Information Avoidance, Echo Chambers and Uninformed Decisions

Pasqualina Arca
University of Manchester

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Abstract. We study how agents select information sources in a model with potentially delusional agents. Agents have access to multiple sources of information of different informativeness about the underlying state of the world. Before investing in a common project with uncertain productivity, agents select which information sources to pay attention to. Once they receive the signals about the future productivity of the project and before the investment decision takes place, they decide whether to exactly recall the signal received or to engage in costly denial. We show that if in the low state the expected productivity is negative, multiple equilibria coexist: one in which agents are fully informed and one where agents pay attention only to the information source most likely to reveal favourable information. From a welfare perspective, full information is always desirable both in the high state and in the low state. This model provides a rationale for the raise of echo chambers.

1. Introduction

Technologies such as the internet have eased the access to information and news to a continuously growing fraction of the society. This is so because the internet has dramatically reduced the cost of acquiring information from a wide range of sources. In fact, with the increasing variety of new media choices, the scale at which individuals are exposed to opinions, news and information is larger than what was possible with the traditional media. On the one hand, the growth of online news and social media allows individuals to be exposed to diverse viewpoints and opinions as well as to a variety of information. On the other hand, it might induce people to focus their attention only on a subset of information sources. As such, the way information is transmitted, processed and consumed has brought to the attention of academics and scholars the negative consequences of the increased variety of media platforms. In particular, it is claimed that social media and Internet “filter bubbles” can create echo chambers, (Pariser, 2011). Although there is not consensus in the literature on a formal definition of “echo chamber”, the term is commonly used to describe a phenomenon in which consumers of information and opinions get stuck in a chamber with like-minded people. In the chamber, the opinions, information and beliefs get repeated and confirmed like an echo, rather than foster dialogue and critical reasoning (Jamieson and Cappella, 2008). Echo
chambers are considered harmful for societies to the extent that the information and
beliefs shared can produce more extreme opinions and increase polarisation (Sunstein,
2002). Echo chambers and the polarization of views may shape decisions regarding
many aspects of social life, ranging from those based on opinions (about politics or
religion, for example) to those based on objective facts that have a well established
consensus.

The phenomenon of echo chambers has been recently discussed in relation to the
UK Brexit referendum and the US presidential elections. Considering objective facts,
such as global warming evidence or the usefulness of vaccinations, there is evidence
of a fervent debate on the validity of these scientific arguments, with a fraction of
the population showing ideological division and disagreement about the importance of
implementing greenhouse gas emission reduction’s policies or vaccinating to reduce the
risk of the spread of diseases. Many scholars are persuaded that these phenomena are
due to the existence of echo chambers.

From a psychological perspective, the echo chamber phenomenon seems connected
to the psychological mechanism called \textit{selective exposure}. According to the selective
exposure hypothesis, individuals tend to favour information that aligns with their pre-
exiting views while avoiding contradictory information.

In this paper, we aim to provide a theoretical explanation of the interaction between
the echo chamber and the selective exposure phenomena. Our theory is based on and
extends Benabou (2013). In that model, agents with anticipatory utility must decide
whether to undertake a common project. Ex-ante, they receive information about the
project’s benefits but can strategically decide to engage in denial at a psychological
cost. We augment Benabou’s model by adding an initial stage where agents can select
which information sources to pay attention to. For instance, an agent may select a
subset of newspapers among all available newspapers, or may follow some pundits
on twitter but not others. Crucially, when choosing the information sources to pay
attention to, agents take into account the fact that they may ex-post have to engage
in costly denial. This may create an incentive to avoid information sources that are
more prone to convey “bad news”. In other words, neglecting some information sources
can spare the agent from bearing the psychological cost of suppressing bad news ex-
post. Moreover, they also take into account that others may be similarly selective
in their choices of information sources. As in Benabou’s model, multiple equilibria
coexist (for some parameter values) in our framework. There is typically an equilibrium
where agents look at all available information and undertake the project only if it is
worthwhile given the available information. However, there also exists an equilibrium
where agents only pay attention to a subset of information sources (those that are less
likely to convey bad news). This equilibrium is similar to Benabou’s Mutually Assured
Delusion (MAD) equilibrium, although we stress an important difference. The MAD
equilibrium can be tested only indirectly, since it is not easy to observe whether agents engage in self delusion. In contrast, the sources of information that people look at are in principle observable. In terms of welfare, we show that the equilibrium where agents pay attention to all information sources dominates the equilibrium where agents are selective. While this result might appear obvious at first glance, we show that this is the case even when we include agents’ anticipatory feelings in the welfare measure.

The multiple equilibria result is in line with the empirical studies about the presence of echo chamber and polarisation in social media in the debate about politics, climate change policy and the importance of vaccination. Multiple equilibria can explain contradictory empirical evidence about the existence of echo chambers. For example Williams, McMurray, Kurz, and Lambert (2015) show that social media discussions on climate change often occur within polarising “echo chambers”, but also within “open forums”, namely mixed-attitude communities that reduce polarisation and stimulate debate. Examinations of selective exposure have shown that individuals do tend to confront with information and ideas they find supportive and consistent with their existing beliefs (Iyengar and Hahn, 2009). Garrett, Carnahan, and Lynch (2013) study Americans’ use of online sources of political information. According to their empirical results, they argue that even though individuals seek ideologically consistent news sites, they are not systematically avoiding other news sites. Other scholars have found evidence of echo chambers on Twitter (Barbera, Jost, Nagler, Tucker, and Bonneau, 2015; Himelboim, McCreery, and Smith, 2013), while others have shown that the trend does not persist on Facebook (Bakshy, Messing, and Adamic, 2015). Lawrence, Sides, and Farrell (2010) examine political polarisation among blog readers and find that they gravitate toward blogs that accord with their political beliefs. Few read blogs on both the left and right of the ideological spectrum. This empirical evidence provides strong support to our results. In fact, in our paper we show that selective exposure (in terms of information avoidance) is the result of an active role of the agents that strategically choose which information to pay attention to. At the investment stage, our equilibrium results can be interpreted as situations in which individuals, trapped inside the echo chamber, undertake uninformed decisions such as sustaining policies that do not reduce the greenhouse gas emissions, or deciding not to vaccinate. In the politics sphere, uninformed decisions may favour extreme candidates, populism and potentially harmful decisions such as leaving the EU in the Brexit referendum.

2. Related literature

This paper contributes to the broad literature on information avoidance. Information avoidance has been extensively studied in many research areas, such as medicine, communication, organisational behaviour and psychology. Sweeny, Melnyk, Miller, and Shepperd (2010) provide a survey of these literatures and define information avoidance
as any behaviour intended to prevent or delay the acquisition of available but potentially unwanted information. According to them three reasons are at the base of why people may chose to avoid information. More information may induce a change in beliefs, it may demand undesired action or it may cause unpleasant emotions. There exists also empirical research in psychology that documents the tendency of people not to attend, i.e to ignore, information. A long-standing body of work links this phenomenon to the selective exposure hypothesis. According to this hypothesis people tend to selectively process, interpret and recall data in a way that leads to more favourable beliefs about their personal characteristics or future prospects. Several recent papers show in a rigorous way that people tend to respond in an asymmetrical way to good and bad news. For example Karlsson, Loewenstein, and Seppi (2009) examine the degree to which people choose to expose themselves differentially to additional information after conditioning on prior positive and negative news. They develop a model of selective attention in which individuals first receive incomplete information and then decide whether to acquire and attend to definitive information. Their results show that for reasonable parameter values, individuals exhibit an ostrich effect. That is, agents avoid exposing themselves to information that might cause psychological discomfort. A comprehensive review, both theoretical and empirical, about information avoidance has been recently documented by Golman, Hagmann, and Loewenstein (2017). Their focus is on situations in which people avoid information even when it is free and could improve decision making. In particular, they refer to a phenomenon which they call ‘active’ information avoidance. To be classified as such, information avoidance requires that the individual is aware of the existence of that information and that information if freely accessible.

Our work is in some sense related to the growing literature on models of opinion polarisation. Dixit and Weibull (2007) show how the beliefs of Bayesians with different priors can polarise when new information arrives. Benoit and Dubra (2016) argue that findings of group attitude polarisation in psychological studies can be rationalised using purely Bayesian models. Fryer, Harms, and Jackson (2015) show that opinion polarisation can persist when Bayesian agents have limited memory. Ortoleva and Snowberg (2015) explore how overconfidence drives polarisation and affects political behaviour. In a recent work Gentzkow, Wong, and Zhang (2018) argue that ideological divisions, like the ones displayed by recent debates over global warming, evolution, and vaccination, may arise when Bayesian agents have small biases in information processing and they are uncertain which sources they can trust. In this scenario, increasing the amount of information available may deepen ideological differences. All these papers show that polarisation is a result of Bayesian agents that either have limited memory, bias in information processing or different prior.
Our paper, however, does not intend to explain opinion polarisation but provides a rationale for the formation of echo chambers in which opinions may later on polarise. In fact, our paper can explain why “information bubbles” or “echo chambers”, in which agents pay attention to only the same information or “voice”, emerge, inducing agents to undertake uninformed decisions. The formation of echo chambers is not the result of biases or limited memory of Bayesian agents. It is instead the result of agents that strategically choose not to pay attention to potentially bad news in order to avoid the psychological cost of denying them.

