen by country $i$’s policymaker in period 2 be $y_i$.

At the beginning of period 2, there are two candidates: the incumbent and a challenger who is the congruent type with probability $q$. Let the valence advantage of the incumbent be $q$, which follows a uniform distribution $U[-\varepsilon,\varepsilon]$ where $\varepsilon > 0$.

Voter $i$’s utility is given by

\[
-L(|x_i - \omega_i|) - kL(|y_i - \omega_i'|) + 1_i \theta,
\]

where $1_i$ is the indicator function that is denoted by one if the incumbent is reelected. As in the objective function, the voter’s optimal policy for the issue in period 2 is $\omega_i' \in \{1,2\}$. However, since the issue is different, its relevance is also different. $k > 0$ represents the importance of the issue in period 2.\textsuperscript{54} The prior probability of $w_i' = 1$ is denoted by $r \in [0,1]$. To exclude learning about $\omega_i'$ and focus on that about $\omega_i$, we, for simplicity, assume that $\omega_i'$ is determined independently across countries. The voter decides whether to reelect the incumbent or not based on this expected utility. When the voter is indifferent between the incumbent and the challenger, the incumbent is reelected.

The congruent type’s utility is given by

\[
-L(|x_i - \omega_i|) + 1_i [\lambda_i - kL(|y_i - \omega_i'|)],
\]

\textsuperscript{53}Note, however, that the welfare improves in the unstable world because tyranny is curbed to some extent. See the discussion at the end of the previous section.

\textsuperscript{54}Another interpretation is that the policy is irreversible to some extent. Suppose that both are the same issue (and thus $\omega_i = \omega_i'$). If we consider the situation in which the policy determined in period 1 can be changed in period 2 only with a certain probability, we obtain a similar objective function and the result.
where $1_i$ is the indicator function which takes one if the politician is reelected in period 2, and $\lambda_i \geq 0$ represents the office-seeking motivation. On the other hand, the non-congruent type’s utility is

$$-L(|x_i|) + 1_i[\lambda_i - kL(|y_i|)].$$

The voters do not want to re-elect the non-congruent type politician to avoid the implementation of undesirable policies in the second period. This situation is analogous to the model of Acemoglu, Egorov, and Sonin (2013), in which voters try to avoid electing corrupt elite politicians.

Under this setting, the reelection probability of the incumbent is equal to

$$\Pr(\theta \geq (q - \pi)k[r + l(1 - r)]) = \frac{1}{2} - \frac{qk}{2\varepsilon} [r + l(1 - r)] + \frac{\pi_i(x_i)k}{2\varepsilon} [r + l(1 - r)],$$

under the assumption that $\varepsilon \geq (1 - q)k[r + l(1 - r)]$. Hence, ignoring constants, the incumbent’s objective at the beginning of period 1 is to maximize

$$-L(|x_i - x_i^*|) + \frac{\lambda_i k}{2\varepsilon} [r + l(1 - r)]\pi_i(x_i),$$

where $x_i^* = \omega_i$ for the congruent type while $x_i^* = 0$ for the non-congruent type. Hence, by defining $b_i \equiv \frac{\lambda_i k}{2\varepsilon} [r + l(1 - r)]$, the present dynamic election model is reduced to the original model. In particular, the congruent type $L$ has low office-seeking motivation (i.e., small $\lambda_i$) so that $b_i$ is sufficiently low.

### 7.2 Applications

Although our model is presented in the context of populist extremism, it can also be applied to various political and non-political contexts in which players face principal-agent problems.

#### 7.2.1 Policy Diffusion

Our results indicate the diffusion of radical policies across countries. Empirically, various policies have been found to be correlated across space. As summarized in Table 3, the examples include, but not limited to, taxation, regulation, and public spending. While various explanations are possible for the policy diffusion, our results also indicate a novel explanation about it and yield some interesting results. In this subsection, we particularly discuss the implications about yardstick competition and the diffusion of parties’ policy positions. The discussion about other examples of policy diffusion (political polarization, corporate income taxation, and minimum wages) can be seen in Appendix B8.

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55 As in Kartik and Van Weelden (2019), we normalize the payoff when they are not reelected as zero. Using a slightly different model, analogous results are obtained if the unelected candidate derives utility from policy.

56 Since politicians are unaccountable in period 2, the expected payoff when the voter reelects the incumbent is $-(1 - \pi_i(x_i))k[r + l(1 - r)]$, while that when the voter elects the challenger is $-(1 - q)k[r + l(1 - r)]$. The former is larger than or equal to the latter if and only if $\theta \geq (q - \pi)k[r + l(1 - r)]$.

57 Note that $b < l - 1$ holds under weak conditions. To illustrate, suppose that $\lambda_i k/\varepsilon$ for the non-congruent type and the...
Yardstick competition. As an illustration, consider personal income taxation. Besley and Case (1995) show that in the absence of factor mobility, policies are spatially contagious because of voters’ benchmarking behavior. In their model, excessive spending may be curbed when citizens can observe the political outcomes of other jurisdictions because it serves as a useful benchmark for ousting bad (rent-seeking) politicians in their election. Our model, on the other hand, reveals a new disadvantage of voters’ benchmarking in elections.

Suppose that when $\omega = 1(2)$, a jurisdiction experiences moderate (low) fiscal needs. Policy 1 includes a moderate tax rate, policy 2 includes a low tax rate, and policy 0 includes a high tax rate. Here, following Besley and Case (1995), we assume that policy 0 is preferred by a “Leviathan” who seeks fiscal control and is unambiguously harmful to voters. The utility loss of voters depends on the difference between their fiscal needs and the implemented tax rate.

Our propositions predict that sub-optimally low tax rates may be contagious even when fiscal needs are not significantly low. They imply that information flow between jurisdictions may induce excessively low tax rates, leading to the collapse of the welfare state. The congruent type $H$ sets a lower tax rate than the country’s fiscal needs to signal that they are not a “Leviathan”. Moreover, this process spreads because a low tax rate in jurisdiction 1 makes jurisdiction 2 lower its tax rate through voters’ downward belief updating about the fiscal needs. Thus, if the belief about fiscal needs happens to decline in one jurisdiction, this process is contagious. This result reveals an important side-effect of yardstick competition. Typically, yardstick competition is regarded as a virtue of a decentralized system. However, incorrect information may be transmitted through political agency problems. This result contributes to the large body of literature discussing the benefits and drawbacks of decentralized political systems.

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Table 3: Examples of studies on policy diffusion.

<table>
<thead>
<tr>
<th>Policy issue</th>
<th>Examples of studies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Personal income taxation</td>
<td>Besley and Case (1995); Eugster and Parchet (2019)</td>
</tr>
<tr>
<td>Corporate income taxation</td>
<td>Devereux, Lockwood, and Redoano (2008); Devereux and Loretz (2013)</td>
</tr>
<tr>
<td>Environmental regulation</td>
<td>Levinson (2003); Konisky (2007)</td>
</tr>
<tr>
<td>Welfare spendings</td>
<td>Brueckner (2000); Dahlberg and Edmark (2008)</td>
</tr>
<tr>
<td>Immigration policy</td>
<td>Rayp, Ruyssen, and Standaert (2017)</td>
</tr>
<tr>
<td>Minimum wages</td>
<td>Li, Kanbur, and Liu (2018)</td>
</tr>
<tr>
<td>Tobacco control</td>
<td>Pachenco (2012)</td>
</tr>
<tr>
<td>Debt issuance</td>
<td>Borck et al. (2015)</td>
</tr>
</tbody>
</table>

Although our model is different from that of Besley and Case (1995), an analogous mechanism is at work. To illustrate, suppose $\omega = 2$ for all countries and that a country moves from (NE) equilibrium to (E1) equilibrium by observing policy 2 in the previous country. Note that policy 2 in the previous country conveys information about the true state and serves as a benchmark. In this case, politicians’ behavior do not change, but voters are now more confident that policy 1 is taken by the non-congruent type. This process allows voters to detect the non-congruent type with higher probability.

congruent type $H$ is one. Then, $b < l - 1$ can be rewritten as $l > (r/2 + 1)/(r/2 + 1/2)$, which always holds when $l > 2$.
**Parties’ policy positions.** Our argument is related to the policy diffusion through parties’ policy positions, which has recently been brought into the spotlight by Böhmelt et al. (2016). On the left-right scale of party positions, Böhmelt et al. (2016) show the interdependence across countries. While Böhmelt et al. (2016) and subsequent works primarily focus on the heuristic emulation of foreign successful parties’ positioning by domestic parties, we show that voters’ learning may also induce this diffusion because parties change their policy positions in response to voters’ belief updating. Party policy diffusion could be the case not just because of learning by parties, but because voters learn from parties in other countries and change opinions, which in turn changes parties’ positions.\(^{59}\)

This result implies that controlling for voters’ preferences, mainly the median voters’ position, may cause an over-control problem in empirically detecting the diffusion effect. To see this point, let \(y_{it}\) denote a party’s position in country \(i\) at time \(t\). The canonical regression equation in this literature is

\[
y_{it} = \phi y_{it-1} + \beta X_{it-1} + \rho \sum_{j \neq i} w_{ij} y_{je-1} + \epsilon_{it},
\]

where \(X_{it-1}\) are the lagged control variables, \(w_{ij}\) is the spatial weight, and \(y_{je-1}\) is party \(j\)’s position in the year before the last election held in their country before time \(t\) (Böhmelt et al. 2016, Equation 1). The parameter of interest is \(\rho\), which captures the effect of foreign parties’ positions on party \(i\)’s position. Now, suppose that \(X_{it-1}\) includes the median voters’ position. Then, given that it is influenced by foreign party’s positioning, \(X_{it-1}\) can be written as a function of \(\sum_{j \neq i} w_{ij} y_{je-1}\), implying that the term \(\beta X_{it-1}\) absorbs parts of the effects which should be captured by the coefficient \(\rho\). Thus, as long as party policy diffusion through voters’ updating is an effect of interest, control variables regarding voters’ preference should be carefully chosen to avoid over-controlling for these effects.

### 7.2.2 Applications to Non-Political Issues

Our model can be also applied to non-political settings where agency problems and reputation concerns of agents co-exist (Ely and Välimäki 2003). Examples include doctors and patients.\(^{60}\) Some “bad-type” doctors might recommend new drugs because they are bribed by pharmaceutical companies. Then, some “good-type” doctors, who seek to provide the best care to patients, might refrain from using new drugs to distinguish themselves from bad-type doctors and acquire higher reputation. Our results indicate that such signaling behavior may be contagious and induce large welfare loss.\(^{61}\) Appendix B8 contains applications to irreversible investment and the allocation of scientific grants.

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\(^{59}\)This argument is similar to the idea of Pacheco (2012), but we put it into the context of party policy diffusion and formalize the idea.

\(^{60}\)Ely and Välimäki (2003) also provide examples of car mechanics and lawyers.

\(^{61}\)In the model of Ely and Välimäki (2003), the signaling motive might be so strong that the market is unsustainable. Our results of contagion implies that the welfare loss might be quite large even without the market collapse.
8 Concluding Remarks

The aim of this study was to investigate the so-called domino effect of populist extremism. In particular, we explored whether populist extremism spreads across countries and, if so, whether it eventually stops. To this end, we constructed a multi-country model in which each country’s politician sequentially implements a policy. In the model, voters face information asymmetries about the incumbent politician’s type (congruent type vs. non-congruent type) as well as the state of the world. The novelty of our model is that voters’ learning about the state of the world is affected by policies implemented in foreign countries. The distorted learning pattern affects the policy outcome, which in turn affects voters’ learning in the subsequent country.

In the benchmark single-country model, we showed that populist extremism can arise in the presence of high reputation concerns. Notably, the condition for the emergence of extremism depends on voters’ beliefs about the state of the world. Based on this static result, we analyzed the multi-country model. It was shown that extremism is contagious across countries, at least in the short-term. Interestingly, whether this domino effect stops in the long-term depends on the cohesion of the state of the world across countries. We showed that extremism eventually stops spreading under the perfect correlation, whereas either the convergence towards extremism or cycles of extremism hold when the state of the world follows a Markov process without absorbing states. Overall, the results suggested that the dysfunction of democracy in one country endangers democracy in other countries.

