Tax policy, bubbles and unemployment

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Abstract

Following the recent bubble asset crash, a great recession of economic activity took place and waves of job destruction were increasing after a period of a growing bubble accompanied by increasing levels of employment and GDP. In this paper, a model with tax policy is presented to demonstrate that asset bubbles decrease the unemployment level and increase the economic activity. We consider an OLG model with endogenous labor and transfers financed by tax burden on capital and labor incomes. Our results indicate that the bubble promotes capital stock and reduces unemployment if the income redistribution is in favor of young households, the tax rate on labor income is low, and the capital tax rate is high. Indeed, in the presence of bubble, a high level of tax burden on capital income modifies significantly the cost of using capital, which incites firms to substitute capital to labor. The income distribution effect and/or the low labor income tax allow the economy to sustain a higher capital stock.

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1 Introduction

The economic world has experienced multiple financial crises, such as Japan’s crisis of the early 1990s or subprime crisis of 2007-2008 in the USA. They were the results of the bursting of asset bubbles. In particular, the last crisis was followed by a period of lower growth and higher unemployment. In contrast, periods of growing bubbles are associated with higher employment and capital.

First works on speculative bubbles and economic activity, like Tirole (1985) and Weil (1987), showed that deterministic and stochastic bubbles have a recessionary effect on GDP, by absorbing a share of over-saving which leads to lower capital. This is the so-called crowding-out effect of bubble, but is hard to reconcile with the observations, which show that speculative bubbles promote economic expansions. Thus, recent works exhibit different mechanisms explaining that bubbly episodes are characterized by the boom of productive capital and its bursting causes depression, meaning that there is a crowding-in effect of the bubble. For instance, Martin and Ventura (2012) or Hirano and Yanagawa (2017) have relied on the existence of heterogeneous investment projects, Fahri and Tirole (2012) focus on the liquidity role of bubbles, Kocherlakota (2009) assumes that the bubble is a collateral in the credit constraint, Raurich and Seegmuller (2019) make a distinction between liquid bubbles and illiquid capital.

Most of the literature on rational bubbles has mainly been interested on the impact of asset bubbles on capital accumulation, but what about the relationship between speculative bubbles and labor or unemployment? During the period of the asset bubbles’ crash, in particular the bursting of the Japanese asset price bubble and the bubble in US, the observations have shown that these periods were marked by a dramatic increase of the unemployment rate, where waves of job destruction were growing. According to the World Bank, the unemployment rate in Japan increased from 2.1% in 1991 to 4.8% in 2000, and increased in the US from 4.7% in 2007 to 9.7% in 2010. This leads us to say that fluctuations of employment are caused by the appearance and the burst of bubbles. Despite these observations, the relationship between rational bubbles and employment has not been extensively investigated in macroeconomics. In this paper, we fill this gap, finding a mechanism that will allow us to show the positive effect of speculative bubbles on employment and economic activity.

Our paper contributes to the few literature that investigated this issue provided some mechanism explaining that asset bubbles diminish the unemployment rate. Among others, Shi and Suen (2014) have shown that considering labor as endogenous, asset bubbles can promote capital, because a higher interest rate at the bubbly steady state may be in accordance with higher capital when labor is higher too. The main mechanism goes through the saving rate. It should strongly increase with respect to the interest rate, which requires a high elasticity of intertemporal substitution in consumption of the CES utility these authors consider. Miao, Wang and Lifang (2015) have incorporated endogenous credit constraints. Under optimistic beliefs, the firms can borrow more because their value used as a collateral is higher. In such a situation, they are hiring more. Hashimoto, Im and Kunieda (2017) rely on the existence of heterogeneous
investment projects. The mechanism is close to the one introduced in Martin and Ventura (2012), but is extended to a model with unemployment. The bubble has a liquidity role. Investors having the projects with the higher return sell the bubble to those which have projects with lower returns to increase their investment, capital and employment. Finally, Kocherlakota (2011) has combined overlapping generations to a matching model on the labor market. Following a crash of the bubble, the increase of unemployment is due to the zero lower bound of the interest rate imposed by the monetary authority, which restricts the liquidities in periods of recession.

Our approach is different than these previous ones because it is based on the effects of tax policy. We argue that it is relevant because the fiscal policy affects the labor market and the disposable incomes used to buy assets. In addition, to our knowledge, no one has investigated the relationship between asset bubbles and unemployment focusing on the role of taxes. Considering a two-period overlapping generations model, capital and labor income taxation finance public spending and transfers. Capital income tax affects the level of the interest rate, while the labor income tax and/or the transfers modifies the revenue at young age. Both the interest rate and the income when young are important for the existence of bubbles. The first determines the growth rate of the bubble, while the second is an essential determinant of the savings, which is used in part to buy the asset bubbles.

At the first period of life, each agent works, consumes and saves, and at the second period, he only consumes. There is portfolio choice between the investment in productive capita and in a purely speculative asset. Since individual labor supply is indivisible, we determine a reservation wage above which each household decides to enter into the labor market and offers one unit of labor. Therefore, the supply of labor is endogenous and perfectly elastic. We consider also that the tax burden on capital and labor income finance the government spending and transfer to the households. Two cases are investigated. Either the redistribution system is nil and only the tax burden on labor income affects the income level of each households, or the government spending is nil and transfers are positive meaning that the revenue of young households is affected by the labor income tax and the transfer. Finally the technology benefits from a capital externality, which is arbitrarily small. Such a technology is introduced to be able to define a steady state in the presence of indivisible labor (Coimbra, Lloyd-Braga and Modesto (2005)).