3. The Model

Our model builds on the framework by Benabou (2013). In particular we use the same model set-up but with a different information structure. Specifically, while in Benabou (2013) agents receive only one exogenous signal about an underlying state of the world, in our set-up we allow agents to choose the information source they want to pay attention to. Technology. A group of risk neutral agents, \( i \in \{1, \ldots, n\} \), are engaged in a joint project or other activities generating spillovers. Time is discrete and covers three periods, \( t = 0, 1, 2 \). At \( t = 1 \), each agent chooses effort \( e^i = \{0, 1\} \), which costs \( ce^i \), \( c > 0 \). At \( t = 2 \), she will reap utility

\[
U^i_2 = \theta \left[ \alpha e^i + (1 - \alpha) e^{-i} \right]
\]

where \( e^{-i} = \frac{1}{n-1} \sum_{j\neq i} e^j \) is the average effort of others and \( (1 - \alpha) \in [0, 1 - 1/n] \) represents the degree of interdependence, reflecting the joint nature of the enterprise. The payoff structure of the final period is very simple and it is exactly the same of Benabou (2013): there is no interdependence between effort decisions. This implies that there are not in built complementarities in the payoff of the agent, but only externalities, given by \( 1 - \alpha \), without strategic interaction.

The state of nature is uncertain and it is \( H \) (high) with (prior) probability \( \mu \) and \( L \) (low) with probability \( (1 - \mu) \). The project productivity \( \theta \) is uncertain with expected value equal to \( \theta_H \) conditional to the state being \( H \) and \( \theta_L \) conditional to the state being \( L \). We denote \( \Delta \theta = \theta_H - \theta_L > 0 \) and we assume \( \theta_H > 0 \) without loss of generality.

Information structure. There are two different information sources about the state of the world, information source \( I_1 \) and information source \( I_2 \). Each information source delivers either a signal about the state of the world or it is silent about it. We assume that signals are perfectly correlated within the same information source, that is agents observing the same information source receive the same signal. These information sources are costless, in the sense that the information is freely available to any agent that would like to observe it.

Specifically, \( I_1 \) sends the following signals
- $s_H$ with probability $p_1$ and $\emptyset$ (the empty signal) with probability $(1 - p_1)$ if the state is $H$, 
- $\emptyset$ with probability 1 if the state is $L$, 
and $I_2$ sends 
- $\emptyset$ with probability 1 if the state is $H$, 
- $s_L$ with probability $p_2$ and $\emptyset$ with probability $(1 - p_2)$ if the state is $L$.

Information source $I_1$ is the favourable information in the sense that either provides good news or leaves the agent uncertain about the state of the world. On the contrary, information source $I_2$ is potentially disappointing because it may convey bad news to the agent.

At the beginning of period 0 each agent chooses the information source to which she will pay attention. We assume that an agent pays always attention to the more favourable information\(^1\). However, because this information source is noisy, she can opt to pay attention to more information to reduce this noise. Formally she can (i) choose $I_1$ or (ii) choose both $I_1$ and $I_2$. Notice that when an agent $i$ chooses to pay attention to both information sources, she will receive the following combination of signals 
- $(\emptyset, \emptyset)$ with probability $1 - \mu p_1 - (1 - \mu) p_2$, 
- $(s_H, \emptyset)$ with probability $\mu p_1$ and 
- $(\emptyset, s_L)$ with probability $(1 - \mu) p_2$, 

where the first signal refers always to a signal delivered by $I_1$ and the second one refers always to a signal delivered by $I_2$.

Preferences. Period 1 payoff (the investment stage) includes the cost of effort, $-ce^i$, but also the anticipatory utility experienced from thinking about one’s future prospects, $sE_1^i[U_2^i]$, where $s \geq 0$ parametrises the psychological and health effects of hopefulness, dread, and similar emotions.

At the start of period 1, an agent $i$ chooses effort to maximise the expected present value of payoffs, discounted at rate $\delta \in (0, 1)$:

$$U_1^i = -ce^i + sE_1^i \left(U_2^i\right) + \delta E_1^i \left(U_2^i\right)$$

(2)

Actual beliefs in period 1 will depend on the information source chosen in period 0 and how objectively or subjectively the agent processed the signals received as described in the next paragraph. Therefore, the strategic interaction between agents is not at the effort decision stage, but at the information source choice stage. In period 0, an agent $i$ aims to maximise the discounted utility of all payoffs by choosing which information source to pay attention to, that is

$$U_0^i = -M^i + \delta E_0^i \left[-ce^i + sE_1^i \left(U_2^i\right)\right] + \delta^2 E_0^i \left(U_2^i\right),$$

(3)

\(^1\)This is a simplifying assumption that we impose for tractability of the model. We could also allow agents to choose only $I_2$. However, our results would not be affected.
where $E^i_t$ denotes expectations at $t = 0, 1$ and $M^i$ the date-1 costs of her cognitive strategy.

Cognitive Strategy and Beliefs. In period 0, once the information source has been chosen, agents receive a signal. Upon observing the signal, each agent chooses how to interpret it, whether to keep it in mind or not to think about it, etc. Denoting with $\sigma \in \{s_H, s_L, \emptyset\}$ any signal received at period 0 and with $\hat{\sigma}^i \in \{\hat{s}_H, \hat{s}_L, \hat{\emptyset}\}$ the signal recalled at the beginning of period 1, formally an agent can:

i. accept the facts realistically, truthfully encoding $\hat{\sigma}^i_j = \sigma_j$, into memory or awareness$^2$.

ii. engage in denial, censoring or rationalization, encoding
- $\hat{s}_H$ when she receives $\emptyset$ from $I_1$,
- $\hat{\emptyset}$ when she receives $s_L$ from $I_2$.

We assume that denial is costly and that for each signal censored the agent bears an immediate cost $m_j \geq 0$, with $j \in \{1, 2\}$. In particular $m_1$ is the cost of censoring a signal that comes from information source $I_1$; while $m_2$ is the cost of denying a signal from information source $I_2$. We do not put any restriction on the cost of denial, allowing for both $m_1 = m_2$ and $m_1 \neq m_2$.

It is worth highlighting that, differently from Benabou (2013), in our set-up we consider the possibility that an agent engages in denial only when the signal received is the less favourable one among the two possible she can receive from each information source. That is, agents can change the signal from no signal to good signal if she observes information source $I_1$ and from bad to no signal if she observes information source $I_2$, but not vice-versa. However, based on Benabou (2013) results, we could generalise the model to a framework where the cognitive strategy considers also the case of censoring the more favourable signal. He shows that it is never optimal to deny a good signal for a positive cost of denial.

Specifically, in our model, the agent’s cognitive strategy functions as follows

- $\lambda^i_1 \equiv Pr(\hat{\emptyset}|\emptyset)$ is the probability that the agent will process correctly the signal $\emptyset$ received from information source $I_1$. Thus, $\lambda^i_1 = 1$ means that from period 0 to period 1 the agent carries the same information.

- $\lambda^i_2 \equiv Pr(\hat{s}_L|s_L)$ is the probability that the agent will process correctly the the signal $s_L$ received from information source $I_2$. So, $\lambda^i_2 = 1$ means that from period 0 to period 1 the agent carries the same information.

Although the model allows cognitive mixed strategy, we restrict our attention to equilibria in pure strategies. That is, we only look at the case where the agents either deny the signal with probability 1 or are completely realist.

$^2$So for example if an agent receive the signal $s_H$ and encodes it truthfully, at the beginning of period 1 she will observes $\hat{s}_H$. Thus $\hat{s}_H = s_H$.

$^3$The complement of these two probabilities is respectively $(1 - \lambda^i_1) \equiv Pr(\hat{s}_H|\emptyset)$ and $(1 - \lambda^i_2) \equiv Pr(\hat{\emptyset}|s_L)$. 

Thus, given period-0 agents’ cognitive strategy, the period-1 information set of an agent \(i\) may be different from her period-0 information set. We assume that agents are rational, in the sense that they are aware of their tendency to deny bad signals and they will take this into account when they formulate their posterior beliefs. We also assume that, when an agent observes both information sources, the denial strategies they will take this into account when they formulate their posterior beliefs. We also rational, in the sense that they are aware of their tendency to deny bad signals and

\[
Pr(s_H|\hat{s}_H, \lambda_1^i) = \frac{\mu_{p_1}}{\mu_{p_1} + (1 - \mu_{p_1})(1 - \lambda_1^i)} \equiv r(\lambda_1^i).
\]

In the case where the agent observes both \(I_1\) and \(I_2\), given the assumption that denial strategies on \(I_1\) and on \(I_2\) are set independently ex-ante, she might recall \((\hat{\varnothing}, \hat{\varnothing})\), \((\hat{s}_H, \hat{\varnothing})\) or \((\hat{s}_H, \hat{s}_L)\). Therefore if she recalls \((\hat{s}_H, \hat{\varnothing})\) her posterior beliefs are

\[
Pr(s_H, \varnothing|\hat{s}_H, \hat{\varnothing}) = \frac{\mu_{p_1}}{\mu_{p_1} + (1 - \mu_{p_1} - (1 - \mu)p_2)(1 - \lambda_1^i)(1 - \lambda_2^i)} \equiv v(\lambda_1^i, \lambda_2^i)
\]

\[
Pr(\varnothing, \varnothing|\hat{s}_H, \hat{\varnothing}) = \frac{(1 - \mu_{p_1} - (1 - \mu)p_2)(1 - \lambda_1^i)}{\mu_{p_1} + (1 - \mu_{p_1} - (1 - \mu)p_2)(1 - \lambda_1^i)(1 - \lambda_2^i)} \equiv q(\lambda_1^i, \lambda_2^i)
\]

\[
Pr(\varnothing, s_L|\hat{s}_H, \hat{\varnothing}) = \frac{(1 - \mu)p_2(1 - \lambda_1^i)(1 - \lambda_2^i)}{\mu_{p_1} + (1 - \mu_{p_1} - (1 - \mu)p_2)(1 - \lambda_1^i)(1 - \lambda_2^i)} \equiv p(\lambda_1^i, \lambda_2^i).
\]

If she recalls \((\hat{\varnothing}, \hat{\varnothing})\) her posterior beliefs are

\[
Pr(\varnothing, \varnothing|(\hat{\varnothing}, \hat{\varnothing}), \lambda_2^i) = \frac{1 - \mu_{p_1} - (1 - \mu)p_2}{1 - \mu_{p_1} - (1 - \mu)p_2 + (1 - \mu)p_2(1 - \lambda_2^i)} \equiv q(\lambda_2^i).
\]

Finally, if she recalls \((\hat{s}_H, \hat{s}_L)\), the agent understands immediately that she has been delusional, inferring that the state is \(L\).