Before concluding this paper, we discuss the remaining challenges for future research. First, populism has various aspects and some characteristics fall outside our scope of research. Examining whether similar patterns of the propagation occurs for other aspects of populism may be worthwhile. Second, studying learning patterns in more complex networks might be also beneficial. Given recent developments on these issues, it might be promising to introduce a network structure. Third, although our model assumes Bayesian rationality to focus on the fundamental contagion mechanism, voters might not be Bayesian rational in reality. Extending the model to non-Bayesian updating cases might be worthwhile. These issues are left to future work.

Appendix A Omitted Proofs

A1 Proof of Lemma 1

(i): Suppose that there exists such an equilibrium. In this equilibrium, \( \pi(1) = 1 \) holds since the non-congruent type never chooses 1 (see (v)). Given this, consider the deviation incentive of the non-congruent type. S/he deviates from 0 to 1 if \(-1 + b < 0\). This holds since \( b > 2 \). Hence, there is no such equilibrium.

(ii): The non-congruent type loses \( l \) by taking 2 instead from 0. Since \( b < l - 1 \), it never does so whatever belief the voter holds.

(iii): From (ii), \( \pi(2) = 1 \). Given this, \( \alpha^*(2; 2) = 1 \) must hold because it is the ideal policy for the congruent type and it ensures the high reputation.
From (ii), $\pi(2) = 1$. Given this, $x^*(0; \omega) > 0$ can be the case only when $\pi(0) = 1$. Note that taking 2 also brings the same utility loss of 1 and ensures the high reputation. However, when $\pi(0) = 1$, the non-congruent type chooses 0 so that $\pi(0) \neq 1$. This is contradiction.

(v): The congruent type loses at least one by taking the policy different from $\omega$. Since $b_L < 1$, this is not optimal for the congruent type $L$. ■

A2 Proof of Proposition 1

(a) First, observe that from this equilibrium strategy,

$$\pi(2) = 1; \quad \pi(1) = \frac{q_L p}{q_L p + (1 - q)}. \quad (3)$$

Given this, the congruent type $H$ has no deviation incentive from 2 to 1 when $\omega = 1$ if and only if

$$-1 + b \geq \frac{q_L p}{q_L p + (1 - q)} \iff p \leq \frac{(b - 1)(1 - q)}{q_L}. \quad (4)$$

Note that the congruent type $H$ obviously has no deviation incentive when $\omega = 2$.

Next, consider the non-congruent type’s incentive. S/he has no incentive to deviate from 1 to 0 if and only if

$$-1 + b \frac{q_L p}{q_L p + (1 - q)} \geq 0 \iff p \geq \frac{1 - q}{(b - 1)q_L}. \quad (5)$$

Note that the non-congruent type has no incentive to choose 2 from Lemma 1.

Combining (4) and (5) yield the condition.

(b) First of all, from Lemma 1, $\beta^*(2; \omega) = 0$.

The non-congruent type mixes 1 and 0 only when s/he is indifferent between these two policies i.e.,

$$-1 + b \frac{pq_L}{pq_L + \beta^*(1)(1 - q)} = 0 \iff \beta^*(1) = \frac{(b - 1)pq_L}{1 - q}. \quad (6)$$

This $\beta^*(1) < 1$ if and only if

$$p < \frac{1 - q}{(b - 1)q_L}.$$

Given this, examine whether the congruent type $H$ has a deviation incentive. The congruent type $H$ has no incentive to deviate from 2 to 1 if and only if

$$-1 + b \geq \frac{pq_L}{pq_L + \beta^*(1)(1 - q)} \iff b \geq 2,$$

which holds since $b > 2$. 39
In this equilibrium,

\[ \pi(2) = 1; \quad \pi(1) = \frac{pql + p\alpha^*(1;1)q_H}{pql + p\alpha^*(1;1)q_H + (1-q)}. \]

Given this, the congruent type \( H \) mixes 1 and 2 when \( \omega = 1 \) if and only if

\[ b - \frac{pql + p\alpha^*(1;1)q_H}{pql + p\alpha^*(1;1)q_H + (1-q)} = -1 + b \Leftrightarrow \alpha^*(1;1) = \frac{(b-1)(1-q)}{pq_H} - \frac{q_L}{q_H}. \]  

(7)

Here, the derived \( \alpha^*(1;1) \) is less than one if and only if

\[ p \geq \frac{(b-1)(1-q)}{q}. \]

(8)

In addition, it is larger than zero if and only if

\[ p \leq \frac{(b-1)(1-q)}{q_L}. \]

(9)

Combining these two inequalities, we have the condition.

Lastly, examine the non-congruent type’s incentive. Since the deviation incentive of the non-congruent type is minimized when \( \pi(0) = 0 \), s/he has no deviation incentive when

\[ -1 + b - \frac{pql + p\alpha^*(1;1)q_H}{pql + p\alpha^*(1;1)q_H + (1-q)} \geq 0 \Leftrightarrow b \geq 2, \]

which holds since \( b > 2 \).

(d) First, the equilibrium belief is given by

\[ \pi(2) = 1; \quad \pi(1) = \frac{pq}{pq + (1-q)}. \]

Given this belief, the congruent type \( H \) has no incentive to deviate from 1 to 2 when \( \omega = 1 \) if and only if

\[ -1 + b \leq \frac{pq}{pq + (1-q)} \Leftrightarrow p \geq \frac{(b-1)(1-q)}{q}. \]

(10)

Note that the congruent type \( H \) obviously has no deviation incentive when \( \omega = 2 \).

Next, consider the non-congruent type’s deviation incentive. S/he has no incentive to deviate from 1 to 0 if and only if

\[ -1 + b - \frac{pq}{pq + (1-q)} \geq 0 \Leftrightarrow p \geq \frac{1-q}{(b-1)q}, \]

(11)

This is because the deviation incentive is minimized when \( \pi(0) = 0 \).

Combining (10) and (11), we have the lemma. Note that because \( b > 2 \),

\[ \frac{(b-1)(1-q)}{q} \geq \frac{1-q}{(b-1)q}. \]

(e) Denote the set of policies chosen by the congruent type \( H \) with a positive probability given the state \( \omega \) by \( X^c_H(\omega) \equiv \{ x \in \{0,1,2\} : \alpha^*(x;\omega) > 0 \} \), and denote the element of \( X^c(\omega) \) by \( x^c(\omega) \). Similarly, define \( X^n \)
and \( x^*_N \) for those of the non-congruent type corresponding to the above notions.

Step 1. We start by investigating the conditions under which equilibria fully separate such that \( X^*_C(\omega) \cap X^*_N = \emptyset \) for all \( \omega \). Prove that there is no separating equilibrium except for (E1) and (E2) equilibria. From Lemma 1 (i) and (ii), if such an equilibrium exists, either (I) \( \beta^*(1) = 1 \), or (II) \( \beta^*(1) + \beta^*(0) = 1 \), \( \beta^*(1) \in (0, 1) \), and \( \beta^*(0) \in (0, 1) \). Then, from Lemma 1 (iv), \( \alpha^*(2; \omega) = 1 \) holds in any fully separating equilibrium.

Step 2. Next, we explore semi-separating equilibria such that \( X^*_C(\omega) \cap X^*_N \neq \emptyset \) for some \( \omega \), but \( X^*_C(\omega) \neq X^*_N \) for some \( \omega \). Prove that there is no separating equilibrium except for (E3) and (NE) equilibria. From Lemma 1 (i) and (ii), if such an equilibrium exists, either (I) \( \beta^*(1) = 1 \), or (II) \( \beta^*(1) + \beta^*(0) = 1 \), \( \beta^*(1) \in (0, 1) \), and \( \beta^*(0) \in (0, 1) \).

Case (I). \( \alpha^*(1; 1) > 0 \) must hold in semi-separating equilibria because \( \alpha^*(2; 2) = 1 \) from Lemma 1 (iii). When \( \alpha^*(1; 1) = 1 \), this is the equilibrium in (a). When the congruent type mixes 1 and 2, that is the equilibrium in (b).

Case (II). As in case (I), \( \alpha^*(1; 1) > 0 \) must hold. Consider the case where \( \alpha^*(1; 1) = 1 \) and the case where the congruent type \( H \) takes a mixed strategy one by one.

Case (II-1). \( \alpha^*(1; 1) = 1 \). The non-congruent type mixes 0 and 1 if and only if

\[
-1 + \frac{pq}{pq + \beta^*(1)(1-q)} = 0 \iff \beta^*(1) = \frac{(b-1)pq}{1-q}.
\]

Given this, the congruent type \( H \) has no incentive to deviate from 1 to 2 when \( \omega = 1 \) if and only if

\[
b \geq -1 + b \iff b \leq 2,
\]

which does not hold. Hence, there is no such an equilibrium.

Case (II-2). Mixed strategy. The congruent type \( H \) mixes 1 and 2 when \( \omega = 1 \) if and only if

\[
b \geq \frac{pqL + \alpha^*(1; 1)qH}{pqL + \alpha^*(1; 1)pqH + \beta^*(1)(1-q)} = -1 + b.
\]  \tag{12}

Similarly, the non-congruent type mixes 0 and 1 if and only if

\[
-1 + \frac{pqL + \alpha^*(1; 1)qH}{pqL + \alpha^*(1; 1)pqH + \beta^*(1)(1-q)} = 0.
\]  \tag{13}

(12) and (13) simultaneously hold only when \( b = 2 \). Hence, this equilibrium does not exist.

Step 3. Lastly, there is no equilibrium in which for any \( \omega \), \( X^*_C(\omega) = X^*_N \) because \( \alpha^*(2; 2) = 1 \) but \( \beta^*(2) = 0 \) (Lemma 1).

From steps 1-3, we have (e). ■

A3 Proof of Lemma 2

Step 1. Prove that there is an equilibrium such that \( \alpha^*(2; 2) = 1 \) and \( \beta^*(0) = 1 \). Consider a belief system such that \( \pi(1) = 0 \). Then, it is straightforward that no one has deviation incentive.
Step 2. Prove that there exist no other equilibria. From Lemma 1, $\alpha^*(2; 2) = 1$ and $\beta^*(2) = 0$. Hence, the candidate of other equilibria is that $\beta^*(1) > 0$. Prove by contradiction. When $\beta^*(1) > 0$, $\pi(1) = 0$ since $p = 0$. Thus, the non-congruent type has no incentive to choose 1, which contradicts $\beta^*(1) > 0$. ■

A4 Proof of Lemma 3

From Theorem 1, $p_{i+1}(p_i, 2)$, $p_{i+1}(p_i, 1)$, and $p_{i+1}(p_i, 0)$ are increasing in $p_i$ and $p_{i+1}(p_i, 2) < p_{i+1}(p_i, 0) < p_{i+1}(p_i, 1)$. Note that both $Pr(x_i = 2|p_i)$ and $Pr(x_i = 0|p_i)$ are weakly decreasing in $p_i$.

Fix $\tilde{p}$. Depending on the value of $p_i$, we can potentially have the following cases. The cases are ordered so that a case with a smaller number corresponds to that under smaller $p_i$.

Case 1. $p_{i+1}(p_i, 1) < \tilde{p}$: In this case, $Pr(p_{i+1} < \tilde{p}|p_i) = 1$.

Case 2. $p_{i+1}(p_i, 0) < \tilde{p} \leq p_{i+1}(p_i, 1)$: In this case, $p_{i+1} < \tilde{p}$ if and only if $x_i = 0$ or 2. Hence, $Pr(p_{i+1} < \tilde{p}|p_i) < 1$. In addition, since both $Pr(x_i = 2|p_i)$ and $Pr(x_i = 0|p_i)$ are weakly decreasing in $p_i$, $Pr(p_{i+1} < \tilde{p}|p_i)$ is weakly decreasing in $p_i$.

Case 3. $p_{i+1}(p_i, 2) < \tilde{p} \leq p_{i+1}(p_i, 0)$: In this case, $p_i < \tilde{p}$ if and only if $x_i = 2$. Hence, $Pr(p_{i+1} < \tilde{p}|p_i)$ is smaller than that of Case 2. In addition, since $Pr(x_i = 2|p_i)$ is weakly decreasing in $p_i$, $Pr(p_{i+1} < \tilde{p}|p_i)$ is weakly decreasing in $p_i$.

Case 4. $\tilde{p} \leq p_{i+1}(p_i, 2)$: In this case, $Pr(p_{i+1} < \tilde{p}|p_i) = 0$ regardless of the value of $p_i$.