A high tax on capital income increases the interest rate. Facing such a raise of capital use, firms substitute capital by labor. Moreover, the supply of capital stock increases, due to the low level of labor income tax and/or the high level of transfers to the young households which buy the assets. At equilibrium, on the one hand, the unemployment decreases due to the substitution effect, and on the other hand, the aggregate capital stock increases too (the increase of capital supply dominates the decrease of capital demand). We prove that this mechanism is more powerful in the bubbly than in the bubbleless economy. This explains that a high capital income tax, a low labor income tax and a high transfer to young households promote the crowding-in effect of the bubble.
Our paper is organized as follows. Next section presents the model, which consists of three agents: households, firms and government. Sections 3 and 4 determine the conditions under which the bubble can exist and exhibits a new mechanism explaining why asset bubbles are productive. Section 5 gives a numerical illustration of our results. Section 6 provides concluding remarks. Some technical details are relegated to an Appendix.

2 Model

The economy is populated by three types of agents: Households, firms and a government. All markets are perfectly competitive.

2.1 Households

We consider overlapping generations living two periods. At each period of time, \( t = 0, 1, 2, 3, \ldots + \infty \), a new generation of identical consumers \( N \) is born. Population size is constant. When young, a household earns a wage income \( w_t \) taxed at a constant rate \( \tau_L \) and can benefit from a lump-sum transfer \( T_{1t} \). This income is shared between consumption and saving, through two types of assets. Indeed, each saver makes a portfolio choice between the investment in productive capital \( s_t \) and the holding of \( m_t \) units of an intrinsically useless paper asset, for instance “money”, which has a positive value \( P_t \geq 0 \). We define a bubble as the difference between the market price and the market fundamental. Since this asset is intrinsically useless, i.e. its market fundamental is zero, there is a bubble when it is positively valued, i.e. \( P_t > 0 \). When old, the household consumes all her remunerated savings, which is the sum of the after-tax return of productive investment \( (R_{t+1}(1 - \tau_K)s_t)^1 \) and the return from speculative asset \( (P_{t+1}m_t) \), where \( R_{t+1}, \tau_K \) and \( P_{t+1} \) denote the gross return from productive capital, the tax rate on capital income and the price of the intrinsically worthless asset at time \( t + 1 \), respectively. She may also receive a lump-sum transfer \( T_{2t+1} \).

The preferences of the representative household born at period \( t \) are represented by a Cobb Douglas utility function and a linear desutility of labor:

\[
U = c_{1t}^{\alpha}c_{2t+1}^{1-\alpha} - Vd_t, \quad \alpha \in (0, 1)
\]

where \( c_{1t} \) and \( c_{2t+1} \) are the consumption levels when young and old respectively, \( V > 0 \) denotes the disutility of work, and \( d_t = \{0, 1\} \) is the individual labor supply. Each individual chooses to supply one unit of labor or does not work, depending on the comparison between the household’s reservation wage and the wage proposed by the firm. If this last one is higher or equal to the reservation wage, the individual decides to participate to the labor market and offers one unit of work.

\(^1\)As often in overlapping generations economy where the period is long, we assume complete depreciation of capital after one period of use.
A household faces two budget constraints:

\[ c_{1t} + s_t + P_t m_t = (1 - \tau_L)w_t d_t + T_{1t} \]
\[ c_{2t+1} = (1 - \tau_K)R_{t+1} s_t + P_{t+1} m_t + T_{2t+1} \]

The household program is solved in two steps. First, assuming that she supplies labor, i.e. \( d_t = 1 \), we find the consumptions and savings:

\[ c_{1t} = \alpha \left[ (1 - \tau_L)w_t + T_{1t} + \frac{T_{2t+1}}{R_{t+1}(1 - \tau_K)} \right] \]
\[ c_{2t+1} = (1 - \alpha)(1 - \tau_K)R_{t+1} \left[ (1 - \tau_L)w_t + T_{1t} + \frac{T_{2t+1}}{R_{t+1}(1 - \tau_K)} \right] \]
\[ s_t + P_t m_t = (1 - \alpha)[(1 - \tau_L)w_t + T_{1t}] - \frac{\alpha T_{2t+1}}{R_{t+1}(1 - \tau_K)} \]

and the no arbitrage condition between capital and bubble:

\[ R_{t+1}(1 - \tau_K) = \frac{P_{t+1}}{P_t} \]

When there is a bubble, the after-tax returns of productive capital is equal to the returns of holding the speculative bubble. In other words, the bubble must grow at the after-tax interest rate for all \( t \geq 0 \).

Second, we determine if a household supplies labor comparing the utility when \( d_t = 1 \) and \( d_t = 0 \). We deduce the reservation wage \( w_t \), which is the take-home pay required to make a worker indifferent between working and remaining unemployed.

\[ w_t = \frac{V}{\alpha^\alpha((1 - \tau_L)\frac{R_{t+1}}{R_{t+1}(1 - \tau_K)})^{1 - \alpha}(1 - \tau_L)} \]

This equation shows that, we have an endogenous reservation wage, which depends positively on the disutility of work \( V \) and capital taxation \( \tau_K \), and negatively on the future interest rate \( R_{t+1} \). A household will choose to work if \( w_t \geq w_t \).

2.2 Firms

A firm produces the unique final good using capital \( K_t \) and labor \( L_t \). The firm also benefits from an arbitrarily small externality from the average capital stock \( K_1 \). For simplicity, the production function is Cobb-Douglas:

\[ Y_t = A K_t^\alpha K_1^\beta L_t^{1-\beta} \]

where \( A > 0 \) denotes scaling factor and \( \beta \in (0,1) \). The production function exhibits constant returns to scale at the individual level, but there are an

\[ ^2 \text{See Appendix A for details.} \]
increasing returns to scale at the social level, due to a capital externality $\nu > 0$. Note that $\nu$ is assumed to be arbitrarily small and close to zero, meaning that returns to scale are close to be constant. Taking the externality as given, the maximization of profits $Y_t - w_tL_t - R_tK_t$ yields:

$$w_t = (1 - \beta)A K_t^{\nu} K_t^\beta L_t^{-\beta} \quad \text{and} \quad R_t = \beta A K_t^{\nu} K_t^{\beta-1} L_t^{1-\beta}$$

(10)

### 2.3 Government

The government revenue is financed by tax burden on capital and labor income. It is used to finance government spending $G_t$, and the transfers $T_{1t}$ and $T_{2t}$ intended to each young and old household, respectively. The budget is balanced at each period, so that:

$$\tau_K R_t K_t + \tau_L w_t L_t = G_t + NT_{1t} + NT_{2t}$$

(11)

From the government budget constraint, we can distinguish two cases: The government spends all its revenue in public spending ($T_{1t} = 0, T_{2t} = 0$) or uses it for the transfers only ($G_t = 0$). We distinguish these two cases because it allows to clearly see the implications of the transfers on the existence and the features of a bubble. In each case, we investigate the effects of a speculative bubble on unemployment and production taking into account the tax policy.