Timing. At \(t = 0\) agents choose which information source to pay attention to and after they receive the corresponding signal they decide how to process it. At \(t = 1\) agents choose an action \(e = \{0, 1\}\) at cost \(ce\) and have anticipatory feelings about \(U_2\). At \(t = 2\) agents get their final payoff \(U_2\).

4. Model Solution

4.1. The effort choice. We solve the model by backward induction. In period \(t = 1\), conditional on the recalled signal, an agent \(i\) chooses effort \(e\) to maximise (2). Notice that, given \(U_2\), an agent \(i\)'s effort decision only depends on her beliefs about \(\theta\). That
is, she exerts effort only if \((s + \delta)\alpha E_1(\theta) > c\), independently of the effort decision of the other agents. The following parametric restriction

**Assumption 1.**

\[
\theta_L < \frac{c}{(\delta + s)\alpha} < \frac{c}{\delta\alpha} < \mu\theta_H + (1 - \mu)\theta_L
\]  

(8)

ensures that, without denial taking place, if the agent knew the true state, she would not exert effort in the bad-news state and she would exert effort in the good-news state. In contrast, if she were to choose an action based only on the prior, she would exert effort. In this set-up, as we allow agents to select the information sources they want to listen to, if denial does not take place, an agent \(i\)'s incentive to exert effort, conditional on having received a signal from the more favourable information source \(I_1\), is

- \(e = 1\) if she receives the signal \(s_H\), because she knows that the state is \(H\) w.p. 1 and therefore \(\theta = \theta_H\) and
- \(e = 1\) if she receives the signal \(\emptyset\), as long as

\[
\frac{c}{(s + \delta)\alpha} < \underbrace{Pr(H|\emptyset)\theta_H + Pr(L|\emptyset)\theta_L}_{E(\theta|\emptyset)}. 
\]  

(9)

If she chooses to listen to both information sources, then

- \(e = 0\) if she receives \((\emptyset, s_L)\), because she knows that the state is \(L\) and therefore \(\theta = \theta_L\) and
- \(e = 1\) if she receives both \((s_H, \emptyset)\) and \((\emptyset, \emptyset)\) as long as condition (9) holds.

To make things simple we restrict our attention to situations where effort is not exerted only when the agent receives the bad signal, i.e. when she receives \(s_L\).

**Assumption 2.** *Condition (9) is always satisfied.*

This assumption implies that an agent \(i\) exerts effort whenever she receives an empty signal, that is both in the case she pays attention to only \(I_1\) and in the case she pays attention to both \(I_1\) and \(I_2\), and encode it truthfully. Moreover notice that with the above assumption, an agent \(i\) exerts effort also whenever she engages in denial of the signal received.

**Lemma 1 (Optimal Action).** *For all recalled signals profiles which do not include \(\hat{s}_i = \hat{s}_L\), an agent \(i\) always exerts effort.*

*Proof.* A formal proof is provided in Appendix A. \(\square\)

To understand the undergoing mechanism of the effort choice, notice that at the recalling stage the agent does not remember the signal received the period before but is aware of her tendency to deny signals. She just remembers the information sources observed. Given that the agent is a Bayesian updater, she also takes into account her tendency to deny signals when updating her beliefs about the state of the world.
Therefore, when denial occurs with probability 1, the conditional expected productivity of \( \theta \) is equivalent to what would be the expected value of \( \theta \) having not observed the information source of the denied signal. Moreover, assumptions 1 and 2 imply that effort is not exerted only when agents know that the state is low with probability one. Thus, systematic denial of the signal \( s_L \) is equivalent, for the agent, to not knowing when the state is low.

### 4.2. Ignorance as equilibrium.

At the beginning of period 0 agent \( i \) chooses which information source to observe in order to maximise the discounted utility of all payoffs given by (3). When selecting an information source, an agent will take into account:

a) the fact that she might ex-post engage in denial, b) how other agents process the information they receive. In particular, agent \( i \) needs to form beliefs about what information sources other agents will pay attention to and how they will process the signals received in order to predict whether others will exert effort or not. As argued above, the way an agent processes the information received affects her own effort choice, which in turn affects the final payoffs of all the other agents in the economy and, through the anticipatory utility term, their anticipatory feelings. This is because the final period payoff - expression (1) - depends both on \( i \)'s effort and on the effort of other agents. This spillover effect determines agent \( i \)'s anticipatory feelings. For instance, agent \( i \) will feel upbeat if she expects both the state of nature to be high and the others to exert effort. On the other hand, she will be less enthusiastic if others exert effort when she expects the low state of nature.

In Benabou (2013), if an agent expects others to suppress bad news (and thus refrain from exerting effort), she may have higher incentives to suppress bad news. This happens when \( \theta_L < 0 \). In this case, the fact that others are suppressing bad news induces them to exert effort, which in turn generates a negative spillover. Confronted with anticipatory feelings, the agent thus chooses to suppress bad news (which in turn leads her to exert effort as well). Benabou (2013) calls this mechanism the MAD principle.

In our model, agents receive a signal only after having chosen the information source. If the signal delivered by the information source chosen is a bad news and if an agent expects the other agents to exert effort, then she has an incentive to deny the bad news. However denying a signal is costly. If, on the one hand, the agent increases her anticipatory utility by being delusional and exerting effort, on the other hand she bears the psychological cost of suppressing the bad signal. However, the agent could avoid the risk of facing the bad news altogether by carefully selecting her information sources. This mechanism, as in Benabou (2013), spills over onto the other agents who become more willing to avoid the information source that might potentially carry the bad news and thus they become more likely to exert effort.
This is the case of agents observing only $I_1$, which represents the information source more favourable. Any agent that chooses to observe $I_1$ will always exert effort regardless of how the signal received is processed. Moreover, for a cost of denial sufficiently high, an agent will always encode truthfully the signal received by $I_1$. Clearly, an agent that were to observe $I_2$ together with $I_1$ will become more informed about the state of the world, and in particular this will help her to better predict the bad state and to choose the “right” action, on the ex-post perspective. However, due to the anticipatory preferences of the agent, when she receives a bad signal, she has the tendency to deny it and this is more likely to happen if the agent thinks that the other agents in the economy will always exert effort. Thus, in the end, an agent trying to become more informed will remain trapped by her way of processing the extra information she decides to receive; she benefits by being delusional because she perceives the extra information as good news but at the same time she immediately pays the mental cost of suppressing the bad news. The only way an agent has to avoid this trap is to decide to remain ignorant observing only $I_1$. We formalise this equilibrium in the following proposition.

**Proposition 1.** Define $\theta_L \equiv \frac{1}{\alpha(s+\delta)} \left[ c - s\Delta \theta \left( \frac{(1-p_1)\mu}{1-\mu p_1} - 1 \right) \right]$ and $\bar{\theta}_L \equiv \frac{c}{\alpha(s+\delta)}$ and assume $\mu > \frac{1}{2-p_1}$ to ensure $\bar{\theta}_L > \theta_L$. Then, for any $\theta_L \in (\theta_L, \bar{\theta}_L)$ there exists a non empty interval $[\underline{m}, \overline{m}]$ such that for any cost of denial $m_1, m_2 \in [\underline{m}, \overline{m}]$ there exists an equilibrium where all agents (i) look only at information source $I_1$ and (ii) exert effort.

**Proof.** Proof in the Appendix A

Formal proof is provided in the Appendix. Here we just describe how we constructed this equilibrium. We first assume that there exists an equilibrium in which all agents observe only information source $I_1$ and study agents’ optimal cognitive strategy. We then check the optimal cognitive strategy of an agent $i$ if she were to deviate and observe both $I_1$ and $I_2$. Finally, we compare the utility from the deviation to the utility in the candidate equilibrium, and we show that the deviation is not profitable.

This proposition thus shows that agents prefer to restrict attention only to some information, the more favourable one, if paying attention to more information would bring them to costly deny bad news leaving them to lower ex-ante utility. In other words, it is pointless to deviate and acquire more information if one then has to deny it. Neglecting information saves the psychological cost of denial. This happens (i) for values of $\theta_L$ that can be both negative and positive and (ii) when the prior probability of a high state is high enough.

**4.3. Full information as equilibrium.** In this section we provide sufficient conditions under which there exists an equilibrium where agents are fully informed, that is they observe both information sources and are always realist.
If an agent decides to listen to all the information available, that is both $I_1$ and $I_2$, she can receive the following pairs of signals: $(s_H, \emptyset)$, $(\emptyset, \emptyset)$ and $(\emptyset, s_L)$. In the first two cases the agent will always exert effort regardless the way she processes the signals. In the latter case instead, she does not exert effort if she does not deny the signal $s_L$ and she exerts effort if the signal $s_L$ is denied. Moreover, when the agent receives bad news, she would like the other agents to exert effort if $\theta_L > 0$ and abstain from it if $\theta_L < 0$. If the cost of denying the signals received is sufficiently high, the agent knows that she will process correctly any signal received, implying that if the state is bad she will not exert effort. If she expect the other agents to observe both information sources as well, then she knows that also the other agents will not exert effort when the signal delivered by $I_2$ is a bad news. Therefore if the agent cannot deny the bad news because it is too costly to do so, than it is better for her to become fully informed rather than observing only $I_1$.

**Proposition 2.** For any $\theta_L < 0$, there exists a threshold cost of denial $\hat{m}$ such that, for any $m_1, m_2 \geq \hat{m}$ there exists an equilibrium where all agents (i) look at both information sources and (ii) are always realist.