From these cases, $Pr(p_{i+1} < \tilde{p}|p_i)$ is weakly decreasing in $p_i$. ■

A5 Proof of Proposition 3

(a) Without loss of generality, we focus on the case where $k = 1$. Lemma 3 directly shows that $Pr(p_2 \geq \tilde{p})$ is weakly increasing in $p_1$.

Moreover, invoking Lemma 3 again, for each $\tilde{p} \in (0, 1]$, $Pr(p_2 < \tilde{p}|p_1)$ is weakly decreasing in $p_1$. Here,

$$Pr(p_3 \geq \tilde{p}|p_1) = \sum_{p'_2 \in \text{Supp}(p_2|p_1)} [Pr(p_3 \geq \tilde{p}|p_2 = p'_2) \cdot Pr(p_2 = p'_2|p_1)], \quad (14)$$

where for $i \geq 2$,

$$\text{Supp}(p_i|p_1) \equiv \{p_i \in [0, 1] | Pr(p_i = p'_i|p_1) > 0\}.$$  

In addition, $Pr(p_3 \geq \tilde{p}|p_2 = p'_2)$ is non-decreasing in $p'_2$ since from Lemma 3, for each $\tilde{p} \in (0, 1]$, $Pr(p_3 < \tilde{p}|p_2)$ is weakly decreasing in $p_2$. Compare $p_1 = p_H$ and $p_1 = p_L$ where $p_H > p_L$. From Lemma 3, $Pr(p_2 < \tilde{p}|p_1 = p_H) \leq Pr(p_2 < \tilde{p}|p_1 = p_L)$ for each $\tilde{p} \in (0, 1]$, meaning that the distribution of $p_2$ under $p_1 = p_H$ first-order stochastically dominates that under $p_1 = p_L$. Hence, from the property of the first-order stochastically dominance, (14) is weakly increasing in $p_1$. Thus, we have proven the assertion for $i = 2$.

The result for $i = 2$ directly implies that $Pr(p_3 < \tilde{p}|p_1)$ is weakly decreasing in $p_1$. Having this result at hand, we can repeat the same argument for all $i \geq 3$ to show that

$$Pr(p_{i+1} \geq \tilde{p}|p_1) = \sum_{p'_i \in \text{Supp}(p_i|p_1)} [Pr(p_{i+1} \geq \tilde{p}|p_i = p'_i) \cdot Pr(p_i = p'_i|p_1)] \quad (15)$$

---

62 When $x_i = 0$ is the off-equilibrium path, we ignore the case where $x_i = 0$.

63 This is a finite set.

64 In our model, the distribution of $p_2$ is a discrete distribution. However, even in a discrete case, the property holds (Courtault, Crettez, and Hayek 2006).
is weakly increasing in $p_1$.

(b) From (a), $\Pr(p_1 \leq \bar{p}|p_2)$ is weakly increasing in $p_2$. Furthermore, $p_2(2) < p_2(1)$ and $p_2(2) < p_2(0)$. Hence, we have the first part of (b). Next, prove the second part.

Case 1. $\frac{1-q}{(b-1)qL} \leq \bar{p}$: Consider $p_1 \in (0, \bar{p})$ that is sufficiently close to $\bar{p}$. Since $x_1 = 0$ is an off-equilibrium path, it suffices to show that $\Pr(p_1 \geq \bar{p}|x_1 = 2) < \Pr(p_1 \geq \bar{p}|x_1 = 1)$. For such $p_1$, $p_2(2, p_1) < \bar{p} < p_2(1, p_1)$. Thus, $\Pr(p_i \geq \bar{p}|x_1 = 2) < 1 = \Pr(p_i \geq \bar{p}|x_1 = 1)$.

Case 2. $\frac{1-q}{(b-1)qL} > \bar{p}$: In this case, for any $p \in (0, \bar{p})$, (E2) equilibrium is realized. Hence, we need to prove both $\Pr(p_i \geq \bar{p}|x_1 = 2) < \Pr(p_i \geq \bar{p}|x_1 = 1)$ and $\Pr(p_i \geq \bar{p}|x_1 = 2) < \Pr(p_i \geq \bar{p}|x_1 = 0)$.

(i) $\Pr(p_i \geq \bar{p}|x_1 = 2) < \Pr(p_i \geq \bar{p}|x_1 = 1)$. Consider $p_1 \in (0, \bar{p})$ that is sufficiently close to $\bar{p}$. Then, as in Case 1, $\Pr(p_i \geq \bar{p}|x_1 = 2) < 1 = \Pr(p_i \geq \bar{p}|x_1 = 1)$.

(ii) $\Pr(p_i \geq \bar{p}|x_1 = 2) < \Pr(p_i \geq \bar{p}|x_1 = 0)$. Again, consider $p_1 \in (0, \bar{p})$ that is sufficiently close to $\bar{p}$.

First, derive the upper bound of $\Pr(p_i \geq \bar{p}|x_1 = 2)$. There are three cases depending on the value of $x_2$.

Next, derive the lower bound of $\Pr(p_i \geq \bar{p}|x_1 = 0)$. There are three cases depending on the value of $x_2$.

Now, country 2 is in (E2) equilibrium so that $p_2(2) < p_2(1)$ implies $\Pr(x_2 = 1|x_1 = 2) < \Pr(x_2 = 1|x_1 = 0)$.

Hence, the right-hand side of (16) is strictly smaller than that of (17). That is, $\Pr(p_i \geq \bar{p}|x_1 = 2) < \Pr(p_i \geq \bar{p}|x_1 = 0)$.

From cases 1 and 2, we obtain the second part of (b).

A6 Proof of Proposition 5

(a) Prove by contradiction. Suppose that there exists $p^* \neq 1$ such that $\Pr(\lim_{N \to \infty} p_N = p^*) = 1$. If this gives us a contradiction, Lemma 4 implies that $\Pr(\lim_{N \to \infty} p_N = 1) = 1$.

Case 1. $p^* \in (0, 1)$: Since almost sure convergence implies convergence in probability, the following must hold: $\forall \epsilon > 0, \forall \delta > 0, \exists N^*(\epsilon, \delta)$ s.t.

$$\forall N \geq N^*(\epsilon, \delta) \quad \Pr(|p_N - p^*| \geq \epsilon) < \delta. \quad (18)$$

Let

$$\theta_1(\epsilon) \equiv \min \left\{ \frac{1 + (b-1)(p^* - \epsilon)}{b}, \frac{(p^* - \epsilon)(q_L + (1-q))}{(p^* - \epsilon)q_L + (1-q)}, \frac{p^* - \epsilon}{(p^* - \epsilon)q + (1-q)} \right\}.$$
Fix $\delta = \frac{q_0}{1+q_0}$ and $\epsilon$ such that $\theta_1(\epsilon) \geq p^* + \epsilon$.\(^{65}\) Consider $N \geq N^*(\epsilon, \delta)$. Then, Theorem 1 implies that when $|p_N - p^*| < \epsilon$, the updated belief is $p_{N+1}(1) > \theta_1(\epsilon) \geq p^* + \epsilon$. Since the probability that policy 1 is realized is at least $q_L$, $\Pr(|p_{N+1} - p^*| \geq \epsilon) \geq (1 - \delta)q_L \geq \delta$, which contradicts (18).

Case 2. $p^* = 0$: First, observe that when $\omega = 1$, $p_N > 0$ always holds. Hence, for $N \geq 2$, $p_N(1) > 0$. In particular, $p_N(1) > 1/b$ from the proof of Proposition 4. Furthermore, at least with probability $q_L$, $x_N = 1$ for any $p_N \in (0,1]$. Hence, $\Pr(p_N > 1/b) \geq q_L$ for all $N$, meaning that $\Pr(\lim_{N \to \infty} p_N \geq 1/b) > 0$.

From cases 1 and 2, $p^* \neq 1$ leads to a contradiction. That is, $p^* = 1$.

(b) Prove by contradiction. Suppose that there exists $p^* \neq 0$ such that $\Pr(\lim_{N \to \infty} p_N = p^*) = 1$.

Case 1. $p^* \in (0,1)$: As in case 1 of (a), the following must hold: $\forall \epsilon > 0, \forall \delta > 0, \exists N^*(\epsilon, \delta)$ s.t. (18).

Let

$$\theta_2(\epsilon) \equiv \max \left\{ 1 - \frac{(1 - (p^* + \epsilon))q}{(1 - (p^* + \epsilon))q_L + q_H}, 1 - \frac{(1 - (p^* + \epsilon))q}{q - (b - 1)(1 - q)}, 0 \right\} > 0.$$

Fix $\delta = \frac{q_0}{1+q_0}$ and $\epsilon$ such that $\theta_2(\epsilon) \leq p^* - \epsilon$.\(^{66}\) Consider $N \geq N^*(\epsilon, \delta)$. Then, Theorem 1 implies that when $|p_N - p^*| < \epsilon$, the updated belief is $p_{N+1}(2) < \theta_2(\epsilon) < p^* - \epsilon$ i.e., $|p_{N+1}(2) - p^*| > \epsilon$. Since the probability that policy 2 is realized is at least $q_L$, $\Pr(|p_{N+1} - p^*| \geq \epsilon) \geq (1 - \delta)q_L \geq \delta$, which contradicts (18).

Case 2. $p^* = 1$: As in case 1 of (a), the following must hold: $\forall \epsilon > 0, \forall \delta > 0, \exists N^*(\epsilon, \delta)$ s.t. (18). Fix $\epsilon = 1 - \bar{q}$ and $\delta = \frac{q_0}{1+q_0}$. Consider $N \geq N^*(\epsilon, \delta)$. Then, Theorem 1 implies that when $|p_N - 1| < \epsilon$, $p_{N+1} = 0$ with at least probability $q_L$. This is because $p_N \geq \bar{q}$ and thus the congruent type $L$ takes policy 2. Hence, $\Pr(|p_{N+1} - 1| \geq \epsilon) \geq (1 - \delta)q_L \geq \delta$, which contradicts (18).

From cases 1 and 2, $p^* \neq 0$ leads to a contradiction. That is, $p^* = 0$.

### A7 Proof of Proposition 6

(a) First, observe that (2) is increasing in $p_i$ because $p_{i+1}^*$ is increasing in $p_i$ when $p_i < \bar{p}$. Hence, the necessary and sufficient condition for (a) is equivalent that when $p_i = \tilde{p}$, $p_{i+1} \leq \bar{p}$ holds.

Case 1. $\frac{1-q}{b-qL} \leq \bar{p}$.

$p_{i+1} \leq \bar{p}$ can be rewritten as

$$p_{i+1}(\bar{p}, 1) \leq \bar{p} \iff \frac{\bar{p}qL + \bar{p}(1-q)}{\bar{p}qL + 1 - q}(\theta_1 + \theta_2 - 1) + 1 - \theta_2 \leq \bar{p}$$

$$\iff qL\bar{p}^2 + [(2 - \theta_1 - \theta_2)(1 - q) - \theta_1qL]\bar{p} - (1 - q)(1 - \theta_2) \geq 0. \quad (19)$$

Here, the left-hand side of (19) is

$$qL \frac{1 - \theta_2}{2 - \theta_1 - \theta_2} \left( \frac{1 - \theta_2}{2 - \theta_1 - \theta_2} - \theta_1 \right) = qL \frac{1 - \theta_2}{(2 - \theta_1 - \theta_2)^2} \left( \theta_1 + \theta_2 - 1 \right)(\theta_1 - 1) < 0$$

when $\bar{p} = p_S$ while it is $-((\theta_1 + \theta_2 - 1)(\theta_1 - 1)) > 0$ when $\bar{p} = \theta_1$. Hence, there exists $p_E \in (p_S, \theta_1)$ such that if and only if $\bar{p} \geq p_E$, (19) holds.

Case 2. $\frac{1-q}{b-qL} > \bar{p}$.

\(^{65}\)Such $\epsilon > 0$ exists because $\theta_1(\epsilon)$ is continuous with respect to $\epsilon$ and $\theta_1(0) > p^*$.

\(^{66}\)Such $\epsilon > 0$ exists because $\theta_2(\epsilon)$ is continuous with respect to $\epsilon$ and $\theta_2(0) < p^*$. 