### 3 Economy without tax transfer

In this section, capital and labor taxation only finance the public spending, i.e. $T_{1t} = T_{2t} = 0$ for all $t \geq 0$.

Aggregate supply of intrinsically worthless asset is normalized to 1 while each young household buy $m_t$ units of asset bubbles, at price $P_t$. At equilibrium, we get:

$$L_t m_t = 1$$

(12)

Equilibrium in capital market requires that aggregate savings of young households is to use to buy the asset bubble and the future capital stock. Since there is a continuum of identical firms, $K_t = \overline{K}_t$ at equilibrium. We deduce that:

$$K_{t+1} + P_t = (1 - \alpha)(1 - \tau_L)(1 - \beta) A K_t^{\nu} k_t^\beta$$

(13)

where $k_t = K_t/L_t$ denotes capital per worker.

We focus on equilibrium with unemployment, i.e. $w_t = w_1$ and $L_t < N$. From (8) and (10), we get:

$$[\beta A K_{t+1}^{\nu} k_{t+1}^{\beta-1}]^{1-\alpha} = \frac{V}{\alpha(1-\alpha)(1-\tau_K)(1-\beta)(1-\beta) A K_t^{\nu} k_t^\beta}$$

(14)

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3Public spending neither affect the households' preferences, nor the technology.
Definition 1 Given the initial capital stock $K_0 > 0$, an intertemporal equilibrium is a sequence $(K_t, k_t, P_t) \in \mathbb{R}_+^3$ satisfying (7), (10), (13) and (14).

A stationary equilibrium is an intertemporal equilibrium $K_t = K$, $k_t = k$ and $P_t = P$ for all $t$, satisfying:

$$V = \alpha^\alpha (1 - \alpha)^{1-\alpha} (1 - \beta)(1 - \tau_L)(AK^\nu)^{2-\alpha} k^{\beta-(1-\alpha)(1+\frac{\beta}{A})}$$

$$1 = (1 - \alpha)(1 - \tau_L)(1 - \beta)AK^\nu k^{\beta-1} - \frac{P}{K}$$

$$P = (1 - \tau_K) \beta AK^\nu k^{\beta-1} P$$

There are two types of stationary equilibria, the bubbleless one without bubbles, i.e. $P = 0$, and the bubbly one with bubbles, i.e. $P > 0$.

We study the existence of the steady states in the economy without transfer. We are especially concerned with the condition for the existence of a bubbly steady state.

When there is no bubble in the economy (i.e. $P = 0$), we use equations (15)-(17) to deduce that there is a unique bubbleless steady state ($\tilde{K}, \tilde{k}$), given by:

$$\tilde{k} = \frac{V(1 - \alpha)}{\alpha^\alpha} \left( \frac{1 - \beta}{\beta(1 - \tau_K)} \right)^{1-\alpha}$$

$$\tilde{K} = \left[ \frac{1}{A} \right]^{\frac{1}{A}} \left[ \frac{V}{\alpha^\alpha(\beta(1 - \tau_K))^{1-\alpha}} \right]^{\frac{1}{\beta - 1}} \left[ \frac{1}{1 - \alpha} \right]^{\frac{1}{\beta}} \left[ \frac{1}{1 - \beta} \right]^{\frac{1}{(1 - \beta)(1 - \tau_L)}}$$

The values of $\tilde{k}$ and $\tilde{K}$ allows us to determine labor:

$$\tilde{L} = \left[ \frac{1}{A} \right]^{\frac{1}{A}} \left[ \frac{V}{\alpha^\alpha(\beta(1 - \tau_K))^{1-\alpha}} \right]^{\frac{1}{\beta}} \left[ \frac{1}{1 - \alpha} \right]^{\frac{1}{\beta} + 1} \left[ \frac{1}{1 - \beta} \right]^{\frac{1}{(1 - \beta)(1 - \tau_L)}}$$

Let $\tilde{R}$ denotes the interest rate at the bubbleless steady state. We show that:

Proposition 1 A unique bubbly steady state exists if and only if $(1 - \tau_K) \tilde{R} < 1$, which is equivalent to $\frac{1 - \tau_K}{1 - \tau_L} < 1 - \alpha$.

Proof. See Appendix B. □

According to Proposition 1, $(1 - \tau_K) \tilde{R} < 1$, is a necessary condition for the existence of bubbly steady state. It involves the ratio $\frac{1 - \tau_K}{1 - \tau_L}$, meaning that the respective levels of the labor and capital tax rates play a role. When $\frac{1 - \tau_K}{1 - \tau_L} = 1$, the two taxes are identical. The existence of the bubble does not depend on the level of the two taxes. Our investigation highlights that it is relevant to focus on differentiated taxes.

The bubble is more likely to appear, when $\tau_K$ is high and/or $\tau_L$ is low. Indeed, when the tax burden on capital income increases, the return from
productive investment becomes less attractive, which promotes the appearance of bubble. The after-tax income is enhanced with a low level of labor income tax, which promotes savings and, therefore, the existence of a bubble.

Note that the condition to have a bubble can be rewritten as \( \beta (1 - \tau_K) < (1 - \alpha)(1 - \tau_L)(1 - \beta) \), or equivalently \( \beta (1 - \tau_K) Y_t < (1 - \alpha)(1 - \tau_L)(1 - \beta) Y_t \), which means that there exists a bubble if the returns of one unit per GDP invested is lower than its cost per unit of GDP, which is equal to savings per GDP. Finally, this is a usual condition to have a bubble, except that, in our model, it involves the fiscal instruments.