*Proof.* Proof in the Appendix A. □

This proposition shows that, when in the low state future prospects are bad, if the cost of denying both information sources is sufficiently large, agents choose to be fully informed and accept reality. The logic of this equilibrium mirrors the one for the selective exposure equilibrium presented in the previous section. Under this scenario, an agent still would like to avoid bad news. However, different from the previous section, other agents now pay attention to $I_2$. If others are realist, then they will refrain from exerting effort when faced with bad news. This implies that there will be no negative spillover. As a result, the gain in anticipatory utility from denial or avoidance of $I_2$ is smaller than in the previous case. In other word, selective exposure is contagious. An agents want to be selective only if others are selective. And this happens when $\theta_L < 0$. This creates strategic complementarity and thus ultimately multiple equilibria.

**Proposition 3.** Assume $\mu > \frac{1}{2-p_1}$. For any $\theta_L \in (\theta_L, \overline{\theta}_L)$ if the following condition holds

$$-\theta_L(1 - \alpha) > (\theta_H - \theta_L)r(I_1, I_2)$$

then for any $m_1, m_2 \in [\underline{m}, \overline{m}]^2$ there exists an equilibrium where all agents observe only $I_1$ and are realist and an equilibrium where all agents observe both $I_1$ and $I_2$ and are realist.

*Proof.* Proof in Appendix A. □

This result, which shows multiplicity of equilibria in terms of different awareness of the same reality, reflects the MAD principle of Benabou (2013). According to the MAD
principle, $\theta_L < 0$ generates complementarity in the cognitive strategy of the agents leading to multiple equilibria. While in Benabou (2013) the multiplicity arises in terms of the cognitive strategy, in our paper the multiplicity arises in terms of the information sources observed. However the explanation of this phenomenon has the same intuition as in Benabou (2013). Looking at equation (10), the LHS can be interpreted as the net benefit of an agent $i$, from observing both information sources, to believe that the project is highly productive ($\theta_H$ rather than $\theta_L$) when there are no spillovers (i.e. when $\alpha = 1$) or, equivalently, fixing everyone’s action $e = 1$. In fact, when an agent $i$ observes both $I_1$ and $I_2$ and receives $(\emptyset, \emptyset)$, she does not know the state of the nature, but she only knows that with probability $r(I_1, I_2)$ the productivity is $\theta_H$ and with probability $(1 - r(I_1, I_2))$ the productivity is $\theta_L$. Therefore the net expected gain of thinking the state is $H$ when it is not is $r(I_1, I_2)\theta_H + (1 - r(I_1, I_2))\theta_L - \theta_L \equiv r(I_1, I_2)(\theta_H - \theta_L)$.

The RHS of (10) represents the expected loss that is inflicted to the agent $i$ by the other agents, through the spillover effect $(1 - \alpha)$, if the state is $L$ and they exert effort anyway because they observe only information source $I_1$. Therefore, when the expected loss inflicted by others is higher than the expected benefit an agent will get being more informed, ignorance becomes contagious and the agent $i$ prefers to remain ignorant too. This “complementarity” therefore leads to multiple equilibria.

5. Welfare Analysis

In this section we rank the equilibria obtained in the previous section in terms of welfare. Welfare is computed at period $t = 0$.

Consider the case of the low state of the world. In the equilibrium where agents observe only $I_1$ they will receive the signal $\emptyset_1$ and equilibrium welfare is

$$U^*_{L,I_1} = \delta[-c + (s + \delta)(r(I_1)\Delta\theta + \theta_L)]$$

(11)

where $r(I_1) = \frac{\mu(1-p_1)}{1-\mu p_1}$. In the other equilibrium where all agents observe both $I_1$ and $I_2$, agents receive either $(\emptyset_1, \emptyset_2)$ or $(\emptyset_1, s_L)$ and equilibrium welfare is

$$U^*_{L,I_1,\&I_2} = (1 - p_2)\delta[-c + (s + \delta)(r(I_1, I_2)\Delta\theta + \theta_L)]$$

(12)

where $r(I_2) = \frac{\mu(1-p_1)}{1-\mu p_1(1-\mu)p_2}$. A comparison of the equilibrium welfare in the two equilibria shows that welfare is higher in the equilibrium where both $I_1$ and $I_2$ are observed provided the following holds

$$-(1 - \alpha)\theta_L > r(I_1, I_2)r(I_1)(\theta_H - \theta_L) - \left(\frac{c}{s + \delta} - \alpha\theta_L\right).$$

(13)

Consider now the case of the high state of the world. In the equilibrium where agents observe only $I_1$, they receive $s_H$ with probability $p_1$ and $\emptyset_1$ with probability $1 - p_1$, with equilibrium welfare corresponding to

$$U^*_1 = p_1\delta[-c + (s + \delta)\theta_H] + (1 - p_1)\delta[-c + (s + \delta)(\Delta\theta r(I_1) + \theta_L)].$$

(14)
In the equilibrium where agents observe both information sources, they receive \((s_H, \emptyset_2)\) and \((\emptyset_1, \emptyset_2)\) with probability \(p_1\) and \(1 - p_1\) respectively with equilibrium welfare being equal to
\[
U^*_{I_1, I_2} = p_1 \delta(-c + (s + \delta)\theta_H) + (1 - p_1)\delta[-c + (s + \delta)(\Delta \theta r(I_1, I_2) + \theta_L)].
\] (15)

**Proposition 4.** (i) If condition (10) holds, welfare in the low state is higher in the equilibrium where agents observe both information sources.
(ii) In the high state welfare is always higher in the equilibrium were agents observe both information sources.

**Proof.** Proof in Appendix A. □

The result of part (i) seems counter intuitive. In fact, the anticipatory preferences of an agent should balance out the loss she will incur in the future when wrongly choosing \(e = 1\) with the benefit she receives from the anticipatory utility, proportional to the value of the anticipatory preferences parameter \(s\). However, notice that we are considering only values of \(\theta_L\) negative and parameters space of the cost of denials that sustain multiplicity of equilibria. In this situation being more informed always dominate ignorance, because full information will impede agents to undertake the wrong action, i.e. \(e = 1\), with some positive probability, which has negative externalities. Moreover, the cost of denial that makes realism or denial sustainable, is negatively related to the anticipatory preferences parameter \(s\). That is, lower values of the cost of denial that make denial sustainable, correspond to higher values of \(s\). However, our multiple equilibria exists for parameters space of the cost of denial small enough, or equivalently anticipatory preferences larger enough, such that agents choose information sources whose signals will not be denied. Therefore, in our equilibria agents are always realist because, for the same parameter space of the cost of denial, it they were to observe a signal that later on would be denied, that information source wouldn’t be chosen in equilibrium. Thus, in the low state the equilibrium with full information Pareto dominates the equilibrium with ignorance, because in the former equilibrium there is a positive probability that agents do not exert effort when it is better not to do it. Intuitively, if anticipatory feelings are very strong, then the selective exposure could be optimal. However, we are considering a parameter space where the two equilibria coexist. In this parameter space, anticipatory utility cannot be too large, otherwise the equilibrium with full information cannot exist. We cannot prove this result directly though, as we are able to characterise the equilibrium only under the circumstance of \(\theta_L < 0\).

6. CONCLUSIONS AND FINAL REMARKS

This paper develops a model that explains how strategic information avoidance leads to the formation of echo chambers, where ignorance spreads inside them. In settings
where avoidance of bad news have negative externalities, these news become harder to accept, resulting in a contagious collective ignorance in which agents undertake “harmful” uninformed decisions. Examples of this phenomenon include the well known echo chamber of no-vaccination movement and the many ideological echo chambers in the political sphere, which leads to polarisation of opinion and extremism. This paper, to the best of our knowledge, is the first that explains the mechanism that links the formation of echo chambers to the selective exposure theory. In fact, in our model the echo chamber emerges in equilibrium as the result of the strategic selection of the information to pay attention to, in order to avoid the psychological cost of denying ex-post uncomfortable news.

This paper belongs to a broad research agenda that looks at the recent debate on how social media and the variety of news’ markets affect individual choice of news consumption. However, our results are not robust enough to be able to explain why in the real world, even though we do observe the presence of echo chambers, we also observe that a fraction of the society relies on diversified information. Possible explanations of this evidence is that people might have different beliefs about the state of the world, or that people and groups are heterogeneous in many other aspects that shape their information choice. Our model is not rich enough to capture these elements.

The model could be further extended to account for different groups with some heterogeneity between groups. Such a model, would be a step further able to explain ideological polarisation. Under this perspective, when agents have anticipatory preferences and can engage in denial of bad signals, we could address whether group polarisation occurs and whether it is symmetrical or not. For example we expect to find that only one group polarises while the other group does not. The opposite scenario would be one where one group polarises towards one information source, or “opinion”, and the other group polarises towards the other information source.

In the context of social media and plurality of online and physical outlets in which information is consumed, the providers of news play an active role in selecting the signals to deliver through their platforms. Along this direction, another possible extension of the model is to introduce endogenous information sources in which a sender has to optimally select the signal to deliver, taking into account that consumers of information have anticipatory preferences and can engage in denial of the signal received. The model would allow to identify not only how news consumption shapes individual ignorance, but also how and whether ignorance or full awareness depends on the equilibrium production of news.
A. Omitted Proofs

Proof of Lemma 1. Lemma 1 is an if and only if statement. That is, for any recalled sigm profile that does not include \( \hat{s}_L \) an agent \( i \) exerts effort and for any recalled sigm profile that does include \( \hat{s}_L \) the agent does not exert effort. At the recalling stage, depending on the information source chosen, an agent recalls one of the following sigm profiles: \( \hat{\sigma}, \hat{s}_H, (\hat{\sigma}, \hat{\sigma}), (\hat{s}_H, \hat{\sigma}), (\hat{\sigma}, \hat{s}_L) \) or \( (\hat{s}_H, \hat{s}_L) \). At the recalling stage, when she decides whether to exert effort, she also takes into account her period \( t = 0 \) denial strategy. Notice that, given that an agent can change the sigm from bad to good and not vice-versa, an agent \( i \) recalling both \( \hat{\sigma} \) and \( (\hat{\sigma}, \hat{s}_L) \) is never in denial. In all the other recalled sigm profiles an agent \( i \) may or may not be in denial of the sigm/s received.