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The condition \( p_{i+1} \leq \bar{p} \) can be rewritten as
\[
\begin{align*}
  p_{i+1}(\bar{p}, 1) \leq \bar{p} & \iff \frac{1 + (b-1)\bar{p}}{b} (\theta_1 + \theta_2 - 1) + 1 - \theta_2 \leq \bar{p} \\
  & \iff \bar{p} \geq p_E = \frac{\theta_1 + \theta_2 - 1 + b(1 - \theta_2)}{\theta_1 + \theta_2 - 1 + b(2 - \theta_1 - \theta_2)}.
\end{align*}
\] (20)

Here, we can easily verify that \( p_E \in (p_S, \theta_1) \).

From cases 1 and 2, we have (a).

(b) From (a), when \( p < \bar{p}, \Pr(p_N < \bar{p}) = 1 \) for all \( N \).

Next, consider \( p \geq \bar{p} \). From (a), \( \Pr(p_N < \bar{p}) = \Pr(\exists i \leq N : p_i < \bar{p}) \). Hence, it suffices to show that \( \lim_{N \to \infty} \Pr(\forall i \leq N : p_i \geq \bar{p}) = 0 \). Observe that when \( p_i \geq \bar{p} \) and \( x_i = 2, p_{i+1}(2) = 1 - \theta_2 < p_S < \bar{p} \). In addition, \( x_i = 2 \) at least with probability \( (1 - \theta_1)q_L \). Thus,
\[
\Pr(\forall i \leq N : p_i \geq \bar{p}) \leq [1 - (1 - \theta_1)q_L]^N.
\]
This goes to zero as \( N \to \infty \). Hence, \( \lim_{N \to \infty} \Pr(\forall i \leq N : p_i \geq \bar{p}) = 0 \).

(c) (i). Prove that \( \frac{\partial p_E}{\partial \theta_1} > 0 \). Consider case 1, first. Applying the implicit function theorem to (19) with equality yields
\[
\frac{\partial p_E}{\partial \theta_1} = -\frac{-q_HP_E}{2q_LP_E + (2 - \theta_1 - \theta_2)(1 - q) - \theta_1 q_L}.
\]
Here, since
\[
p_E[q_LP_E + (2 - \theta_1 - \theta_2)(1 - q) - \theta_1 q_L] = (1 - q)(1 - \theta_2)
\]
holds, the denominator is positive. In addition, the numerator is negative. Hence, \( \frac{\partial p_E}{\partial \theta_1} > 0 \) in case 1.

Next, consider case 2.
\[
\frac{\partial p_E}{\partial \theta_1} = \frac{b(1 - \theta_1)}{[\theta_1 + \theta_2 - 1 + b(2 - \theta_1 - \theta_2)]^2} > 0.
\]
From cases 1 and 2, we have \( \frac{\partial p_E}{\partial \theta_1} > 0 \).

(ii). Prove that \( p_E \) is weakly increasing in \( q_L \). \( \frac{1 - q}{(b-1)q_L} \) is decreasing in \( q_L \) so that there exists \( \bar{q}_L \) such that case 2 holds for \( q_L < \bar{q}_L \), while case 1 holds for \( q_L \geq \bar{q}_L \). Furthermore, \( p_E \) under case 2 when \( q_L \to \bar{q}_L \) is equal to \( p_E \) under case 1 when \( q_L = \bar{q}_L \).

Hence, it suffices to prove that \( \frac{\partial p_E}{\partial q_L} \geq 0 \) for \( q_L \geq \bar{q}_L \). Applying the implicit function theorem to (19) with equality yields
\[
\frac{\partial p_E}{\partial q_L} = -\frac{p_E(p_E - \theta_1)}{2q_LP_E + (2 - \theta_1 - \theta_2)(1 - q) - \theta_1 q_L}.
\]
Here, the denominator is positive. In addition, the numerator is negative since \( p_E < \theta_1 \) from (a). Hence, \( \frac{\partial p_E}{\partial q_L} > 0 \) for case 1. ■

A8 Proof of Proposition 7

(a) Observe that when \( p_i \geq \bar{p} \) and \( x_i = 2, p_{i+1}(2) = 1 - \theta_2 < \bar{p} \). In addition, \( x_i = 2 \) at least with probability \( (1 - \theta_1)q_L \). Thus,
\[
\Pr(\forall i \text{ s.t. } M \leq i \leq N : p_i \geq \bar{p}) \leq [1 - (1 - \theta_1)q_L]^{N-M}.
\]
This goes to zero as \( N \to \infty \). Hence, we have (a).

(b) Case 1. \( \frac{1-q}{(b-1)q} > \bar{p} \). Consider how many times \( x = 1 \) must be observed at most to reach \( p_i \geq \bar{p} \).

Define

\[
p_C = \frac{b[\bar{p} - (1 - \theta_2)]}{(b - 1)(\theta_1 + \theta_2 - 1)} - \frac{1}{b - 1},
\]

Note that \( p_C < \bar{p} \) holds since \( \bar{p} < p_E \) by the assumption.

(i) Suppose that \( p_i \geq p_C \). Then, \( p_{i+1}(1) \geq \bar{p} \).

(ii) Suppose that \( p_i < p_C \). Then,

\[
p_{i+1} - p_i = \frac{\theta_1 + \theta_2 - 1 + b(1 - \theta_2) - p_i(\theta_1 + \theta_2 - 1 + b(2 - \theta_1 - \theta_2))}{b} \geq \frac{\theta_1 + \theta_2 - 1 + b(1 - \theta_2) - \bar{p}[\theta_1 + \theta_2 - 1 + b(2 - \theta_1 - \theta_2)]}{b} > 0.
\]

Here, the first inequality comes from the fact that the function is decreasing in \( p_i \) and the last inequality comes from the assumption that \( \bar{p} < p_E \). Thus, when \( (x_i, \ldots, x_{i+K^*-1}) = (1, \ldots, 1) \), \( p_{i+K^*} \geq \bar{p} \) holds, where \( K^* \) is the smallest integer \( K \) satisfying

\[
K\frac{\theta_1 + \theta_2 - 1 + b(1 - \theta_2) - \bar{p}[\theta_1 + \theta_2 - 1 + b(2 - \theta_1 - \theta_2)]}{b} > \bar{p}.
\]

Hence, when \( (x_1, \ldots, x_{i+K^*-1}) = (1, \ldots, 1) \), \( p_{i+K^*} \geq \bar{p} \).

From (i) and (ii), \( p_{i+K^*} \geq \bar{p} \) holds for all \( p_i \in (0, 1) \) when \( (x_i, \ldots, x_{i+K^*-1}) = (1, \ldots, 1) \).

Here, divide \( \{M, \ldots, N\} \) into subgroups \( \{M, \ldots, M + K^*-1\}, \{M + K^*, \ldots, M + 2K^* - 1\}, \ldots, \{M + (L + 1)K^*, \ldots, N\} \) where \( L \) is the quotient when \( N - M + 1 \) is divided by \( K^* \). Then, from the above discussion,

\[
\Pr(p_i < \bar{p} \forall i \text{ s.t. } M \leq i \leq N) \\
\leq \Pr(\forall k \in \{0, \ldots, L - 1\}, \exists i \in \{kK^* + M, \ldots, M + (k + 1)K^* - 1\}; x_i \neq 1) \\
\leq (1 - (1 - \theta_2)q_1^K)^L.
\]

The first inequality comes from the fact that when \( (x_i, \ldots, x_{i+K^*-1}) = (1, \ldots, 1) \), \( p_{i+K^*} \geq \bar{q} \). The second inequality comes from the fact that at least with probability \( (1 - \theta_2)q_1L \), \( x_i = 1 \).

Therefore, \( \lim_{N \to \infty} \Pr(\forall i \text{ s.t. } M \leq i \leq N: p_i < \bar{p}) = 0 \) because \( L \to \infty \).

Case 2. \( \frac{1-q}{(b-1)q} \leq \bar{p} \). Similarly, we have \( \lim_{N \to \infty} \Pr(\forall i \text{ s.t. } M \leq i \leq N: p_i < \bar{p}) = 0 \).

A9 \quad \text{Proof of Proposition 8}

Without loss of generality, suppose that \( N \) is even.

From (b) of Proposition 7, \( \lim_{N \to \infty} \Pr(\forall i \text{ s.t. } 1 \leq i \leq N/2: p_i < \bar{p}) = 0 \). Combining this with the fact that \( \theta_1 = 1 \), we have \( \lim_{N \to \infty} \Pr(p_{2N} \geq \bar{p}) = 1 \). In addition, \( \lim_{N \to \infty} \Pr(\omega_{2N} = 1) = 1 \).

Next, suppose that \( \omega_{2N} = 1 \) and \( p_{2N} \geq \bar{p} \). Then, in countries \( i \geq N/2 \), only policy 1 is taken. Hence, for all \( \varepsilon > 0 \), \( \lim_{N \to \infty} \Pr(|p_N - 1| > \varepsilon) = 0 \).

Combining these arguments yields the proposition.
A10 Proof of Proposition 9

It suffices to prove that the following statement holds: When \( p_i \geq \bar{p} \), \( \Pr(\forall k \in \{i, \ldots, j\} : p_k \geq \bar{p}|p_i) \) is weakly increasing in \( \theta_1 \) for each \( j \geq i+1 \). We prove step by step.

**Step 1.** We first prove that when \( p_i \geq \bar{p} \), \( \Pr(p_{i+1} \geq q|p_i) \) is increasing in \( \theta_1 \) for each \( q \geq \bar{p} \). First of all, \( \Pr(p_{i+1} \geq \bar{p}|p_i) \) is increasing in \( \theta_1 \) because the probability of \( x_i \) being 1 is increasing in \( \theta_1 \) and \( p_{i+1}(1) \) is also increasing in \( \theta_1 \). Furthermore, \( \Pr(p_{i+1} \geq q|p_i, p_{i+1} \geq \bar{p}) \) is increasing in \( \theta_1 \) since \( p_{i+1}(1) \) is increasing in \( \theta_1 \). Combining them yields the above property.

**Step 2.** We next prove that \( \Pr(p_{i+2} \geq q, p_{i+1} \geq \bar{p}|p_i) \) is weakly increasing in \( \theta_1 \) for each \( q \geq \bar{p} \). To this end, first observe that

\[
\Pr(p_{i+2} \geq q, p_{i+1} \geq \bar{p}|p_i) = \sum_{p'_{i+1} \in \text{Supp}(p_{i+1}|p_i) \cap [\bar{p}, 1]} [\Pr(p_{i+2} \geq q|p_{i+1} = p'_{i+1}) \cdot \Pr(p_{i+1} = p'_{i+1}|p_i)].
\]

As in the proof of Proposition 3 (a), the right-hand side is weakly increasing in \( \theta_1 \).

**Step 3.** We can repeat the argument same as in Step 2 to show that for \( j \geq i+2 \)

\[
\Pr(p_j \geq q, \forall k \in \{i, \ldots, j-1\} : p_k \geq \bar{p}|p_i) = \sum_{p'_{j-1} \in \text{Supp}(p_{j-1}|p_i) \cap [\bar{p}, 1]} [\Pr(p_j \geq q|p_{j-1} = p'_{j-1}) \cdot \Pr(p_{j-1} = p'_{j-1}, p_k \geq \bar{p} \forall k \in \{i, \ldots, j-2\}|p_i)].
\]

where

\[
\text{Supp}(p_{j-1}|p_i) = \{p'_{j-1} | \Pr(p_{j-1} = p'_{j-1}, p_k \geq \bar{p} \forall k \in \{i, \ldots, j-2\}|p_i) > 0\}.
\]

Hence, we have the proposition. ■

A11 Proof of Fact 6

Consider case 1 in the proof of Proposition 6, first. Applying the implicit function theorem to (19) with equality yields

\[
\frac{\partial p_E}{\partial \rho} = \frac{(1-q)[s^*-(1+s^*)p_E] - p_E q L}{2q_L p_E + (1+s^*)\rho(1-q) - (1-\rho)q L}.
\]

Here, since

\[
p_E[q_L p_E + (1+s^*)\rho(1-q) - (1-\rho)q L] = (1-q)s^* \rho > 0
\]

holds, the denominator is positive. In addition, the numerator is negative since \( p_E > p_S = \frac{s^*}{1+s^*} \) from (a). Hence, \( \frac{\partial p_E}{\partial \rho} < 0 \) in case 1.