Using (15)-(17), the bubbly steady state is given by:

\[
k^* = \frac{V \beta (1 - \tau_K)}{\alpha^\alpha(1 - \alpha)\beta(1 - \tau_L)}
\]

\[
K^* = \left[ \frac{1}{A} \right]^{\frac{1}{\nu}} \left[ \frac{V}{\alpha^\alpha(1 - \alpha)\beta(1 - \tau_L)} \right]^{\frac{1}{\nu}} \left[ \frac{1}{\beta(1 - \tau_K)} \right]^{\frac{\beta}{\nu} + 1}
\]

\[
P^* = \left[ \frac{V}{\alpha^\alpha(1 - \alpha)\beta(1 - \tau_L)} - \beta(1 - \tau_K) \right]^{-\frac{\beta}{\nu} - 1} \left[ \frac{1}{\beta(1 - \tau_K)} \right]^{\frac{\beta}{\nu} + 1}
\]

Since \( L = K/k \), we also get:

\[
L^* = \left[ \frac{1}{A} \right]^{\frac{1}{\nu}} \left[ \frac{V}{\alpha^\alpha(1 - \alpha)\beta(1 - \tau_L)} \right]^{-\frac{\beta}{\nu} - 1} \left[ \frac{1}{\beta(1 - \tau_K)} \right]^{\frac{\beta}{\nu} + 1}
\]

Comparing the bubbly and bubbleless steady states, we will be able to deduce which steady state is characterized by higher levels of capital and employment. If these aggregates are higher at the bubbly steady state, the crowding-in effect of the bubble dominates and we say that the bubble is productive. If it is the opposite, the crowding-out effect dominates and the bubble is unproductive. The results are summarized in the following proposition:

**Proposition 2**

1. If \( \frac{\beta}{1-\beta} \frac{1-\tau_K}{1-\tau_L} < \frac{\beta + \nu}{1-\beta - \nu} < 1 - \alpha \), there is a stationary non productive bubble, because \( K^* < \bar{K} \) and \( L^* < \bar{L} \).
2. If \( \frac{\beta}{1-\beta} \frac{1-\tau_K}{1-\tau_L} > \frac{\beta + \nu}{1-\beta - \nu} < 1 - \alpha \), there is a stationary productive bubble, because \( K^* > \bar{K} \) and \( L^* > \bar{L} \).

**Proof.** See Appendix C.

According to Proposition 2, the nature of bubble (productive or not) depends on the parameters \( \alpha, \beta \) and \( \nu \). If the saving rate \( (1 - \alpha) \) exceeds the ratio \( \frac{\beta + \nu}{1-\beta - \nu} \),
the crowding-out effect of the bubble dominates. Both the capital stock and the employment level are lower when there is a bubble. As we already argued, this crowding-out effect of the bubble is hard to reconcile with the empirical evidence. In addition, the saving rate is usually not so high and the condition $1 - \alpha < \frac{\beta}{1-\beta}$ seems more plausible.\textsuperscript{4} In such a configuration, when $\tau_K$ is significantly higher than $\tau_L$, a productive bubble may appear (case 3 of Proposition 2).

At the bubbleless steady state, a high tax rate on capital $\tau_K$ means a high reservation wage $w$ (see equation (8)) and, therefore, a high wage. Moreover, a low $\tau_L$, induces a weak interest rate $\tilde{R}$ (see equation (B.4) in Appendix B). Facing these levels of the wage and the interest rate, firms tend to favor the use of capital to labor. At equilibrium, the employment level falls. Since capital accumulation comes from aggregate savings, the capital stock also falls.

At the bubbly steady state, the level of capital per worker, $k^*$, comes from an arbitrage condition between the speculative asset and capital. A high $\tau_K$ leads to a high interest rate $R^*$. The reservation wage drops following the reduction in $\tau_L$ (see equation (8)) and, therefore, the wage also. Firms substitute capital by labor and, at equilibrium, the employment level increases. As the capital stock is mainly determined aggregate savings, which increases with labor, the capital stock increases too.

To summarize, we end up with $K^* > \tilde{K}$ and $L^* > \tilde{L}$ when $\tau_K$ is high and $\tau_L$ low. The capital stock and labor at the bubbleless steady state are quite low, whereas at the bubbly steady state, the levels of capital and labor are high.

4 Economy with tax transfers

Now, there is no more public spending, but the tax rates on capital and labor income finance the transfers shared between young and old households. The share of government revenue distributed to young (old) households is $\theta (1 - \theta)$. This analysis of the implications of such a policy on the existence and features of a bubble would allow us to clearly identify the role of the lump-sum transfers to young and old households.

Since all firms are identical, in equilibrium $K_t = \overline{K}_t$. Using (6)-(8), (10) and (11), the equilibrium system writes:

$$K_{t+1} = \frac{\beta(1 - \tau_K)(1 - \alpha)(1 - \tau_L)(1 - \beta) + \theta(\beta \tau_K + (1 - \beta) \tau_L)}{\beta(1 - \tau_K) + \alpha(1 - \theta)(\beta \tau_K + (1 - \beta) \tau_L)} A K_t^{1-\beta} L_t^{1-\beta} - P_t$$  \hspace{1cm} (25)

$$P_{t+1} = (1 - \tau_K) \beta A K_{t+1}^{1+\beta-1} L_{t+1}^{1-\beta} P_t^{1-\beta}$$  \hspace{1cm} (26)

$$[\beta A K_{t+1}^{1+\beta-1} L_{t+1}^{1-\beta}]^{1-\alpha} = \frac{V}{\alpha(1 - \alpha)}$$  \hspace{1cm} (27)

Definition 2 Given the initial capital stock $K_0 > 0$, an intertemporal equilibrium is a sequence $(K_t, L_t, P_t) \in \mathbb{R}_+^3$ satisfying (37)-(39).\textsuperscript{9}