We first prove that effort is always exerted if the recalled sigm profile does not include \( \hat{s}_L \), that is when the agent recalls one of the following \( \hat{\sigma}, \hat{s}_H, (\hat{\sigma}, \hat{\sigma}), (\hat{s}_H, \hat{\sigma}) \). If the agent recalls \( \hat{\sigma} \), by Assumption 2 she will exert effort because \( E(\theta|\hat{\sigma}) = E(\theta|\emptyset) > \frac{c}{\alpha(s + \delta)} \).

In the other cases in which the agent may have denied the sigm, given (2) an agent \( i \) exerts effort whenever

\[
E(\theta|\hat{\sigma}_1, \lambda_1^i) > \frac{c}{\alpha(s + \delta)}
\]

if she pays attention only to Information source \( I_1 \) and

\[
E(\theta|\hat{\sigma}_1, \hat{s}_H, \lambda_1^i, \lambda_2^i) > \frac{c}{\alpha(s + \delta)}
\]  

(16)

if she pays attention to both information sources. Therefore, if an agent \( i \) recalls \( \hat{s}_H \), then

\[
E(\theta|\hat{s}_H, \lambda_1^i) = \frac{Pr(s_H|\hat{s}_H)E(\theta|s_H) + Pr(\emptyset|\hat{s}_H)E(\theta|\emptyset)}{\mu p_1 + (1-\mu p_1)(1-\lambda_1^i)} \theta_H + \frac{(1-\mu p_1)(1-\lambda_1^i)}{\mu p_1 + (1-\mu p_1)(1-\lambda_1^i)} E(\theta|\emptyset).
\]  

(17)

Equation (17) is increasing in \( \lambda_1^i \), and at \( \lambda_1^i = 0 \) it reduces to \( E(\theta) \), which by Assumption 1 implies \( e = 1 \). Consider now the case of an agent \( i \) observing both information sources. If she recalls \( (\hat{\sigma}_1, \hat{\sigma}_2) \), it might be that either she is recalling the true sigm or she has denied \( s_L \). Thus the expected productivity of \( \theta \) will be

\[
E(\theta|(\hat{\sigma}_1, \hat{\sigma}_2), \lambda_1^i) = Pr((\emptyset_1, \emptyset_2)|(\hat{\sigma}_1, \hat{\sigma}_2), \lambda_1^i) E(\theta|(\emptyset_1, \emptyset_2)) +
\]

\[
+ Pr((\emptyset_1, s_L)|(\hat{\sigma}_1, \hat{\sigma}_2), \lambda_1^i) E(\theta|(\emptyset_1, s_L))
\]

\[
= \frac{1-\mu p_1 - (1-\mu)p_2}{1-\mu p_1 - (1-\mu)p_2 + (1-\mu)p_2(1-\lambda_2^i)} \theta_L + \frac{(1-\mu)p_2(1-\lambda_2^i)}{1-\mu p_1 - (1-\mu)p_2 + (1-\mu)p_2(1-\lambda_2^i)} \theta_L
\]  

(18)

Equation (18) is increasing in \( \lambda_2^i \) and at \( \lambda_2^i = 0 \) it reduces to \( E(\theta|\emptyset_1) \) which, by Assumption 2, implies \( e = 1 \). With the same argument we can show that effort is
exerted when an agent \(i\) recalls the signal \((\hat{s}_H, \hat{s}_L)\). In this case it might be that either he has received the true signal, he has denied only \(\emptyset_1\) or he has denied both \(\emptyset_1\) and \(s_L\). It is easy to see that \(E(\theta|\hat{s}_H, \hat{s}_L, \lambda_1^1, \lambda_2^1)\) is increasing both in \(\lambda_1^1\) and \(\lambda_2^1\) with \(E[\theta](\hat{s}_H, \hat{s}_L, 0, 0) = E(\theta)\). Therefore, Assumption 1 implies \(e = 1\) when agent \(i\) recalls \((\hat{s}_H, \hat{s}_L)\).

We now prove that effort is not exerted if the recalled signal profiles includes \(\hat{s}_L\), that is when an agent \(i\) recalls both \((\emptyset, \hat{s}_L)\) and \((\hat{s}_H, \hat{s}_L)\). If the agent recalls \((\emptyset, \hat{s}_L)\), according to her posterior beliefs she knows that the state of the world is \(L\), therefore she has received the true signal, he has denied only \(s_L\). An agent’s net incentive to deny the signal. On the contrary, if she receives the empty signal \(\emptyset\), she decides to be realist, she obtains the inter-temporal utility

\[
U_i^0 = \frac{\delta}{\theta_L - \lambda_1^1} \left( \frac{1 - \mu_p}{1 - \mu_p} \theta_H + \lambda_1^1 \theta_L \right).
\]

because, \(Pr[(s_H, s_L)|(\hat{s}_H, \hat{s}_L), \lambda_1^1] = \frac{0}{\frac{\delta}{\theta_L - \lambda_1^1} \left( \frac{1 - \mu_p}{1 - \mu_p} \theta_H + \lambda_1^1 \theta_L \right)} = 0\). Therefore an agent \(i\) recalling \((\hat{s}_H, \hat{s}_L)\) will not exert effort. \(\square\)

**Proof of Proposition 1.** Suppose that an agent \(i\) chooses to observe information source \(I_1\) only. Consider a symmetric equilibrium, where everybody else is observing \(I_1\). An agent \(i\) will receive the signal \(s_H\) with probability \(\mu_{p_1}\) and \(\emptyset\) with probability \(1 - \mu_{p_1}\). Upon receiving \(s_H\), the agent knows the state is \(H\). Accordingly, she has no incentive to deny the signal. On the contrary, if she receives the empty signal \(\emptyset\) she might deny it, recalling \(\hat{s}_H\) at \(t = 1\).

The optimal cognitive strategy of an agent \(i\) when receiving \(s_L\) is as follows; if she decides to be realist, she obtains the inter-temporal utility

\[
U_{0R|o}^i = \delta \left[ -c + (s + \delta)E(\theta|\emptyset) \right]
= \delta \left[ -c + (s + \delta) \left[ Pr(H|\emptyset) \theta_H + Pr(L|\emptyset) \theta_L \right] \right]
= \delta \left[ -c + (s + \delta) \left[ \frac{(1 - p_1) \mu_p}{1 - \mu_{p_1}} \theta_H + \lambda_1^1 \theta_L \right] \right].
\]

Whereas if the agent reacts denying the signal, she obtains

\[
U_{0D|o}^i = -m_1 + \delta \left[ -c + s \left[ r(\lambda_1^1) \theta_H + (1 - r(\lambda_2^1)) E(\theta|\emptyset) \right] + \delta E(\theta|\emptyset) \right].
\]

Notice that, for an agent, realism or denial of a signal from \(I_1\) is independent of other agents’ cognitive strategy. This is so because, when observing only information set \(I_1\), agents always exert effort independently of the recalled signal. An agent’s net incentive
to deny reality is thus:

\[ U_{1D}(\emptyset) - U_{1R}(\emptyset) = -m_1 + \delta s \left[ r(\lambda_1^i)(\theta_H - E(\theta|\emptyset)) \right] \\
= -m_1 + \delta sr(\lambda_1^i) \left[ \theta_H - \left( \frac{\mu(1 - p_1)}{1 - \mu p_1} \theta_H + \frac{1 - \mu}{1 - \mu p_1} \theta_L \right) \right] \\
= -m_1 + \delta sr(\lambda_1^i) \left( \frac{1 - \mu}{1 - \mu p_1} \theta_H - \frac{1 - \mu}{1 - \mu p_1} \theta_L \right) \\
= -m_1 + \delta sr(\lambda_1^i) \left( \Delta \theta \frac{1 - \mu}{1 - \mu p_1} \right). \tag{21} \]

Let us define \( \Gamma(\lambda_1^i, m_1) \) the RHS of equation (21). Thus, the optimal strategy for agent \( i \) is:

a) \( \lambda_1^i = 1 \) if \( \Gamma(1, m_1) \leq 0 \), which means

\[ m_1 \geq \delta s \Delta \theta \frac{1 - \mu}{1 - \mu p_1} \equiv \overline{m}(I_1) \tag{22} \]

b) \( \lambda_1^i = 0 \) if \( \Gamma(0, m_1) \geq 0 \), which means

\[ m_1 \leq \delta s \Delta \theta \frac{\mu p_1 (1 - \mu)}{1 - \mu p_1} \equiv \underline{m}(I_1). \tag{23} \]

Notice that \( \underline{m}(I_1) < \overline{m}(I_1) \). This means that denial is always optimal if the cost of denial \( m_1 \) is very small; vice-versa if the cost of denial is high, then the optimal strategy is to be realist. Moreover, notice that when the probability of receiving \( s_H \) is very small, that is \( p_1 \to 0 \), denial is never optimal.

**Lemma 2.** In any equilibrium where all agents observe \( I_1 \) only, the equilibrium cognitive strategy is to be realist, that is \( \lambda = 1 \), if \( m_1 > \overline{m}(I_1) \).