Next, consider case 2.

\[
\frac{\partial p_E}{\partial \rho} = \frac{-b}{[1-(1+s^*)\rho + b(1+s^*)\rho]^2} < 0.
\]

From cases 1 and 2, we have \( \frac{\partial p_E}{\partial \rho} < 0 \). ■

References


Appendix B  Additional Discussions (Not for Publication)

B1  Further Discussion on the Domino Effect of Extremism

In Section 5.3, we have highlighted the domino effect by considering the effect of the policy implemented in country 1 given that the country is in the extremism equilibrium. Although it is assumed that country 1 is in the extremism equilibrium in this case, another interpretation of the domino effect is that a sudden shock in country 1 induces the extremism equilibrium, and it has a contagion effect on subsequent countries. We obtain a similar result even if we adopt this second interpretation.

To illustrate, let us consider an exogenous shock that makes country 1 be in the extremism equilibrium. The benchmark case is that $b = \bar{b}$ such that $p_1 > \bar{p}$. In this case, extremism never arises in all countries. Next, as a hypothetical situation, suppose that only country 1’s $b$, denoted by $b_1$, changes from $b$ to $\bar{b}$. We assume that $\bar{b}$ is sufficiently large so that $p_1 < \bar{p}$. This hypothetical situation represents the case in which country 1 receives an exogeneous shock that makes only country 1 face the extremism equilibrium. Notice that all the players (including those in other countries) know the value of $b_1$.

This exogenous shock has a domino effect. We assert the following proposition.

**Proposition 10.** Suppose $\omega = 1$. For $i \neq 1$,

$$Pr(p_i \geq \bar{p} | b_1 = b) = 1 > Pr(p_i \geq \bar{p} | b_1 = \bar{b}).$$

**Proof.** First, when $p_i < \bar{p}$, the congruent type $H$ chooses policy 2, and $p_{i+1}(2) < p_i$. Hence, $Pr(p_i \geq \bar{p} | b_1 = \bar{b}) \leq \frac{1}{\bar{b} - \bar{p}} < 1$. On the contrary, when $p_i \geq \bar{p}$, policy 2 is never chosen so that $p_{i+1} \geq \bar{p}$ always holds. Hence, $Pr(p_i \geq \bar{p} | b_1 = b) = 1$. Combining them, we have the proposition. ■

When country 1 does not receive the shock, every country does not suffer from extremism. On the contrary, when country 1 recieves the shock so that the country is in the extremism equilibrium, all the subsequent countries are captured in the extremism equilibrium with a positive probability. It indicates a domino effect of extremism.

This domino effect is even quantitatively large. To illustrate, in Figure 9, we plot the distribution of the duration of the domino effect given $x_1 = 2$. According to the figure, the probability that the domino effect does not stop yet even after 20 countries is around 0.1, suggesting the severe contagion effect. Note that we focus on the case in which there is a shock only to country 1 and people in other countries correctly anticipates the extremism in country 1. These assumptions tend to weaken the domino effect and the severer contagion may arise once they are relaxed.

In Table 4, we also demonstrate that the severeness of extremism is sensitive to the relative size of $q_L$ similarly with the result in the main analysis. In particular, when $q_L$ is quite small, long-lasting extremism is likely to occur.

B2  Welfare Consequences of Social Learning

We have one remark on the long-term result in Section 5. In standard models of social learning, the success of social learning directly implies the improvement of the welfare. In contrast, social learning may damage welfare in our model. From Fact 2, when $p$ is too low, voters’ welfare is the lowest because tyranny is severe (i.e., the voter cannot discipline the non-congruent type). However, when $\omega = 2$, $p$ goes to zero as a result of social

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\(^{67}\) $\bar{p}$ is increasing in $b$. 

52
Figure 9: Domino effect of populist extremism.

Notes: The figure shows the first country in the non-extremism equilibrium. The parameter values are $l = 4$, $q_H = 0.55$, $q_L = 0.1$, $\omega = 1$, $b = 2.1$, and $p_1 = \bar{p}$. We suppose that country 1 is in the extremism equilibrium and causes the domino effect by assuming that $b$ becomes 2.2 only in country 1 and that policy 2 is implemented in country 1. We simulate the economy 100,000 times in calculating the histogram. When the extremism occurs for all countries $i \leq 20$ and extremism arises in country 21, it is shown as 21 in the figure.

<table>
<thead>
<tr>
<th>$\psi$</th>
<th>0.8</th>
<th>0.82</th>
<th>0.84</th>
<th>0.86</th>
<th>0.88</th>
<th>0.9</th>
<th>0.92</th>
<th>0.94</th>
<th>0.96</th>
<th>0.98</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>0.071</td>
<td>0.082</td>
<td>0.095</td>
<td>0.116</td>
<td>0.131</td>
<td>0.164</td>
<td>0.230</td>
<td>0.330</td>
<td>0.470</td>
<td>0.681</td>
</tr>
</tbody>
</table>

Table 4: Frequency of long-lasting extremism

Notes: The table shows the frequency that extremism takes place in country 21. See Table 1 for the definition of $\psi$. The parameter values are $l = 4$, $q = 0.65$, $\omega = 1$, $b = 2.1$, and $p_1 = \bar{p}$. We assure that country 1 is in the extremism equilibrium by assuming that $b$ becomes 2.2 only in country 1. We also suppose that policy 2 is implemented in country 1. We simulate the economy 100,000 times in calculating the frequency. All numbers are rounded up to three decimal places.
learning. This directly implies the following welfare implication. When \( \omega = 2 \), for sufficiently large \( N \), voters’ utility in country \( N \) when the history is unobservable (i.e., \( h^N = \emptyset \)) is strictly higher than the case in which voter \( N \) observes history. Hence, social learning is not necessarily welfare-improving.

### B3 Long-Run Distribution in the Markovian Environment

This subsection is devoted to the further discussion on the Markovian environment. The probability of the implementation of each policy in period \( i \) depend on \( p_i \) and \( \omega_i \). Because \( p_i \) is a continuous variable, the relevant states are infinite so that in the general case, it is impossible to calculate the steady state probability of the extremism equilibrium (at least for us). Still, in a special case, we can obtain analytically the steady state probability of the extremism equilibrium (at least for us). Still, in a special case, we can obtain analytically the steady state probability of the extremism equilibrium (at least for us). Still, in a special case, we can obtain analytically the steady state probability of the extremism equilibrium (at least for us). Still, in a special case, we can obtain analytically the steady state probability of the extremism equilibrium (at least for us). Still, in a special case, we can obtain analytically the steady state probability of the extremism equilibrium (at least for us). Still, in a special case, we can obtain analytically the steady state probability of the extremism equilibrium (at least for us). Still, in a special case, we can obtain analytically the steady state probability of the extremism equilibrium (at least for us). Still, in a special case, we can obtain analytically the steady state probability of the extremism equilibrium (at least for us). Still, in a special case, we can obtain analytically the steady state probability of the extremism equilibrium (at least for us). Still, in a special case, we can obtain analytically the steady state probability of the extremism equilibrium (at least for us). Still, in a special case, we can obtain analytically the steady state probability of the extremism equilibrium (at least for us). 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Still, in a special case, we can obtain analytically the steady state probability of the extremism equilibrium (at least for us). Still, in a special case, we can obtain analytically the steady state probability of the extremism equilibrium (at least for us). Still, in a special case, we can obtain analytically the steady state probability of the extremism equilibrium (at least for us). Still, in a special case, we can obtain analytically the steady state probability of the extremi

\[
T = \begin{pmatrix}
\theta_1 & 0 & 1 - \theta_1 & 0 \\
\theta_1(1 - q_H) & \theta_1q_H & (1 - \theta_1)(1 - q_H) & (1 - \theta_1)q_H \\
(1 - \theta_2)(1 - q) & (1 - \theta_2)q & \theta_2(1 - q) & \theta_2q \\
(1 - \theta_2)(1 - q) & (1 - \theta_2)q & \theta_2(1 - q) & \theta_2q
\end{pmatrix},
\]

(22)

Since this is aperiodic and irreducible, the Markov chain governed by this transition matrix converges to the unique steady state distribution. Let \( f(k) \) be the steady-state probability that state \( k \) occurs and let \( f \equiv (f(1), f(2), f(3), f(4)) \).

---

68 Note that the first condition implies \( \overline{p} < 1 \).

69 For example, \( l = 4, b = 2.1, q = 0.8, q_L = 0.7, \theta_1 = 0.95, \) and \( \theta_2 = 0.6 \).

70 \( \bar{p} < p_E \) guarantees that the non-extremism equilibrium never ends so long as policy 1 is implemented.
By solving $fT = f$, we obtain

$$f(1) = \frac{q\theta_2 - q - q_L \theta_1 \theta_2 + q_L \theta_1 - \theta_2 + 1}{q_H \theta_1^2 + q_H \theta_1 \theta_2 - 2q_H \theta_1 - \theta_1 - \theta_2 + 2};$$

$$f(2) = \frac{q (\theta_1 \theta_2 - \theta_1 - \theta_2 + 1)}{q_H \theta_1^2 + q_H \theta_1 \theta_2 - 2q_H \theta_1 - \theta_1 - \theta_2 + 2};$$

$$f(3) = \frac{(1 - \theta_1) [q (1 - \theta_2) (1 - q_H) + (1 - q) (1 - q_L \theta_1)]}{(2 - \theta_1 - \theta_2) (1 - q_H \theta_1)};$$

$$f(4) = \frac{q (1 - \theta_1) [(1 - \theta_1) q_H + \theta_2 (1 - q_H)]}{(2 - \theta_1 - \theta_2) (1 - q_H \theta_1)}. $$

By differentiating the expression for $f(2)$, we immediately obtain the following:

**Proposition 11.** In the steady state, the frequency of the extremism equilibrium under $\omega = 1$ is increasing in $q$ (holding $q_L$ fixed) and is decreasing in $q_L$ (holding $q$ fixed).

Intuitively, larger $q$ implies that the congruent type $H$ frequently appears, which prolongs extremism under $\omega = 1$. In this case, extremism always ends with only one moderate policy since the condition behind Proposition 4 is assumed to be satisfied. Thus, the effect that $p_{i+1}^p(2)$ is increasing in $q_H$, which curbs extremism, is absent. Consequently, larger $q_H$ always implies more frequent extremism under $\omega = 1$, and so more frequent extremism equilibrium under the moderate state. Larger $q_L$ enhances the learning of the correct state, and curbs extremism under $\omega = 1$.

We next analyze the comparative statics about the stability of the moderate state, which is the main focus in Section 6.3. While the numerical analysis presented in Table 2 suggests that the more stability induces the less frequent extremism equilibrium, the analytical result of Proposition 9 is still limited. This is because the result of the duration of extremism is only that conditional on $p_i$, but the distribution of $p_i$ itself depends on $\theta_1$. However, by using the above characterization of the steady state probability distribution, we obtain the following property:

**Proposition 12.** Suppose that $\theta_1$ is sufficiently close to 1. In the steady state, the frequency of the extremism equilibrium $(f(2) + f(4))$ is decreasing in $\theta_1$. The frequency of the extremism equilibrium under $\omega = 1$ ($f(2)$) is also decreasing in $\theta_1$.

**Proof.** See Section B9.

Though this result is only for a special case, it analytically confirms our arguments that the more stable environment induces the smaller fraction of the extremism equilibrium.\(^71\) The stability of the moderate state is a key factor in understanding the severeness of extremism.

### B4 Equilibrium Without the Congruent Type $L$

We have assumed that $q_L > 0$, while it can be arbitrary small. In this subsection, we consider the equilibrium when $q_L = 0$. While the off-path belief formation does not matter in the presence of the congruent type $L$, it does matter in the present case. Hence, we need to focus on reasonable off-path belief formation. For this purpose, we consider the equilibrium satisfying the intuitive criterion, which is a standard criterion for the equilibrium refinement of signaling games (Cho and Kreps 1987). We then obtain the following result as the characterization of the static equilibrium.

\(^{71}\)Note that if $\theta \to 1$ but $\theta_2 < 1$, $f(2) + f(4) \to 0$, which is natural since state 1 becomes the absorbing state in the limit and voters can learn the correct state in most of the time.
Proposition 13. (I) When \( p \geq \bar{p} \), there exist three classes of equilibria: (i) \( \alpha^*(\omega; \omega) = 1 \); and \( \beta^*(1) = 1 \); (ii) \( \alpha^*(2; 2) = 1 \); \( \alpha^*(1; 1) = \frac{(b-1)(1-q)}{pq} \); \( \alpha^*(2; 1) = 1 - \alpha^*(1; 1) \); and \( \beta^*(1) = 1 \); and (iii) \( \alpha^*(2; \omega) = 1 \); and \( \beta^*(0) = 1 \).