\textsuperscript{4}Note that since we assume $\nu > 0$ arbitrarily close to 0, $\frac{\beta}{1-\beta}$ and $\frac{\beta \nu + \nu}{\nu + 1 - \beta}$ are very close. Therefore, case 2 of Proposition 2 is unlikely to appear.
A steady state is defined as an equilibrium $K_t = K$, $L_t = L$ and $P_t = P$ for all $t$, satisfying:

$$1 = \frac{\beta(1 - \tau_K)(1 - \alpha)}{\beta(1 - \tau_K) + \alpha(1 - \theta)} \frac{\theta(\beta \tau_K + (1 - \beta) \tau_L)}{AK^{(\alpha + \beta - 1)L(1 - \beta)}} - \frac{P}{K}$$

(28)

$$P = (1 - \tau_K)\beta AK^{(\alpha + \beta - 1)L(1 - \beta)}$$

(29)

$$V = \alpha^\gamma(1 - \alpha)^{1 - \gamma}(1 - \tau_K)^{1 - \gamma}(1 - \tau_L)\beta^{1 - \gamma}(\gamma \beta)^{2 - \gamma} L^{(1 - \alpha)(1 - \beta)}$$

(30)

We study now the bubbleless and bubbly steady states, and compare them to evaluate whether, if a bubble exists, it is productive or not.

When there is no bubble, we have $P = 0$. Using (28) and (30), there exists a unique bubbleless steady state $(\hat{K}, \hat{L})$, given by:

$$\hat{K} = A^{-\nu} \left[ \frac{V}{\alpha^\gamma (1 - \beta)(1 - \tau_L)} \right]^{(1 - \beta)} \left[ \frac{1}{(1 - \alpha)(1 - \tau_K)\beta} \right]^{\beta} \left[ \frac{\gamma}{\beta + 1} \right]^{(1 - \alpha)(1 - \beta)\gamma - (2 - \alpha)}$$

(31)

$$\hat{L} = A^{-\nu} \left[ \frac{V}{\alpha^\gamma (1 - \beta)(1 - \tau_K)} \right]^{(1 - \beta)} \left[ \frac{1}{(1 - \alpha)\beta(1 - \tau_K)} \right]^{\beta + 1} \left[ \frac{\gamma}{\beta + 1} \right]^{(1 - \beta)(1 - \alpha)\gamma - (2 - \alpha)}$$

(32)

where $\gamma \equiv \beta(1 - \tau_K) + \alpha(1 - \theta)(\beta \tau_K + (1 - \beta) \tau_L)$ and $\omega \equiv (1 - \beta)(1 - \tau_L) + \theta(\beta \tau_K + (1 - \beta) \tau_L)$.

We now analyze the existence of a stationary bubble according to the levels of capital taxation, labor taxation and transfers. Let $\hat{R}$ be the interest rate at the bubbleless steady state with transfers. The following proposition establishes the existence and uniqueness of a bubbly steady state:

**Proposition 3** A unique bubbly steady state exists if and only if $(1 - \tau_K)\hat{R} < 1$, which is equivalent to:

$$\lambda \equiv \frac{\beta(1 - \tau_K) + (1 - \theta)(\beta \tau_K + (1 - \beta) \tau_L)}{1 - \beta(1 - \tau_K)} < 1 - \alpha$$

(33)

**Proof.** See Appendix D. ■

As in the model without tax transfer, the existence of the bubble requires that the after-tax interest rate evaluated at the bubbleless steady state should be lower than one. Of course, it is equivalent to a condition which is more complicated than in the model without tax transfer. By inspection of inequality (33), the bubble is more likely to appear, if at least one of the three conditions is satisfied: (i) $\tau_K$ is high, (ii) $\tau_L$ is low, (iii) $\theta$ is high. The effect of the two taxes on the existence of bubble is quite similar than in the model without transfer. The novelty consists in the high transfer to the young households, which allows them to have a sufficiently high income to sustain the existence of the bubble through the purchase of the speculative asset.

Using (28)-(30), the unique bubbly steady state is given by $(K^{**}, L^{**}, P^{**}) \in \mathbb{R}_+^3$, such that:
We study now whether the bubble is productive, i.e. the crowding-in effect of the bubble dominates. We compare the bubbly and bubbleless steady states to analyze if capital and employment are higher when there is a bubble. The results are summarized in the following proposition:

**Proposition 4**

1. If \( \lambda < \frac{\beta + \nu}{1 - \beta - \nu} < 1 - \alpha \), there is a stationary non productive bubble, because \( K^{**} < \hat{K} \) and \( L^{**} < \hat{L} \).
2. If \( \lambda < \frac{\beta}{1 - \beta} < 1 - \alpha < \frac{\beta + \nu}{1 - \beta - \nu} \), \( K^{**} < \hat{K} \) and \( L^{**} > \hat{L} \).
3. If \( \lambda < 1 - \alpha < \frac{\beta}{1 - \beta} \), there is a stationary productive bubble, because \( K^{**} > \hat{K} \) and \( L^{**} > \hat{L} \).

**Proof.** See Appendix E.

Depending on the parameter value of \( \alpha \) and \( \beta \), the nature of bubble can be determined. In fact, if \( \frac{\beta + \nu}{1 - \beta - \nu} < 1 - \alpha \), there is no stationary productive bubble, i.e. the bubble has a crowding-out effect. The bubble lessens capital accumulation and heightens unemployment. On the contrary, if \( 1 - \alpha < \frac{\beta}{1 - \beta - \nu} \), there is a stationary productive bubble, i.e. the crowding-in effect dominates.