Suppose that observing \( I_1 \) is not an equilibrium. Then observing both \( I_1 \) and \( I_2 \) must be a profitable deviation. Suppose then the agent deviates and observes both information sources; her cognitive strategy will depend on the signal received. In particular when receiving \( (s_H, \emptyset) \) she will be realist, when receiving \( (\emptyset, \emptyset) \) she may deny \( \emptyset \) and when receiving \( (\emptyset, s_L) \) she may deny both \( \emptyset \) and \( s_L \), only \( s_L \) or only \( \emptyset \).

Focusing on the optimal cognitive strategy on the signal received from \( I_1 \), the utility of the agent being realist is

\[ U_{0,R(\emptyset,\emptyset)} = \delta [ -c + (s + \delta) [ \Delta \theta r(I_1, I_2) + \theta_L ] ] \tag{24} \]

where \( r(I_1, I_2) \equiv \frac{(1 - p_1)\mu}{1 - \mu p_1(1 - \mu) p_2} \). The utility from denial is

\[ U_{0,D(\emptyset,\emptyset)} = \delta \left\{ s \left[ v(\lambda_1^i, \lambda_2^i) \theta_H + q(\lambda_1^i, \lambda_2^i) [ \Delta \theta r(I_1, I_2) + \theta_L ] \right] + p(\lambda_1^i, \lambda_2^i) \theta_L \right\} + \delta [ -c + \delta (\Delta \theta r(I_1, I_2) + \theta_L ) ] - m_1. \tag{25} \]
The agent’s net incentive to be realist on $I_1$ when she observes $(\emptyset, \emptyset)$ is then

$$U_{0,R(\emptyset, \emptyset)} - U_{0,D(\emptyset, \emptyset)} =$$

$$\delta \left\{ s[(1 - q(\lambda^i_1, \lambda^i_2))[\Delta \theta r(I_1, I_2) + \theta_L] - v(\lambda^i_1, \lambda^i_2)\theta_H - p(\lambda^i_1, \lambda^i_2)\theta_L] \right\} + m_1. \quad (26)$$

Denoting $\Gamma(\lambda^i_1, \lambda^i_2, m_1)$ the RHS of equation (26), the optimal cognitive strategy is then

- $\lambda^i_1 = 1$ if $\Gamma(1, \lambda^i_2, m_1) \geq 0$, that is if

$$m_1 \geq \delta s \Delta \theta (1 - r(I_1, I_2)) \equiv m^d_1(\emptyset, \emptyset) \quad (27)$$

- $\lambda^i_1 = 0$ if $\Gamma(0, \lambda^i_2, m_1) \leq 0$, that is if

$$m_1 \leq \delta s \left[ \frac{\mu}{1 - (1 - \mu)p_2\lambda^i_2}\theta_H + \frac{(1 - \mu)(1 - p_2\lambda^i_2)}{1 - (1 - \mu)p_2\lambda^i_2} - \Delta \theta r(I_1, I_2) - \theta_L \right] \equiv \overline{m}^d_1(\emptyset_1, \emptyset_2; \lambda^i_2) \quad (28)$$

Notice that the threshold calculated in equation (27) does not depend on $\lambda^i_2$. On the contrary, the RHS of equation (28) is increasing in $\lambda^i_2$ and $\overline{m}^d_1(\emptyset_1, \emptyset_2; 1) < \overline{m}^d_1(\emptyset_1, \emptyset_2)$. Therefore full realism and full denial occur for disjoint sets of parameters.

**Lemma 3.** In any equilibrium where all agents observe only $I_1$, the optimal cognitive strategy of an agent deviating and observing both $I_1$ and $I_2$, conditional on receiving $(\emptyset, \emptyset)$, prescribes $\lambda^i_1 = 1$ if $m_1 > m^d_1(\emptyset, \emptyset)$

This lemma just says that, if the cost of denying $I_1$ is sufficiently large, the agent will not deny it.

Consider the case in which the agent receives $(\emptyset, s_L)$. If the agent is realist on both signals, she knows that the state is low and she does not exert effort. However, she receives utility

$$U_{0,R(\emptyset, s_L)}^i = \delta(\delta + s)(1 - \alpha)\theta_L \quad (29)$$

because the other $n - 1$ agents are observing $I_1$ and there they always exert effort. By contrasts if the agents denies only $s_L$, she obtains ex-ante intertemporal utility

$$U_{0,D(\emptyset, s_L)}^i|_{\emptyset, \emptyset} = \delta \left\{ s[q(\lambda^i_2)(r(I_1, I_2)\theta_H + (1 - r(I_1, I_2))\theta_L) + (1 - q(\lambda^i_2))\theta_L] + \delta(\delta \theta_L - c) - m_2. \right\} \quad (30)$$

and in case she denies both $\emptyset$ and $s_L$ she obtains utility

$$U_{0,D(\emptyset, s_L)}^i|_{\emptyset H, \emptyset} = \delta \left\{ -c + s[v(\lambda^i_1, \lambda^i_2)\theta_H + q(\lambda^i_1, \lambda^i_2)[r(I_1, I_2)\Delta \theta + \theta_L] + p(\lambda^i_1, \lambda^i_2)\theta_L] + \delta \theta_L \right\} - (m_1 + m_2). \quad (31)$$

Focusing only on the optimal cognitive strategy on the signal received from $I_1$, upon receiving $(\emptyset, s_L)$, using equations (30) and (31) we calculate the net incentive of being
the optimal cognitive strategy is

- \( \lambda^1_i = 1 \) if \( \Gamma(1, \lambda^2_i, s|0, s_L) \geq 0 \) or equivalently if

\[
m_1 \geq \delta s \Delta \theta [1 - r(I_1, I_2)] q(\lambda^2_i) = m^d_1(0, s_L; \lambda^2_i)
\]

Notice that \( q(\lambda^2_i) \) is increasing in \( \lambda^2_i \) therefore the threshold \( m^d_1(0, s_L; \lambda^2_i) \) is decreasing in \( \lambda^2_i \). The following lemma summarises the conditions under which realism on \( I_1 \) is always optimal.

**Lemma 4.** In any equilibrium where all agents observe only \( I_1 \), the optimal cognitive strategy of an agent deviating and observing both \( I_1 \) and \( I_2 \) prescribes \( \lambda^1_i = 1 \) if \( m_1 > \overline{m}(I_1) \).

**Proof.** Using equation (7), if \( \lambda^2_i = 0 \) equation (32) becomes

\[
\overline{m}^d_1(0, s_L; 0) = \delta s \Delta \theta \frac{1 - \mu}{1 - \mu_{P_1}} = \overline{m}(I_1).
\]

If instead \( \lambda^2_i = 1 \) equation (32) becomes

\[
m^d_1(0, s_L; 1) = \delta s \Delta \theta (1 - r(I_1, I_2)) = m^d_1(0, 0).
\]

Moreover \( \overline{m}(I_1) > m^d_1(0, 0) \). Therefore for any \( m_1 > \overline{m}(I_1) \) the deviating agent is always realist on \( I_1 \), that is \( \lambda^1_i = 1 \), regardless his cognitive strategy on the signal \( s_L \).

The logic is the usual one. This lemma shows that when the cost of denying \( I_1 \) is sufficiently large no agent will never deny it.

Now we analyse the optimal cognitive strategy on the signal received from \( I_2 \). Using equations (29) and (30) the agent’s net incentive of denial is equal to

\[
U^d_{0,R}(0, s_L) - U^d_{0,D}(0, s_L) = -m_2 + \delta[-c + (s + \delta) \alpha \theta_L] + \delta s q(\lambda^2_i) r(I_1, I_2) \Delta \theta.
\]

Denoting \( \Gamma(\lambda^2_i, m_2|0, s_L) \) the RHS of equation (33), then the optimal cognitive strategy is

- \( \lambda^2_i = 0 \) if \( \Gamma(0, m_2|0, s_L) \geq 0 \), which is equivalent to

\[
m_2 \leq \delta [s (\Delta \theta r(I_1) + \alpha \theta_L) + \delta \alpha \theta_L - c] \equiv \overline{m}^d_2(0, s_L)
\]

where \( r(I_1) \equiv \frac{\mu_{P_1}}{1 - \mu_{P_1}} \)

- \( \lambda^2_i = 1 \) if \( \Gamma(1, m_2|0, s_L) \leq 0 \), which is equivalent to

\[
m_2 \geq \delta [s (\Delta \theta r(I_1, I_2) + \alpha \theta_L) + \delta \alpha \theta_L - c] \equiv m^d_2(0, s_L)
\]

Notice that \( \overline{m}^d_2(0, s_L) < m^d_2(0, s_L) \), which means that the optimal \( \lambda^2_i = 0 \) and \( \lambda^2_i = 1 \) belong to two different ranges of the cost of denial. The following summarises the denial strategy on \( s_L \) of an agent that deviates and observes both \( I_1 \) and \( I_2 \).
Lemma 5. In any equilibrium where all gents observe only $I_1$, the optimal cognitive strategy of an agent $i$ deviating and observing both $I_1$ and $I_2$ prescribes $\lambda_2 = 0$ if $m_2 \leq \overline{m}_2(\emptyset, s_L)$.

In other words, if the cost of denying $I_2$ is sufficiently small, then the agent will deny it if she deviates.

We are now able to fully characterise the optimal cognitive strategy of an agent that deviates and observes both $I_1$ and $I_2$ when all other agents are observing only $I_1$. From Lemma 2 to Lemma 4 we know that if $m_1 \geq m(I_1)$ then $\lambda_1 = 1$ and from Lemma 5 that if $m_2 \leq \overline{m}_2(\emptyset, s_L)$ then $\lambda_2 = 0$.

Lemma 6. If the following condition holds

$$\theta_L > \frac{1}{\alpha(\delta + s)} \left\{ c - s \Delta \theta \left[ \frac{2(1 - p_1)\mu}{1 - \mu p_1} - 1 \right] \right\} \quad \text{and} \quad \mu > \frac{1}{2 - p_1} \quad (36)$$

then $m(I_1) < \overline{m}_2(\emptyset, s_L)$ and, for any $m_1$ and $m_2$ in this range, (i) if all agents observe $I_1$ then they are realist and (ii) if an agent deviates and observes both $I_1$ and $I_2$, she never denies $I_1$ and would always deny $I_2$.