(II) When \( p < \bar{p} \), there exist a unique equilibrium strategy: \( \alpha^*(2; \omega) = 1 \); and \( \beta^*(0) = 1 \).

Proof. First of all, from the definition of the intuitive criterion, \( \pi^*(2) = 1 \) must hold when \( x = 2 \) is an off-equilibrium path. Given this, as in the proof of Theorem 1, we show that the equilibria are characterized by the above one. Lastly, we can easily verify that the above equilibria satisfy the intuitive criterion. ■

This result is equivalent to the limit case of Theorem 1 when \( q_L \to 0 \). That is, the characterization of the static equilibria is the same as that when \( q_L > 0 \). Hence, for the emergence of extremism in the static model, we do not have to introduce the congruent type \( L \). This equivalence is unsurprising because the arbitrary small fraction of the congruent type \( L \) just plays a role in determining the off-path belief reasonably. Under the intuitive criterion, such refinement works without this type of politicians.

The reason why we need to have the congruent type \( L \) is that the existence of such politicians gives voters a room for learning from the previously implemented policies in the extremism equilibria. To see this, suppose that, as in the model of Section 5, \( \omega_i = \omega \) for all \( i \) while focusing on the case in which \( \omega = 1 \). We consider the dynamics of beliefs in the absence of the congruent type \( L \). In the extremism equilibrium such that \( \alpha^*(2; \omega) = 1 \) and \( \beta^*(0) = 1 \), voters’ beliefs are never updated because the implemented policy is independent of the state of the world. Hence, when \( p_1 < \bar{p} \), we see no opinion dynamics (i.e., \( p_i = p_1 \) for all \( i \)) and thus, the extremism equilibrium continues forever. On the contrary, when \( p_1 \geq \bar{p} \), the equilibrium is the non-extremism one and beliefs are always updated upwardly so that \( p_N \to 1 \) as \( N \to \infty \). This result is similar to the result outlined in Section 5 in that whether the initial belief induces the spread of extremism or not significantly affects the following dynamics. However, it also has a key distinction. In contrast to the case in which \( q_L > 0 \), there are no dynamics or convergence to the true state when \( p_1 < \bar{p} \). In this regard, the existence of the congruent type \( L \) is crucial, while even an arbitrarily small fraction of this politician type is enough.

B5 Equilibrium with Moderate Reputation Concerns

So far, we have assumed that \( b > 2 \) for the congruent type \( H \) and the non-congruent type. In this subsection, we explore the equilibrium in the single-country model when this does not necessarily hold.

First, assume that \( b \in (1, 2) \). Then, we have the following result.

Proposition 14. (i) When \( p \geq \frac{1-q}{(b-1)q} \), \( \alpha^*(\omega; \omega) = 1 \); and \( \beta^*(1) = 1 \).

(ii) When \( p < \frac{1-q}{(b-1)q} \), \( \alpha^*(\omega; \omega) = 1 \); and \( \beta^*(1) = \frac{(b-1)pq}{1-q} \) and \( \beta^*(0) = 1 - \beta^*(1) \).

Proof. Since \( b < 2 \), (E1), (E2), and (E3) equilibria do not exist from the proof of Proposition 1. Furthermore, from the proof of Proposition 1 (d), the condition for the existence of (NE) equilibrium is replaced with \( p \geq \frac{1-q}{(b-1)q} \). Lastly, the equilibrium (II-2) in the proof of Proposition 1 (e) (Step 2) exists if and only if \( p \leq \frac{1-q}{(b-1)q} \). Combining these arguments yields the theorem. ■

Hence, when \( b < 2 \), only the non-extremism equilibria exist and \( b > 2 \) is needed to analyze interesting cases. This is an intuitive result because, in our model, the spread of extremism is catalyzed by signaling motives of the congruent \( H \) type. In our extremism equilibria, the congruent \( H \) type politicians implements policy 2 regardless of the world-state because they are distinguished from the non-congruent type, thus obtaining higher reputations.
Without sufficiently strong reputation motives, they stop signaling and implement a policy in line with their own preference.\(^{72}\)

However, one might think that this is restrictive because the congruent type \(H\) might have only moderate reputation concerns \(b \in (1, 2)\). This is not the case. To see this, let us allow \(b_1\) of the congruent type \(H\) to be different from that of the non-congruent type. Let \(b_1\) of the congruent type \(H\) (the non-congruent type) be \(b_C\) \((b_N)\) and assume that \(b_C, b_N > 1\). Then, under a certain condition, we obtain a result that is significantly close to that of the previous analysis.

**Lemma 5.** Suppose that \((b_N - 1)/b_N > 1/b_C\).

(a) \((E1)\) equilibrium exists if and only if \(p \leq \frac{(b_C - 1)(1-q)}{q}\)

(b) \((E2)\) equilibrium exists if and only if \(p \leq \frac{(b_C - 1)(1-q)}{q}\)

(c) \((E3)\) equilibrium exists if and only if \(p \leq \frac{(b_C - 1)(1-q)}{q}\)

(d) \((NE)\) equilibrium exists if and only if \(p \geq \max \left\{ \frac{1-q}{(b_N-1)q}, \frac{(b_C-1)(1-q)}{q} \right\} \)

(e) There exist no other equilibria.

**Proof.** By replacing \(b\) in Proposition 1 appropriately, we obtain the proposition. \(\blacksquare\)

This result states that, as long as the non-congruent type’s reputation concern is sufficiently higher than \(b_C\) so that \((b_N - 1)/b_N > 1/b_C\), then our analysis remains essentially unchanged. That is, when the non-congruent type has sufficiently high reputation concerns, the same characterization of equilibria is obtained, even if the congruent type \(H\) does not have high reputation concerns. In particular, extremism equilibria exist for sufficiently low \(p\).

To understand this point intuitively, note first that, as the fraction of the non-congruent type implementing policy 1 increases, the incentive of the congruent type \(H\) to send a signal by taking policy 2 also increases. Here, higher \(b_N\) makes the non-congruent type more likely to implement policy 1 because implementing policy 0 lowers reputation. Hence, higher \(b_N\) encourages the congruent type \(H\) to implement policy 2 for differentiating itself from the non-congruent type, meaning that extremism arises even if \(b_C\) is not high.

Furthermore, this result combined with the non-extremism equilibrium definition yields the following proposition that implies that the same results hold for the multi-country model.

**Proposition 15.** Suppose that \((b_N - 1)/b_N > 1/b_C\). Then, the non-extremism equilibrium exists if and only if \(p \geq \tilde{p}' = \max \left\{ \frac{1-q}{(b_N-1)q}, \frac{(b_C-1)(1-q)}{q} \right\} \).

In many contexts, it is reasonable to assume that the non-congruent type exhibits high reputation concerns while the congruent type \(H\) exhibits moderate reputation concerns. For instance, the non-congruent type might

\(^{72}\)One might still think that \(b > 1\) is enough because the loss when \(x = 2\) in spite of \(\omega = 1\) is just one while the largest reputational cost is \(b\). However, this is not the case due to the existence of the congruent type \(L\). Since this type politician sincerely chooses the voter-optimal policy, there is always a positive probability that the congruent type will implement policy 1. Hence, policy 1 never works as a perfect signal that the incumbent is the non-congruent type, meaning that the reputational cost of choosing policy 1 is strictly less than \(b\). Therefore, \(b > 1\) is not sufficient to create extremism equilibria.

\(^{73}\)\(b\) in \(\beta^*\) is replaced by \(b_N\).

\(^{74}\)\(b\) in \(\alpha^*\) is replaced by \(b_C\).
have more intense office-seeking motivations that influences the development of their preferred policy or engagement in bribery.\textsuperscript{75} Thus, we deem that the condition for the existence of extremism equilibria is not very restrictive—our analysis can be safely applied to various circumstances.

### B6 General Formulation on Reputation Concerns

In the analysis, we have assumed that \( V(\pi) = \pi \) for simplicity. In this subsection, we show that our result still holds under a general setting: we assume that the function \( V : [0, 1] \mapsto [0, 1] \) is strictly increasing, \( V(0) = 0 \), and \( V(1) = 1 \). Let us introduce some notations, which will be used in the following results:

\[
\bar{v} \equiv V^{-1}\left(\frac{b-1}{b}\right); \quad \underline{v} \equiv V^{-1}\left(\frac{1}{b}\right).
\]

Note that these values are uniquely determined because \( V \) is strictly increasing and \( (b-1)/b, 1/b \in (0, 1) \).

Then, we have the following characterization of equilibria, which is almost the same as that of Theorem 1.

**Lemma 6.** Suppose first that \( \frac{(1-q)v}{q(1-\bar{v})} < \frac{(1-q)v}{q(1-\bar{v})} \). Then,

(i) When \( p < \frac{(1-q)v}{q(1-\bar{v})} \), there is a unique class of equilibria: (E2) equilibrium.\textsuperscript{76}

(ii) When \( \frac{(1-q)v}{q(1-\bar{v})} \leq p < \frac{(1-q)v}{q(1-\bar{v})} \), there is a unique class of equilibria: (E1) equilibrium.

(iii) When \( \frac{(1-q)v}{q(1-\bar{v})} \leq p \leq \frac{(1-q)v}{q(1-\bar{v})} \), there exist three classes of equilibria: (E1), (E3),\textsuperscript{77} and (NE).

(iv) When \( p > \frac{(1-q)v}{q(1-\bar{v})} \), there is a unique class of equilibria: (NE) equilibrium.

Next, suppose that \( \frac{(1-q)v}{q(1-\bar{v})} > \frac{(1-q)v}{q(1-\bar{v})} \). Then,

(i) When \( p < \frac{(1-q)v}{q(1-\bar{v})} \), there is a unique class of equilibria: (E2) equilibrium.

(ii) When \( \frac{(1-q)v}{q(1-\bar{v})} \leq p < \frac{(1-q)v}{q(1-\bar{v})} \), there exist three classes of equilibria: (E2), (E3), and (NE).

(iii) When \( \frac{(1-q)v}{q(1-\bar{v})} \leq p \leq \frac{(1-q)v}{q(1-\bar{v})} \), there exist three classes of equilibria: (E1), (E3), and (NE).

(iv) When \( p > \frac{(1-q)v}{q(1-\bar{v})} \), there is a unique class of equilibria: (NE) equilibrium.

**Proof.** See Section B9.

Hence, we have the following proposition:

**Proposition 16.** The non-extremism equilibrium exists if and only if \( p \geq p'' \equiv \frac{(1-q)v}{q(1-\bar{v})} \).\textsuperscript{78}

This means that we have almost the same characterization of equilibria in the single-country model. By using this, we also obtain results of the multi-country model similar with those in the analysis.

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\textsuperscript{75}See a dynamic election model in Section 7.1.

\textsuperscript{76}To be precise, (E2) equilibrium is that \( \alpha^*(2; \omega) = 1, \beta^*(1) = \frac{pq(1-q)}{q(1-\bar{v})}; \beta^*(0) = 1 - \beta^*(1) \).

\textsuperscript{77}To be precise, (E3) equilibrium is that \( \alpha^*(2; 2) = 1; \alpha^*(1; 1) = \frac{(1-q)v}{q(1-\bar{v})} - \frac{q_1}{q_1}; \alpha^*(2; 1) = 1 - \alpha^*(1; 1); \beta^*(1) = 1 \).

\textsuperscript{78}This is increasing in \( b \) because \( \bar{v} \) is increasing in \( b \).
B7 Simultaneous Policymaking

In most yardstick competition models, policymaking by the incumbent politicians is simultaneous across countries (e.g., Besley and Case 1995). While we have not focused on such simultaneous timing because our focus is sequential policymaking and associated dynamics, it might be more appropriate in other contexts.\textsuperscript{79} To analyze this game, suppose that the incumbent politicians in two countries simultaneously choose policies. The state of the world is common across countries and the incumbent’s type is independently determined across countries.