This last result can be understood as follows. For a higher level of \( \tau_K \), the interest rates \( R^{**} \) and \( \hat{R} \) increase. In the two economies, the firms favor the use of labor, due to substitution effect. At equilibrium, we have a raise of employment. Since \( R^{**} > \hat{R} \), which also implies that the reservation wage is lower at a bubbly steady state, the substitution effect is more powerful at the bubbly steady state than at the bubbleless one. We end up by having more labor when there is a bubble. Capital accumulation comes from aggregate savings, which depends positively on the transfer to young households and on the interest rate \( R \) (see equation (6)). As the interest rate and the transfers, which increase with income, are higher at the bubbly steady state, this explains that \( K^{**} > \hat{K} \).

---

\[ K^{**} = \left[ \frac{1}{A} \right]^\frac{1}{\beta} \left[ \frac{V}{\alpha^\alpha (1 - \alpha)^{-\alpha} (1 - \beta)(1 - \tau_L)} \right]^{\frac{1 - \beta}{\nu}} \left[ \frac{1}{\beta (1 - \tau_K)} \right]^\frac{\beta}{\nu} \]  

\[ L^{**} = \left[ \frac{1}{A} \right]^\frac{1}{\beta} \left[ \frac{V}{\alpha^\alpha (1 - \alpha)^{-\alpha} (1 - \beta)(1 - \tau_L)} \right]^{\frac{1 - \beta}{\nu} - 1} \left[ \frac{1}{\beta (1 - \tau_K)} \right]^\frac{\beta + 1}{\nu} \]  

\[ P^{**} = \left[ \frac{1}{A} \right]^\frac{1}{\beta} \left[ \frac{V}{\alpha^\alpha (1 - \alpha)^{-\alpha} (1 - \beta)(1 - \tau_L)} \right]^{\frac{1 - \beta}{\nu}} \left[ \frac{\beta (1 - \tau_K)}{\beta (1 - \tau_K) + \alpha (1 - \theta) [\beta \tau_K + (1 - \beta) \tau_L]} \right] \]  

\[ \left[ \frac{\beta (1 - \tau_K)}{\beta (1 - \tau_K) + \alpha (1 - \theta) [\beta \tau_K + (1 - \beta) \tau_L]} \right]^{\frac{1 - \beta}{\nu}} \left[ \frac{\beta (1 - \tau_K)}{\beta (1 - \tau_K) + \alpha (1 - \theta) [\beta \tau_K + (1 - \beta) \tau_L]} \right]^{\frac{\beta}{\nu}} \]
5 Numerical illustration

Our model with endogenous labor shows that the speculative bubble promotes capital stock and labor. This result is sustained by a high level of capital income tax, a low level of labor income tax and/or a high level of the transfer to the young households. This is now illustrated by the experience of different countries as Chile, Mexico, USA and Israel.

We fix the value of the parameters as follows: \( \alpha = 0.54 \) which means a not too high saving rate, \( \beta = 0.333 \) which is a usual value for the capital share in income, \( \nu = 0.1 \) which implies low increasing returns. In addition, \( A \) and \( V \) and are normalized to 1. We consider two sets of countries. The first one is presented in Table 1, where the distribution system is supposed to be nil (the income and wealth transfers are below the median).

<table>
<thead>
<tr>
<th>Countries</th>
<th>( \tau_K )</th>
<th>( \tau_L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chile</td>
<td>0.20</td>
<td>0.07</td>
</tr>
<tr>
<td>Korea</td>
<td>0.24</td>
<td>0.22</td>
</tr>
<tr>
<td>Mexico</td>
<td>0.3</td>
<td>0.19</td>
</tr>
<tr>
<td>Norway</td>
<td>0.28</td>
<td>0.37</td>
</tr>
<tr>
<td>Slovenia</td>
<td>0.17</td>
<td>0.42</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.22</td>
<td>0.43</td>
</tr>
<tr>
<td>USA</td>
<td>0.39</td>
<td>0.31</td>
</tr>
</tbody>
</table>

Source: OECD Data base.

The second one is presented in Table 2, where there are tax transfers (the redistribution is above the median).

<table>
<thead>
<tr>
<th>Countries</th>
<th>( \tau_K )</th>
<th>( \tau_L )</th>
<th>( \theta )</th>
<th>( 1 - \theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>France</td>
<td>0.38</td>
<td>0.49</td>
<td>0.15</td>
<td>0.85</td>
</tr>
<tr>
<td>Iceland</td>
<td>0.2</td>
<td>0.34</td>
<td>0.49</td>
<td>0.51</td>
</tr>
<tr>
<td>Israel</td>
<td>0.25</td>
<td>0.20</td>
<td>0.57</td>
<td>0.43</td>
</tr>
<tr>
<td>Italy</td>
<td>0.31</td>
<td>0.47</td>
<td>0.13</td>
<td>0.87</td>
</tr>
<tr>
<td>Japan</td>
<td>0.37</td>
<td>0.31</td>
<td>0.1</td>
<td>0.9</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.23</td>
<td>0.31</td>
<td>0.36</td>
<td>0.64</td>
</tr>
</tbody>
</table>

Source: OECD Data base.

The transfers towards the young households, corresponds to the public expenditure on family (family allowances, maternity and parental leave), income maintenance and other cash benefits. Regarding the redistribution towards the old households, it consists the pension.

The calibration results are displayed in Tables 3 and 4. The two tables show the values of capital stock and labor at both bubbly and bubbleless steady
states. The sign of $F$ and $F'$ which appear in these Tables 3 and 4, respectively, indicates the existence or not of the bubble, a negative sign implying that the economy does not exhibit a bubble.