Notice that the condition $\mu > \frac{1}{2 - p_1}$ is required to guarantee that the parameter space of $\theta_L$ under which $m(I_1) < \overline{m}_2(\emptyset, s_L)$ does exist. By Assumption 1, $\theta_L$ is assumed to be smaller than $\frac{c}{\alpha(s+\delta)}$. Therefore in order to have a non empty for $\theta_L$ such that the above optimal cognitive strategies are feasible, the prior probability of the high state must be sufficiently high.

The last thing we need to check is whether the deviation is profitable. By combining the results from Lemmas 2 to 5, for any $m_1, m_2 \in [m(I_1), \overline{m}_2(\emptyset, s_L)]$ the ex-ante inter-temporal utility of an agent $i$ from observing $I_1$ when everybody else is observing $I_1$ is

$$U^i_{0|I_1} = \delta[-c + (s + \delta)(\mu \Delta \theta + \theta_L)], \quad (37)$$

and the ex-ante inter-temporal utility of an agent $i$ that deviates to $I_1$ and $I_2$ is

$$U^{i,\text{dev}}_{0|I_1,I_2} = \delta[-c + (s + \delta)(\mu \Delta \theta + \theta_L)] - (1 - \mu)p_2 m_2. \quad (38)$$

A comparison between (37) and (38) demonstrates that an agent $i$ does not find profitable to deviate from $I_1$. Therefore, observing only $I_1$ is an equilibrium. $\square$

Proof of Proposition 2. We prove this proposition in multiple steps. We first assume that in equilibrium all agents observe both $I_1$ and $I_2$ and are always realist and we calculate the ex-ante inter-temporal utility of an agent $i$. In the second step we assume that, for the same parameters space for which an agent $i$ observes both information sources and is realist, she deviates to observe only $I_1$ and her optimal cognitive strategy is to be realist. We then calculate the agent $i$’s ex-ante inter-temporal utility of deviation and we compare it to the inter-temporal utility of the guessed equilibrium.
strategy. We then show that the deviation is not profitable. In the last step we characterise the parameters space under which an agent $i$ (i) is realist on both $I_1$ and $I_2$ when observing both information sources and (ii) is realist on $I_1$ when deviating and observing only $I_1$.

When an agent $i$ observes both information sources and is realist, she will receive the signal profile $(\emptyset, s_L)$ with probability $(1 - \mu)p_2$ and from Lemma 1 she will not exert effort. Thus, the ex-ante inter-temporal utility of an agent $i$ that observes both $I_1$ and $I_2$ and is realist, if all other agents observe both $I_1$ and $I_2$ and are realist, is

$$U^i_{0|I_1, I_2} = Pr(s_H, \emptyset)[\delta(-c + (\delta + s)E(\theta|s_H, \emptyset)] + Pr(\emptyset, \emptyset)[\delta(-c + (\delta + s)E(\theta|\emptyset, \emptyset)]$$

$$= \delta\{[(s + \delta)(\mu\theta_H + (1 - \mu)(1 - p_2)\theta_L)] - [1 - (1 - \mu)p_2]c\}. \quad (39)$$

Suppose now that an agent $i$ deviates to observe only $I_1$ and in the continuation game she is realist. In this case, from Lemma 1, we know that the agent that deviates will always exert effort both when she receives $s_H$ and when she receives $\emptyset$. However, because the other $n - 1$ agents are observing both $I_1$ and $I_2$ and are always realist, the deviating agent is aware of the fact that, when she receives $\emptyset$ the other $n - 1$ agents might receive $(\emptyset, \emptyset)$ or $(\emptyset, s_L)$. In the first case both the deviating agents and the $n - 1$ agents take the same action $e = 1$. In the latter case, the deviating agent exert efforts while the $n - 1$ agents do not exert effort because they know the state is $L$ with probability 1. Therefore, the deviating agent, when calculating her ex-ante intertemporal expected utility of deviation will take this fact into account, obtaining

$$U^i_{0|I_1} = (1 - \mu p_1)\{\delta[-c + (s + \delta)(r(I_1)\theta_H + (1 - r(I_1))(1 - Pr(s_L|\emptyset)(1 - \alpha))\theta_L)]\} +$$

$$+ \mu p_1[\delta(-c + (s + \delta)\theta_H)]$$

$$= \delta\left\{(s + \delta)\left[\mu\theta_H + (1 - \mu)\left(1 - \frac{p_2}{2 - p_1}(1 - \alpha)\right)\theta_L\right] - c\right\}, \quad (40)$$

where $\frac{p_2}{2 - p_1} = Pr(s_L|\emptyset)$ is the probability that the other $n - 1$ agents receive $s_L$ from information sources $I_2$ when the deviating agent receives $\emptyset$ form $I_1$.

**Lemma 7.** If all agents are observing both information sources and are realist, then it is never profitable to deviate to observe only $I_1$ and be realist if $\theta_L < 0$.

**Proof.** By subtracting equation (40) from (39), the difference reduces to

$$U^i_{0|I_1, I_2} - U^i_{0|I_1} \equiv (s + \delta) \left(\frac{1 - \alpha}{2 - p_1} - 1\right)\theta_L + (1 - \mu)c.$$

We can see that, whether this difference is positive or negative depends on the sign of $\theta_L$. In the case of $\theta_L > 0$ we cannot say that the deviation is profitable, unless under some further parameter restrictions. On the contrary, if $\theta_L < 0$ this difference is always positive, therefore the deviation is not profitable. \hfill \Box
We now identify the sufficient conditions for an agent $i$ to be realist on $I_1$ and $I_2$ when observing both information sources. Remember that when observing both $I_1$ and $I_2$, an agent $i$ can receive $(s_H, \emptyset)$, $(\emptyset, \emptyset)$ or $(\emptyset, s_L)$. When the agent receives $(s_H, \emptyset)$ she will be always realist on both signals. In the other two cases she can deny either both signals or only the signal from $I_2$. We first consider the case in which agents receive $(\emptyset, \emptyset)$. The utility of an agent $i$ being realist is then

$$U_{0,R(\emptyset,\emptyset)}^i = \delta \{ -c + (s + \delta)[r(I_1, I_2)\theta_H + (1 - r(I_1, I_2))\theta_L] \}.$$  \hfill (41)

If the agent $i$ denies the signal $\emptyset$, she obtains utility

$$U_{0,D(\emptyset,\emptyset)}^i = \delta \{ -c + s[v(\lambda_1, \lambda_2)\theta_H + q(\lambda_1, \lambda_2)[\Delta \theta r(I_1, I_2)\theta_H + \theta_L] + +p(\lambda_1, \lambda_2)\alpha \theta_L] + \delta(r(I_1, I_2)\Delta \theta + \theta_L)] \} - m_1.$$ \hfill (42)

Denoting $\Gamma(\lambda_1, \lambda_2, m_1) \equiv U_{0,D(\emptyset,\emptyset)}^i - U_{0,R(\emptyset,\emptyset)}^i$ the agent $i$'s net incentive of denying $\emptyset$ from $I_1$ when receiving $(\emptyset, \emptyset)$, then her optimal cognitive strategy is

- $\lambda_1 = 0$ if $\Gamma(0, \lambda_2, m_1) \geq 0$ which is equivalent to

$$m_1 \leq \begin{cases} 
\delta s[\Delta \theta(r(I_2) - r(I_1, I_2))] = \overline{m}_1(\emptyset, \emptyset; 1) & \text{if } \lambda_2 = 1 \\
\delta s(\Delta \theta[\mu - r(I_1, I_2)] - (1 - \alpha)(1 - \mu)p_2 \theta_L) = \overline{m}_1(\emptyset, \emptyset; 0) & \text{if } \lambda_2 = 0 \\
\end{cases}$$ \hfill (43)

- $\lambda_1^i = 1$ if $\Gamma(1, \lambda_2, m_1) \leq 0$ which is equivalent to

$$m_1 \geq \delta s[\Delta \theta(1 - r(I_1, I_2))] = \overline{m}_1(\emptyset, \emptyset).$$  \hfill (44)

Notice that $m_1(\emptyset, \emptyset) \geq \max\{\overline{m}_1(\emptyset, \emptyset; 1), \overline{m}_1(\emptyset, \emptyset; 0)\}$.