In contrast to the previous game, the non-congruent type’s strategy depends on $\omega$ because it is now payoff-relevant: the policy implemented by the other country’s politician, which is dependent upon $\omega$, affects the reputation. To reflect this modification, let us define $\beta^*(x; \omega)$ be the probability of the non-congruent type choosing $x$ given $\omega$. Furthermore, denote the reputation of politician $i$ given policies in the two countries by $\pi(x_i, x_{-i})$. In this game, characterizing the whole set of equilibria is hard. However, we can derive the condition for the existence of the non-extremism equilibrium.

**Proposition 17.** Suppose Assumption 2. Then, there exists a non-extremism equilibrium for any $p \in (0, 1)$.

**Proof.** See Section B9.

This is contrast to the results in the previous analysis.\textsuperscript{80} When a non-extremism equilibrium exists for some $p$ in the single-country model, there always exists a non-extremism equilibrium independently of the prior belief $p$ in the simultaneous policymaking model. The key is that the non-congruent type might not be able to pretend to be the congruent type when $\omega = 2$. In that case, the congruent type chooses policy 2, meaning that the non-congruent type fails to pretend to be the congruent type by implementing policy 1 so long as the incumbent in the other country is the congruent type. Given this possibility, it is not so attractive for the non-congruent type to argue for policy 1 when $\omega = 2$. Hence, $\beta^*(1; 2) = 0$, implying that $\pi(1, 1)$ is relatively high.\textsuperscript{81} Consequently, the reputation given $x = 1$ is maintained to some extent, and thus the congruent type does not argue for policy 2 to signal the own type. Note, however, that Assumption 2 is essential to derive the proposition. Without Assumption 2, the non-extremism equilibrium might not exist for some $p$. Furthermore, in the non-extremism equilibrium we constructed, the tyranny is severe so that the non-congruent type often takes policy 0 in this equilibrium. Hence, there is no guarantee that the non-extremism equilibrium is always the voter-optimal equilibrium.

Extremism can also arise. To illustrate this point, we focus on (E1) equilibrium wherein $\alpha^*(2; \omega) = 1$, and $\beta^*(1; \omega) = 1$. We show that this equilibrium remains for intermediate values of $p$.

**Proposition 18.** Suppose $\frac{q_u}{1-q_H} > \frac{1}{\tilde{p}}$. Then, there exists $\hat{p} \in (0, 1]$ and $\check{p} \in (0, \hat{p})$ such that (E1) equilibrium exists if and only if $p \in [\check{p}, \hat{p}]$.

**Proof.** See Section B9.

Thus, the extremism equilibrium robustly exists even if some countries act simultaneously. In this sense, the possibility of extremism – the core of our static result – still remains even under simultaneous policymaking.

A natural question is whether the presence of the other country may facilitate the emergence of extremism. We can show the following:

\textsuperscript{79}For example, in many countries, multiple local elections are held simultaneously. As long as we ignore other types of elections such as national ones, simultaneous timing may arguably better approximate such a situation.

\textsuperscript{80}It has been widely known that the simultaneous move game and the sequential move game often yield different results in various multi-sender games including signaling, cheap talk, and Bayesian persuasion games.

\textsuperscript{81}In particular, $\pi(1, 1) = q$. 

59
Fact 7. \( \hat{p} < \frac{(b-1)(1-q)}{q(1-p)} \) holds.\(^{82}\)

Proof. See Section B9.

Hence, (E1) equilibrium is more likely to exist in the absence of the opponent country than in the presence of it, which can be interpreted as the presence of the other country preventing extremism.\(^{83}\) Although our result is restricted to (E1) equilibrium, it illustrates that yardsticking behavior may prevent extremism.

However, this argument requires a significant modification when politicians know each other’s type as in Besley and Case (1995). Suppose that (E1) equilibrium arises in the single-country case.\(^{84}\) When both countries follow this equilibrium strategy,

\[
\pi(1, 2) = \frac{pq_Lq_H}{p(1-q_H)q_H + (1-p)(1-q)} < \pi(1) = \frac{pq_L(1-q_H)}{p(1-q_H)^2 + (1-p)(1-q)^2},
\]

where \( \pi(1) \) is the reputation of taking policy 1 in the single country case. Here, observing policy 2 in the other country makes voters suspect that the optimal policy is more likely to be policy 2, reducing the reputation of taking policy 1. The opposite is the case for observing policy 1 in the other country.

In the single-country case, the incentive constraint for the congruent type \( H \) under \( \omega = 1 \) is

\[
b \pi(1) < -1 + b.
\]

Since \( \pi(1) \) is increasing in \( p \), the incentive constraint may be violated when \( p \) is high. Thus, for high \( p \), (E1) equilibrium is not supported. However, if the politician knows that the opponent is the congruent \( H \) type, \( \pi(1) \) is replaced with \( \pi(1, 2) \), making the incentive constraint more likely to hold. Thus, the extremism in the foreign country may induce extremism. On the other hand, if the opponent is the non-congruent type or the congruent \( L \) type, \( \pi(1) \) is replaced with \( \pi(1, 1) \), making the constraint less likely to hold. Thus, the foreign moderate policy may curb extremism. In Fact 7, both of these forces are at work and one of them happens to be dominant.

Note that the case when the opponent is the congruent \( H \) type is similar to the case wherein the previous country implemented policy 2 in our main model. Under sequential choice, extremism of the foreign country makes voters believe that the optimal policy is more likely to be policy 2, and it induces another extremism through reducing \( p \). On the other hand, under simultaneous choice, extremism in foreign country prevents voters from identifying the non-congruent politicians, which induces extremism through signaling motives. The similar analogy holds for the case that the non-congruent type or the congruent \( L \) type is the opponent. Although this analysis is only suggestive, it implies that the presence of foreign countries might help the emergence of extremism even under the simultaneous policymaking.

B8 Additional Applications

We discuss several applications that were not contained in Section 7.2.

Party Polarization: Our model indicates that extremism in one country may induce party polarization in another country. In other words, when a radical policy is implemented in one country, politicians in the next country are

\(^{82}\)The right-hand side is the upper bound of \( p \) for the existence of (E1) equilibrium.
\(^{83}\)To be precise, \( \hat{p} < \frac{1-q}{(1-p)q} \), meaning that the lower bound of \( p \) for the existence of (E1) equilibrium is smaller under the simultaneous policymaking than in the single-country model. However, this opposite result is not essential because for low values of \( p \), there exists another equilibrium: (E2) equilibrium.
\(^{84}\)It constitutes an equilibrium even if we allow \( \beta \) to depend on \( \omega \).
incentivized to choose not only the radical policy, but also the non-congruent policy, which is an oppositional extreme. To see this point, note that policy 0 is never observed in (NE) equilibrium, while it can be observed in (E2) equilibrium. Thus, if policy 2 in country $i$ induces (E2) equilibrium in country $i + 1$, a party in country $i$ may choose policy 0, meaning that the party responds in the opposite direction to the foreign party’s positioning. Noting that the observation of policy 2 lowers the belief $p$ while policy 0 does not affect it, the polarization may hinder the correct updating of $p$ and result in the contagion of party polarization. This implies that the process of party policy diffusion might be more complicated than previously thought.

This result has two implications. First, it suggests that a right-wing policy in a foreign country may move a domestic party’s positioning to the left. As seen in the Europe, when a society has a more left-wing (right-wing) ideology, some groups may respond in the opposite way and have a more right-wing (left-wing) opinion (Inglehart and Norris 2016). Since the decrease in $p$ can be interpreted as the propagation of a specific ideology, the party polarization in (E2) equilibrium is consistent with it.

Second, the result also suggests that not every party emulates the policy of successful parties. More strikingly, some party may take a position further away from that of successful ones. Supposing temporarily that our model describes the elections of the same country, it indicates that, when the incumbent in period $i$ chooses the radical policy, other politicians might choose policy 0 in the next election. This is consistent with several real-world examples. Rooduijn, De Lange, and Van der Brug (2014) show that in Europe, mainstream parties do not necessarily change their positions even after the electoral success of populist challengers. Furthermore, Eguia and Giovannoni (2019) argue that, in conventional two-party competition, the losing political party sometimes chooses the extreme position, rather than moderating its platforms to win the election. The examples include the US republican party under Barry Goldwater in 1964, the UK labor party under Michael Foot in 1983, and that under Jeremy Corbyn in 2015. These arguments suggest that our result on party polarization effectively explains reality, but empirically testing whether our mechanism indeed works remains for future work.

**Tax Competition for Mobile Capital:** When capital is mobile between jurisdictions, each local government may try to attract it by lowering tax rates, resulting in inefficiently low equilibrium tax rates. Focusing on the mobility of tax bases, theories on tax competition predict such “race to the bottom.” (e.g., Keen and Konrad 2013).

Our model shows that “race to the bottom” may take place even in the absence of mobile factors. Suppose that when $\omega = 1(2)$, capital is not very elastic (very elastic). Policy 1 is moderate tax rate, policy 2 is low tax rate, and policy 0 is high tax rate. Here, it is good to set a low tax rate when capital is mobile because it attracts capital and boosts the economy. Otherwise, a moderate tax rate is good. The worst thing is to set a high tax rate when capital is mobile: it induces capital outflow. However, the non-congruent type has an incentive to choose a high tax rate because s/he is the leviathan who tends to maximize the tax revenue.

In this context, the downward pressure of capital tax rates arises in the absence of the mobility. The key is that voters may start valuing lower tax rates after observing low tax rates in other countries. As is well known, globalization in the sense of increased factor mobility puts the downward pressure on tax rates. However, our result shows that tax rates may be reduced just because every country faces the similar degree of capital elasticity with respect to taxes. Recently, Becker and Davies (2017) also construct a learning model in the context of capital tax competition, and show that countries’ tax rates are correlated in the absence of mobile factors. Their model, however, predicts that the learning is efficient and “race to the bottom” never occurs without capital mobility.

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85 To explain such phenomena, Eguia and Giovannoni (2019) consider the electoral competition by office-seeking parties with endogenous issue ownership. In contrast, we consider neither issue-ownership nor purely office-motivated parties. Instead, by analyzing political agency problems, we obtain an alternative mechanism.

86 Using a similar model, Kasamatsu and Kishishita (2018) explore anti-taxation populism in the framework of tax competition, though there is neither uncertainty on the state of the world in their model.
contrast, we show that inefficiently low equilibrium tax rates may be observed even without capital mobility. This result is also empirically relevant: “race to the bottom” may take place between geographically far apart jurisdictions. To identify the significance of tax competition, the empirical literature adopts a spatial weighting matrix, which identifies the set of competitors for a given country. Although the literature has experimented various spatial weighting matrices, the most popular ones rely on geographical distance between jurisdictions because the mobility is crucially affected by the geography. Our model suggests that regardless of the geographical distance, jurisdictions may react to the tax rates of other jurisdictions. In particular, the dynamic election interpretation of our model implies that jurisdictions react to jurisdictions which recently experience elections. This result might be useful in identifying the spatial weighting matrix for empirically detecting tax competition.

Our theory also predicts that when globalization proceeds and the mobility increases, some parties start proposing extremely high tax rates. Indeed, when \( \omega = 2 \) for all countries, voters’ belief \( p \) approaches zero in the long-run. In such a case, the non-congruent type takes policy 0, which corresponds to the high tax rate. This result might be illuminating in understanding the impact of globalization on taxation.

**Minimum Wage Settings:** Our theory can also be applied to a setting where each country contemplates on the level of minimum wages while voters are uncertain about the desirability of raising minimum wages. This situation is fairly natural as the literature is still inconclusive about the cost and the benefit of minimum wages (e.g., Neumark and Wascher, 2008; Dube, Lester, and Reich, 2010). Suppose that \( \omega \) denotes the desirability of the minimum wage settings, say, the degree of the negative impact on employment. \( \omega = 1 \) is the situation in which minimum wage increase is moderately helpful for poor workers, while \( \omega = 2 \) is the situation when they face the difficult situation and the raise is highly valued. Accordingly, policy 1 is the moderate minimum wage rate, policy 2 is the high minimum wage rate, and policy 0 is the undesirably low minimum wage rate.

In this situation, even if \( \omega = 1 \), undesirably high minimum wages may be contagious across countries. It explains the “race to the top” regarding the minimum wage settings. Here, the key is that each country learn the desirability of minimum wage hikes from minimum wage rates of other countries. Such a social learning process may lead to the minimum wage increase all over the world. It may explain the recent increase in minimum wages among, for example, U.S. cities and some European countries.