<table>
<thead>
<tr>
<th>Countries</th>
<th>$K$</th>
<th>$K^*$</th>
<th>$L$</th>
<th>$L^*$</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chile</td>
<td>193 777.43</td>
<td>197 280.77</td>
<td>204 872.39</td>
<td>230 074.73</td>
<td>0.03</td>
</tr>
<tr>
<td>Korea</td>
<td>767 791.58</td>
<td>--</td>
<td>859 636.95</td>
<td>--</td>
<td>-0.02</td>
</tr>
<tr>
<td>Mexico</td>
<td>760 555.33</td>
<td>773 352.39</td>
<td>805 812.14</td>
<td>897 750.46</td>
<td>0.02</td>
</tr>
<tr>
<td>Norway</td>
<td>3 984 252.05</td>
<td>--</td>
<td>4 800 473.76</td>
<td>--</td>
<td>-0.1</td>
</tr>
<tr>
<td>Slovenia</td>
<td>4 570 748.90</td>
<td>--</td>
<td>6 107 274.08</td>
<td>--</td>
<td>-0.25</td>
</tr>
<tr>
<td>Sweden</td>
<td>6 238 858.78</td>
<td>--</td>
<td>8 166 335.90</td>
<td>--</td>
<td>-0.22</td>
</tr>
<tr>
<td>USA</td>
<td>3 525 567.87</td>
<td>3 563 233.65</td>
<td>3 774 602.5</td>
<td>4 043 475.22</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Source: OECD Data base.

<table>
<thead>
<tr>
<th>Countries</th>
<th>$K$</th>
<th>$K^{**}$</th>
<th>$L$</th>
<th>$L^{**}$</th>
<th>$F'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>France</td>
<td>31 284 426.2</td>
<td>--</td>
<td>82 161 938.40</td>
<td>--</td>
<td>-0.28</td>
</tr>
<tr>
<td>Iceland</td>
<td>2 079 012.26</td>
<td>--</td>
<td>2 506 724.63</td>
<td>--</td>
<td>-0.1</td>
</tr>
<tr>
<td>Israel</td>
<td>666 441.51</td>
<td>667 524.02</td>
<td>706 842.88</td>
<td>714 310.2</td>
<td>0.003</td>
</tr>
<tr>
<td>Italy</td>
<td>17 071 566.74</td>
<td>--</td>
<td>43 575 706.49</td>
<td>--</td>
<td>-0.3</td>
</tr>
<tr>
<td>Japan</td>
<td>3 647 147.5</td>
<td>--</td>
<td>8 201 542.71</td>
<td>--</td>
<td>-0.18</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>1 783 345.69</td>
<td>--</td>
<td>2 542 953.27</td>
<td>--</td>
<td>-0.12</td>
</tr>
</tbody>
</table>

Source: OECD Data base.

As we can see in Table 3, the countries as Chile, Mexico and USA may exhibit a bubble, which is productive. In these countries, the level of capital income tax is sufficiently high with respect to labor income tax. This encourages the investment in productive capital and speculative asset, and promotes also employment.

In Table 4, only Israel may exhibit a bubble. This bubble is productive. Indeed, the value of the capital income tax is high with respect to the labor income one, sustained by sufficiently high transfers to young households.

To summarize, this numerical illustration perfectly illustrates our main results. First, all countries that experience a productive bubble are characterized by a higher tax on capital income than on labor income (Propositions 1 and 2). Second, when there are significant transfers in the economy, the share of transfers to young households should be high enough (Propositions 3 and 4).

### 6 Dynamic analysis

The dynamics are driven by the following three-dimensional dynamic system:
\[ K_{t+1} = \Omega AK_t^{1+\beta}L_t^{1-\beta} - P_t \]  
\[ P_{t+1} = (1 - \tau_K)\beta AK_{t+1}^{1+\beta-1}L_{t+1}^{1-\beta}P_t \]  
\[ |\beta AK_{t+1}^{1+\beta-1}L_{t+1}^{1-\beta}|^{1-\alpha} = \alpha^{\alpha(1-\alpha)^{1-\alpha}(1-\tau_K)^{1-\alpha}(1-\beta)(1-\tau_L)AK_t^{1+\beta}L_t^{1-\beta}} \]

where \( \Omega \) is a constant that takes different values if there are or not some transfer.

We will analyze the dynamics by analyzing the local stability properties in the neighborhood of each steady state. TO BE DONE

7 Conclusion

This paper highlights the roles of tax policy and endogenous labor on the existence and features of rational bubbles. We thus consider that the tax burden on capital and labor incomes finance government spending and transfers. High level of capital income tax makes the investment into capital stock less attractive, which facilitates the existence of a speculative bubble. Moreover, low labor income tax and/or a high transfer to the young investors modifies the income upward, which promotes the appearance of the bubble. We have also exhibited a new mechanism which explains that a bubble is productive. When the bubble exists, the interest rate is high following a high capital income tax. Facing a significant cost of use capital, the firms tend to favor labor. At equilibrium, the unemployment level is low due to this substitution effect. The capital stock is high because it is sustained by a high labor income, a low labor income tax and/or a high transfer to the young households. These findings are illustrated numerically using a panel of countries. Confirming our theoretical results, a productive bubble appears in countries with a higher tax rate on capital than on labor and a large share of transfers to young agents.

Finally, we note that our model with transfers and taxes on capital and labor incomes introduce some distortions that encompass some models with labor market imperfections as a particular case. For instance, it is possible to show that when there are unions which efficiently bargain with firms on employment and wages, the unions’ bargaining power plays a role of capital income taxation and redistributes a share of rental cost of capital to labor income. Therefore, using our analysis, we can deduce the implications of such type of labor market imperfection on speculative bubbles.
Appendix

A  Determination of the reservation wage

Maximizing the utility (1) under the budget constraints (2) and (3), we get:

\[ c_{1t} = \alpha \left[ (1 - \tau_L)w_t d_t + T_{1t} + \frac{T_{2t+1}}{R_{t+1}(1 - \tau_K)} \right] \]

(A.1)

\[ c_{2t+1} = (1 - \alpha)(1 - \tau_K)R_{t+1} \left[ (1 - \tau_L)w_t d_t + T_{1t} + \frac{T_{2t+1}}{R_{t+1}(1 - \tau_K)} \right] \]

(A.2)

Substituting these two consumptions in the utility (1), we obtain an indirect utility which depends on the working time, \( U(d_t) \). We can compute:

\[ U(d_t = 1) = \alpha^\alpha [(1 - \alpha)(1 - \tau_K)R_{t+1}]^{1-\alpha} \]

\[ \left[ (1 - \tau_L)w_t + T_{1t} + \frac{T_{2t+1}}{R_{t+1}(1 - \tau_K)} \right] - V \]

\[ U(d_t = 0) = \alpha^\alpha [(1 - \alpha)(1 - \tau_K)R_{t+1}]^{1-\alpha} \left[ T_{1t} + \frac{T_{2t+1}}{R_{t+1}(1 - \tau_K)} \right] \]

Each individual decides to work if only if

\[ U(d_t = 1) - U(d_t = 0) \geq 0 \]

which is equivalent to \( w_t \geq w_t \), where \( w_t \) is given by (8).