Consider now the case in which agents receive $(\emptyset, s_L)$. In this case the state of the world is low with probability one and if agent $i$ remains realist on both signals, then she obtains ex-ante intertemporal utility $U_{0,R(\emptyset,s_L)}^i = 0$. On the other hand, if the agent $i$ is realist on $\emptyset$ and delusional on $s_L$, she obtains utility

$$U_{0,D(\emptyset,s_L)}^i|_{\emptyset,\emptyset} = \delta \{ s[q(\lambda_2)(r(I_1, I_2)\theta_H + (1 - r(I_1, I_2))\theta_L) + +(1 - q(\lambda_2))\alpha \theta_L] - c + \delta \alpha \theta_L \} - m_2.$$ \hfill (45)

Finally, if the agent denies both signals, she obtains utility

$$U_{0,D(\emptyset,s_L)}^i|_{s_H,\emptyset} = \delta \{ s[r(\lambda_1, \lambda_2)\theta_H + q(\lambda_1, \lambda_2)[\Delta \theta r(I_1, I_2) + \theta_L] + +p(\lambda_1, \lambda_2)\alpha \theta_L] - c + \delta \alpha \theta_L \} - (m_1 + m_2) - m_2.$$ \hfill (46)

Denoting $\Gamma(\lambda_1, \lambda_2, m_1|\emptyset, s_L) \equiv U_{0,D(\emptyset,s_L)}^i|_{s_H,\emptyset} - U_{0,D(\emptyset,s_L)}^i|_{\emptyset,\emptyset}$ the agent $i$'s net incentive of denying the signal from $I_1$ when receiving $(\emptyset, s_L)$, then her optimal cognitive strategy $\lambda_1^i$ is
\[- \lambda_1 = 1 \text{ if } \Gamma(1, \lambda_2, m_1 | \emptyset, s_L) \leq 0 \text{ which is equivalent to } m_1 \geq \begin{cases} \delta s[\Delta \theta - (1 - r(I_1, I_2))] \equiv m_1(\emptyset, s_L; 1) & \text{if } \lambda_2 = 1 \\ \delta s(1 - r(I_1))[\Delta \theta + (1 - \alpha)p_j] \equiv m_1(\emptyset, s_L; 0) & \text{if } \lambda_2 = 0 \end{cases} \quad (46)\]

\[- \lambda_1 = 0 \text{ if } \Gamma(0, \lambda_2, m_1 | \emptyset, s_L) \geq 0 \text{ which is equivalent to } m_1 \leq \begin{cases} \delta s[(1 - r(I_1))(1 - \alpha)p_j] \equiv m_1(\emptyset, s_L; 0) & \text{if } \lambda_2 = 0 \\ \delta s[(r(I_2) - r(I_1, I_2))\Delta \theta] \equiv m_1(\emptyset, s_L; 1) & \text{if } \lambda_2 = 1 \end{cases} \quad (47)\]

**Lemma 8.** Define \( m_1(I_1, I_2) \equiv \max \{m_1(\emptyset, s_L; 0), m_1(\emptyset, s_L; 1)\} \). Whenever agents observe both \( I_1 \) and \( I_2 \) in the first stage, if \( m_1 \geq m_1(I_1, I_2) \), there exists an equilibrium of the continuation game where all agents are realist on \( I_1 \), that is \( \lambda_1 = 1 \).

**Proof.** It follows from
\[ m_1(I_1, I_2) \geq \max\{m_1(\emptyset, s_L; 0), m_1(\emptyset, s_L; 1)\} \geq m_1(\emptyset, s_L; 1) = m_1(\emptyset, \emptyset) \geq \max\{m_1(\emptyset, \emptyset; 0), m_1(\emptyset, \emptyset; 1)\}. \]

Thus, for a cost of denial sufficiently high, all agents are realist on \( I_1 \) when observing both information sources.

We now analyse the conditions under which realism of the signal received from \( I_2 \) is the optimal cognitive strategy of an agent \( i \) when everybody else is realist on both \( I_1 \) and \( I_2 \). Denoting \( \Gamma(\lambda_1, m_2 | \emptyset, s_L) \equiv U_{\emptyset, D}(\emptyset, s_L) |_{\emptyset, \emptyset} - U_{\emptyset, R}(\emptyset, s_L) \) the net incentive of an agent \( i \) of denying the signal from \( I_2 \) when receiving \((\emptyset, s_L)\), then the optimal cognitive strategy \( \lambda_2^i \) for an agent \( i \) is

\[- \lambda_2^i = 1 \text{ if } \Gamma(1, \lambda_2, m_2 | \emptyset, s_L) \leq 0 \text{ which is equivalent to } m_2 \geq \delta s[\Delta \theta (I_1, I_2) + \theta_L] + \delta \alpha \theta_L - c \equiv m_2(I_1, I_2) \quad (48)\]

\[- \lambda_2^i = 0 \text{ if } \Gamma(0, \lambda_2, m_2 | \emptyset, s_L) \geq 0 \text{ which is equivalent to } m_2 \leq \delta s[(r(I_1) \Delta \theta + (1 - (1 - r(I_1))(1 - \alpha)p_j)\theta_L] + \delta \alpha \theta_L - c \equiv m_2(I_1, I_2) \quad (49)\]

**Lemma 9.** Whenever agents observe both \( I_1 \) and \( I_2 \) in the first stage, if \( m_2 \geq m_2(I_1, I_2) \), there exists an equilibrium of the continuation game where all agents are realist on \( I_2 \), that is \( \lambda_2 = 1 \).

**Proof.** This follows from equation (48).

Combining lemmas 8 and 9 we are now able to characterise the social equilibrium cognitive strategies \( \lambda_1 \) and \( \lambda_2 \) when agents observe both information sources.
Lemma 10. Whenever agents observe both $I_1$ and $I_2$ in the first stage, for any $m_1 \geq m_1(I_1, I_2)$ and $m_2 \geq m_2(I_1, I_2)$ there exists an equilibrium of the continuation game
where all agents are realist on both $I_1$ and $I_2$, that is $\lambda_1 = \lambda_2 = 1$.

We now identify the parameter space under which an agent that deviates and observes both
information sources and (ii) if she deviates and observes only $I_1$, she will be always
realist if she receives $s_H$. On the contrary if she receives $\emptyset$, being realist she will obtain utility:

\[ U_{t,R}^d = \delta \left\{ -c + (s + \delta) [r(I_1)\theta_H + (1 - r(I_1)) [1 - Pr(s_L|\emptyset)(1 - \alpha)]\theta_L] \right\} \]

Whenever the agent observes both information sources:

\[ U_{t,D}^d = \delta \left\{ -c + (s + \delta) [r(I_1)\theta_H + (1 - r(I_1)) [1 - Pr(s_L|\emptyset)(1 - \alpha)]\theta_L] \right\} + \delta^2 \left\{ r(I_1)\theta_H + (1 - r(I_1)) [1 - Pr(s_L|\emptyset)(1 - \alpha)]\theta_L \right\} \]

Whereas if she denies the signal she obtains utility:

\[ U_{t,D}^d = \delta \left\{ -c + s \left[ r(\lambda^i_1)\theta_H + (1 - r(\lambda^i_1)) [r(I_1)\theta_H + (1 - r(I_1)) [1 - Pr(s_L|\emptyset)(1 - \alpha)]\theta_L] \right] \right\} + \delta^2 \left\{ r(I_1)\theta_H + (1 - r(I_1)) [1 - Pr(s_L|\emptyset)(1 - \alpha)]\theta_L \right\} \]

Denoting $\Gamma(\lambda^i_1, m_1|I_1^d) \equiv U_{t,D}^d - U_{t,R}^d$ the agent $i$’s net incentive of denying the signal
$\emptyset$ when deviating to $I_1$, then her optimal cognitive strategy of deviation is

- $\lambda^i_1 = 1$ if $\Gamma(1, m_1|I_1^d) \leq 0$, which is equivalent to

\[ m_1 \geq \delta s(1 - r(I_1)) \left( \Delta \theta + (1 - \alpha) \frac{p_2}{2 - p_1} \theta_L \right) \equiv \bar{m}_{1}^d(I_1), \quad (52) \]

- $\lambda^i_1 = 0$ if $\Gamma(1, m_1|I_1^d) \geq 0$, which is equivalent to

\[ m_1 \leq \delta s(1 - r(I_1)) \mu p_1 \left( \Delta \theta + (1 - \alpha) \frac{p_2}{2 - p_1} \theta_L \right) \equiv \underline{m}_{1}^d(I_1). \quad (53) \]

Notice that $\bar{m}_{1}^d(I_1) < \bar{m}_{1}^d(I_1)$ for any value of $\theta_L$ and that $\bar{m}_{1}^d(I_1) < \bar{m}_{1}^d(\emptyset, s_L; 0)$ if $\theta_L > 0$ and the inequality is reversed if $\theta_L < 0$.

Lemma 11. Define $\hat{m} \equiv \max \{\bar{m}_{1}^d(I_1), \bar{m}_{1}(I_1, I_2), \bar{m}_{2}(I_1, I_2)\}$. In any equilibrium where all agents are observing both information sources and are always realist, then for any $m_1, m_2 \geq \hat{m}$ (i) if an agent $i$ observes both $I_1$ and $I_2$ she will be always realist on both
information sources and (ii) if she deviates and observes only $I_1$ she will be always
realist.

Summarising the results from Lemmas 7 to 11 we get Proposition 2. \qed
Proof of Proposition 3. From lemma 6 and Proposition 1 we know that \( m \equiv m(I_1) = \delta s \Delta \theta (1 - r(I_1)) \). Under condition (10), for any \( \theta_L > \hat{\theta}_L \) the threshold value \( \hat{m} \) defined in lemma 11 reduces to \( \hat{m} \equiv \max\{m^d_1(I_1), m_1(I_1, I_2)\} \), where \( m^d_1(I_1) = \delta s (1 - r(I_1)) \left( \Delta \theta + (1 - \alpha) \frac{p_2}{2 - p_1} \theta_L \right) \) and \( m_1(I_1, I_2) = \delta s \Delta \theta (1 - r(I_1, I_2)) \). It can be verified that \( m > \max\{m_1(I_1, I_2), m^d_1(I_1)\} \), which implies that \( \hat{m} < m \). Therefore for any value of \( m_1 \) and \( m_2 \) in the range \([m, \overline{m}]\) both equilibria exist, with the condition \( \mu > \frac{1}{2 - p_2} \) to guarantee that \([m, \overline{m}]\) is a non empty interval.

Proof of Proposition 4. In order to prove part (i), first of all notice that (10) implies \( \theta_L < 0 \). A sufficient condition for welfare being higher in the low state when both information sources are observed is that (13) holds. Thus, when (10) holds, the LHS of (13) is positive and the first part of the RHS of (13) is smaller than the RHS of condition (10). In addition the second term of the RHS of (13) is positive by Assumption 1. Therefore (10) implies (13).

The proof of part (ii) follows directly from the comparison of (14) with (15). It is straightforward to see that welfare is always higher in the equilibrium where agents observe both information sources given that \( r(I_1, I_2) > r(I_1) \).

\( \square \)
References