**Irreversible Investment:** Suppose that principals want to designate an agent in charge of investment decisions. The agent serves for up to two periods, provided that they are not fired at the interim appraisal. Examples may include highway construction and the opening of new factories with high fixed costs. \( \omega = 1 \) corresponds to the situation of usual demand and \( \omega = 2 \) corresponds to the case of high demand. Policy 1 is a moderate investment policy, policy 2 is a very active policy, and policy 0 is no investment (shirking). For the principal, the worst case is to see no investment when it is necessary. Our propositions predict that excessive investment may be contagious even when the demand is not very high, indicating the possibility of the boom in public investment.

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87 Gibbons and Overman (2012) provide a critical introduction about this method and its application to tax competition.
88 A related idea is seen in Davies and Klasen (2018), who use the political similarity for studying overseas development assistance donations. Of course, as discussed by Becker and Davies (2017), the information might be better shared among neighboring jurisdictions. In this sense, geography also matters even in our model.
89 Green and Harrison (2010) also focus on the point that in the context of fairness concerns, a jurisdiction may extract some information from the minimum wage rates of other jurisdictions. In this sense, geography also matters even in our model.
90 Note, however, that the changes in factor mobility play no role in this argument. While several papers argue that factor mobility reduces the level of labor regulation, Fukumura and Yamagishi (2018) argue that such a prediction seems to be inconsistent with the recent European experience, and show that migration of workers may lead to excessively high minimum wage rates by constructing a minimum wage competition model.
Researchers and Scientific Grants: Suppose that a foundation wants researchers to conduct research that is “useful” or “lucrative”. Congruent researchers want to conduct research that is demanded by society, while non-congruent researchers want to research what interests them (but is not demanded by society). Due to information asymmetry, researchers know which research topic is useful while the government does not. The foundation wants to give grants to congruent researchers.

The desirable topic is either \( \omega = 1, 2 \). Possible research topics are 1, 2, and 0.\(^{91} \) Then, our result implies that even congruent researchers may choose socially undesirable research topics to signal that they are congruent. Moreover, such signaling may induce proliferation of this concept, leading to the undesirable herding of the research topic. When \( \omega = 2 \), after social learning, some researchers stop mimicking the congruent type and start conducting research that they are interested in but useless for the society as a whole.

B9  Omitted Proofs

B9.1  Proof of Proposition 12

After simplification, \( f(2) + f(4) \) can be written as

\[
q \frac{q_H(\theta_1^2 + \theta_1 \theta_2 - 2 \theta_1) - \theta_1 + 1 + q_H(1 - \theta_2)}{q_H(\theta_1^2 + \theta_1 \theta_2 - 2 \theta_1) - \theta_1 + (2 - \theta_2)}.
\]

It can be easily verified that the expression is increasing in \( q_H(\theta_1^2 + \theta_1 \theta_2 - 2 \theta_1) - \theta_1 \) if \( 2 - \theta_2 > 1 + q_H(1 - \theta_2) > 0 \). Note that \( 2 - \theta_2 > 1 + q_H(1 - \theta_2) > 0 \) indeed holds. Moreover, marginally increasing \( \theta_1 \) affects the expression only through \( q_H(\theta_1^2 + \theta_1 \theta_2 - 2 \theta_1) - \theta_1 \) appearing both in the numerator and the denominator. Thus, as long as

\[
\frac{\partial}{\partial \theta_1} \left( q_H(\theta_1^2 + \theta_1 \theta_2 - 2 \theta_1) - \theta_1 \right) = q_H(2 \theta_1 + \theta_2 - 2) - 1 < 0,
\]

\( f(2) + f(4) \) is decreasing in \( \theta_1 \). This condition is always satisfied if \( \theta_1 \approx 1 \).

We proceed to the statement about \( f(2) \). Differentiating \( f(2) \) with respect to \( \theta_1 \) yields

\[
\frac{q}{\left( q_H \theta_1^2 - q_H \theta_1 \theta_2 + 2q_H \theta_1 + \theta_1 + \theta_2 - 2 \right)^2} \times \frac{2q_H \theta_1}{q_H (\theta_2 - 1) \left( \theta_1^2 + \theta_1 \theta_2 - 2 \theta_1 + (\theta_1 \theta_2 - \theta_1 - \theta_2 + 1)(-2 \theta_1 - \theta_2 + 2) \right) - (1 - \theta_2)^2}.
\]

The first term is always positive. When \( \theta_1 = 1 \), the second expression becomes \(- (1 - q_H)(1 - \theta_2)^2 < 0 \). Because of continuity, the expression is negative when \( \theta_1 \) is sufficiently close to one. \( \blacksquare \)

B9.2  Proof of Lemma 6

Examine the existence of (E1) equilibrium. The congruent type \( H \) has no deviation incentive from 2 to 1 when \( \omega = 1 \) if and only if

\[
-1 + b \geq bV \left( \frac{q_L p}{q_L p + (1 - q)} \right) \Leftrightarrow p \leq \frac{\bar{v}(1 - q)}{q_L (1 - \bar{v})}.
\]

(27)

Note that the congruent type \( H \) obviously has no deviation incentive when \( \omega = 2 \).

\(^{91}\)Here, the topic 0 is obviously useless for the society, but some researchers want to analyze it just for their pleasure.
Next, consider the non-congruent type’s incentive. S/he has no incentive to deviate from 1 to 0 if and only if
\[ -1 + b V \left( \frac{q_L p}{q_L p + (1 - q)} \right) \geq 0 \Leftrightarrow p \geq \frac{v(1 - q)}{q_L (1 - \bar{v})}. \] (28)

This is because such deviation incentive is minimized when \( \pi(0) = 0 \).

Combining (27) and (28) yield the condition. Note that because \( b > 2 \), the following property holds:\(^92\)
\[ \frac{v(1 - q)}{q_L (1 - \bar{v})} < \frac{\bar{v}(1 - q)}{q_L (1 - \bar{v})}. \]

Similarly, we obtain those corresponding to Proposition 1. ■

**B9.3 Proof of Proposition 17**

It suffices to prove that there is an equilibrium such that \( \alpha^*(\omega; \omega) = 1; \beta^*(1; 1) = 1; \beta^*(1; 2) = 0 \). \( (x_i, x_{-i}) = (1, 2) \) is the off-path. Let us assume that \( \pi(1, 2) = 0 \).

We start by considering the congruent type \( H \)'s incentive. S/he has an incentive to choose policy 1 when \( \omega = 1 \) if and only if
\[ b[(q + (1 - q)\beta^*(1; 1))\pi(1, 1) + (1 - q)(1 - \beta^*(1; 1))\pi(1, 0)] \geq -1 + b \Leftrightarrow b\pi(1, 1) \geq -1 + b. \] (29)

Here,
\[ \pi(1, 1) = \frac{pq}{(1 - p)[\beta^*(1; 2)(1 - q)]^2 + p}. \]

Hence, (29) holds if and only if \( b(1 - q) \leq 1 \), which always holds under Assumption 2. Note that it is obvious that s/he has no deviation incentive when \( \omega = 2 \).

Next, the non-congruent type has an incentive to choose policy 1 when \( \omega = 1 \) if and only if
\[ -1 + b[(q + (1 - q)\beta^*(1; 1))\pi(1, 1) + (1 - q)(1 - \beta^*(1; 1))\pi(1, 0)] \geq 0, \] (30)

which is implied by (29). Furthermore, the non-congruent type has an incentive to choose policy 0 when \( \omega = 2 \) if and only if
\[ -1 + b[(1 - q)\beta^*(1; 2)\pi(1, 1) + (1 - q)(1 - \beta^*(1; 2))\pi(1, 0)] \leq 0 \Leftrightarrow -1 + b(1 - q)\pi(1, 0) \leq 0. \] (31)

This always holds since \( b(1 - q) \leq 1 \). ■

**B9.4 Proof of Proposition 18**

We first analyze the congruent type \( H \). When \( \omega = 1 \), it chooses policy 2 if and only if
\[ b[(1 - q_H)\pi(1, 1) + q_H \pi(1, 2)] \leq -1 + b. \] (32)

Here,
\[ \pi(1, 1) = \frac{pq L(1 - q_H)}{p(1 - q_H)^2 + (1 - p)(1 - q)^2}; \pi(1, 2) = \frac{pq_L q_H}{p(1 - q_H)q_H + (1 - p)(1 - q)q}, \]

where both of which are increasing in \( p \). Note that there is no incentive to deviate when \( \omega = 2 \).

\(^92\bar{v} > \bar{v} \) because \( b > 2 \). Hence, we have this property.
The non-congruent type has no incentive to deviate when $\omega = 1$ if and only if

$$-1 + b[(1 - q_H)\pi(1, 1) + q_H \pi(1, 2)] \geq 0. \quad (33)$$

Similarly, when $\omega = 2$, the incentive compatibility condition for the non-congruent type is given by

$$-1 + b[(1 - q)\pi(1, 1) + q \pi(1, 2)] \geq 0. \quad (34)$$

The strategy constitutes an equilibrium if (32) (33), and (34) are jointly satisfied. Here, the left-hand side of (33) is less than that of (34) because $\pi(1, 1) > \pi(1, 2)$. Hence, (33) implies (34), meaning that (32) and (33) are the necessary and sufficient conditions for the existence of the equilibrium.

Since the left-hand side of (32) is increasing in $p$, there exists $\hat{p} \in [0, 1]$ such that (32) holds if and only if $p \leq \hat{p}$. Note that $\hat{p} > 0$ because (32) holds with strict inequality when $p = 0$. Similarly, there exists $\hat{p} \in [0, 1]$ such that (33) holds if and only if $p \geq \hat{p}$. Note that $\hat{p} > 0$ because (33) never holds when $p = 0$, and $\hat{p} < 1$ because $\frac{q_l}{1 - q_H} > \frac{1}{b}$ is assumed. Furthermore, since $b > 2$, $\hat{p} > \hat{p}$. Combining these arguments yields the proposition. ■

**B9.5 Proof of Fact 7**

We show that given any $p$, $(1 - q_H)\pi(1, 1) + q_H \pi(1, 2) > \pi(1)$, where $\pi(1) = \frac{pqL}{p(1 - q_H) + (1 - p)(1 - q)\frac{q_l}{1 - q_H}}$ is the reputation in (E1) equilibrium after observing policy 1 in the single-country case. This is sufficient because $(1 - q_H)\pi(1, 1) + q_H \pi(1, 2)$ is increasing in $p$.

Here, $(1 - q_H)\pi(1, 1) + q_H \pi(1, 2) - \pi(1)$ is rewritten as

$$\frac{pqL(1 - q_H)}{p(1 - q_H) + (1 - p)(1 - q)\frac{q_l}{1 - q_H}} + \frac{pqLq_H}{p(1 - q_H) + (1 - p)(1 - q)\frac{q_l}{1 - q_H}} - \frac{pqL}{p(1 - q_H) + (1 - p)(1 - q)}$$

$$= \frac{1 - q_H}{A - (1 - p)(1 - q)\frac{q_l}{1 - q_H}} + \frac{q_H}{A + (1 - p)(1 - q)\frac{q_l}{1 - q_H}} - \frac{1}{A}$$

$$= \frac{(1 - q_H)\left[A + (1 - p)(1 - q)\frac{q_l}{1 - q_H}\right] + q_H\left[A - (1 - p)(1 - q)\frac{q_l}{1 - q_H}\right]}{A - (1 - p)(1 - q)\frac{q_l}{1 - q_H}} - \frac{1}{A}$$

$$= \frac{1}{B}\left[A^2 - A^2q_H(1 - p)(1 - q)\frac{q_l}{q_H} + A(1 - q_H)(1 - p)(1 - q)\frac{q_l}{1 - q_H} + (1 - p)^2(1 - q)^2q_l^2\frac{1}{(1 - q_H)(q_H)}\right]$$

$$= \frac{1}{B}(1 - p)^2(1 - q)^2q_l^2\frac{1}{(1 - q_H)q_H} > 0,$$

where $A \equiv p(1 - q_H) + (1 - p)(1 - q)$ and $B \equiv A\left[A - (1 - p)(1 - q)\frac{q_l}{1 - q_H}\right] + A + (1 - p)(1 - q)\frac{q_l}{1 - q_H} > 0$.

Hence, we obtain the fact. ■

**References**