B  Proof of Proposition 1

Using (16) and (17), we have:

\[ 1 = \frac{(1 - \alpha)(1 - \beta)(1 - \tau_L)}{\beta(1 - \tau_K)} - \frac{P}{K^*} \]

(B.3)

Substituting the interest rate at the bubbleless steady state \( \hat{R} = A\beta\hat{K}^\epsilon\hat{k}^{\beta-1} \) into (16) with \( P = 0 \), we get:

\[ \hat{R} = \frac{\beta}{(1 - \alpha)(1 - \beta)(1 - \tau_L)} \]

(B.4)

Substituting (B.4) into (B.3), we get:

\[ 1 = \frac{1}{(1 - \tau_K)\hat{R}} - \frac{\hat{P}}{\hat{K}} \]

Therefore, \( P^* > 0 \) if and only if \( \hat{R}(1 - \tau_K) < 1 \). We easily deduce the proposition.
C Proof of Proposition 2

• Comparison between $\tilde{K}$ and $K^*$:

Using (19) and (22), we can rewrite $\tilde{K}$ as:

$$\tilde{K} = \left(\frac{(1 - \alpha)(1 - \beta)(1 - \tau L)}{\beta(1 - \tau K)}\right)^{(1 - \alpha)(1 - \beta) - \beta}$$

Since $(1 - \tau K)\hat{R} < 1$, $K^* > \tilde{K}$ requires:

$$1 - \alpha < \frac{\beta}{1 - \beta}$$

• Comparison between $\tilde{L}$ and $L^*$:

Using (20) and (24), we can rewrite $\tilde{L}$ as:

$$\tilde{L} = \left(\frac{(1 - \alpha)(1 - \beta)(1 - \tau L)}{\beta(1 - \tau K)}\right)^{(1 - \alpha)[\frac{1}{2} - \frac{\beta}{1 - \beta} - 1]}$$

Since $(1 - \tau K)\hat{R} < 1$, $L^* > \tilde{L}$ requires:

$$1 - \alpha < \frac{\beta + v}{1 - \beta - v}$$

Using these results and Proposition 1, we deduce the proposition.

D Proof of Proposition 3

Using (28) and (29), we have:

$$1 = \frac{(1 - \alpha)[(1 - \tau L)(1 - \beta) + \theta(\beta \tau K + (1 - \beta)\tau L)]}{\beta(1 - \tau K) + \alpha(1 - \theta)(\beta \tau K + (1 - \beta)\tau L)} - \frac{P^{**}}{K^{**}} \quad (D.5)$$

At the bubbleless steady state, $\hat{R} = A\beta \tilde{K}^{v + \beta - 1} \tilde{L}^{1 - \beta}$. Substitute this equation into (28) with $P = 0$, we get:

$$\hat{R} = \frac{\beta(1 - \tau K) + \alpha(1 - \theta)(\beta \tau K + (1 - \beta)\tau L)}{(1 - \tau K)(1 - \alpha)[(1 - \tau L)(1 - \beta) + \theta(\beta \tau K + (1 - \beta)\tau L)]} \quad (D.6)$$

Using (D.5) and (D.6), we obtain:

$$1 = \frac{1}{(1 - \tau K)\hat{R}} - \frac{P^{**}}{K^{**}}$$

We easily deduce that $P^{**} > 0$ if and only if $\hat{R}(1 - \tau K) < 1$. Using again (D.6), we deduce the proposition.
E  Proof of Proposition 4

• Comparison between $\hat{K}$ and $K^{**}$

Using (31) and (34), we can rewrite $\hat{K}$ and $K^{**}$ as:

$$\hat{K} = A^{-\nu} \left[ \frac{V}{\alpha^\nu(1-\beta)(1-\tau_L)} \right]^{(1-\beta)/\nu} \left[ \frac{1}{(1-\alpha)(1-\tau_K)} \tilde{R} \right]^{(1-\alpha)(1-\beta)/\nu}$$

$$K^{**} = \hat{K} [(1-\tau_K)\tilde{R}]^{(1-\alpha)(1-\beta)/\nu}$$

Since $(1-\tau_K)\tilde{R} < 1$, we deduce that $K^{**} > \hat{K}$ requires:

$$1 - \alpha < \frac{\beta}{1 - \beta}$$

• Comparison between $\hat{L}$ and $L^{**}$

Using (32) and (35), we can rewrite $\hat{L}$ and $L^{**}$ as:

$$\hat{L} = \left[ \frac{1}{A} \right]^{\frac{1}{\nu}} \left[ \frac{V}{\alpha^\nu(1-\beta)(1-\tau_L)} \right]^{(1-\beta)/\nu - 1} \left[ \frac{1}{(1-\alpha)(1-\tau_K)} \tilde{R} \right]^{\frac{(1-\alpha)(1-\beta)/\nu}{\nu + 1}}$$

$$L^{**} = \left[ (1-\tau_K)\tilde{R} \right]^{(1-\alpha)(1-\beta)/\nu - (1-\alpha)(1-\beta)/\nu}$$

Since $(1-\tau_K)\tilde{R} < 1$, we deduce that $L^{**} > \hat{L}$ requires:

$$1 - \alpha < \frac{\beta + \nu}{1 - \beta - \nu}$$

Using these results and Proposition 3, we deduce the proposition.

References


