Unemployment with Trade and Firm Heterogeneity*

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Abstract

Specify the setting of footloose capital with firm heterogeneity, this paper examine
the effects of trade liberalization on unemployment through two different mechanisms.
First we embed search frictions into the labor market, and second, we consider fair
wages as the source of unemployment. In the model with labor search frictions, we
find that both labor expected wages and employment rates is higher even in a small
country when the search technology is better. In fair wage settings, the result shows
that the unemployment rate increases in trade cost when the trade freeness is high.
Finally we try to compare the welfare levels under different scenarios, and discover that
unemployment may deteriorate the welfare gains from trade.

Keywords: Unemployment, Search Frictions, Fair Wages, Firm Heterogeneity, Foot-
loose Capital

JEL Classification: F1, J3, L1, R1

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1 Introduction

The fierce debate on the unemployment issue associated with trade liberalization has been going on for a few decades. Improvement in communication equipment and transport technology have dramatically lowered the trade costs; thus, it has gradually become cheaper to transport. At the same time, financial barriers to capital flow have also weakened. Firms can set up factories in different countries to evade tax burdens and other risk. In addition, most governments around the world have made efforts to lower barriers to trade in order to attract investment from other countries. All these facts have facilitated international trade among countries. "Foreign" merchandise has crowded on to the shelves of domestic grocery stores, and consumers enjoy the greater variety.

However, does everyone gain from trade? The deepened interdependence of two countries may transmit the shocks of one country to others through this channel. For example, foreign firms may acquire new technology that can increase production efficiency. This implies that the price of foreign goods may fall. Foreign firms with cheap goods may defeat domestic counterparts. These domestic firms may shut down or move to other countries to access cheaper labor forces, then the domestic workers may suffer from unemployment. In other words, when we assess the benefits of trade, we cannot ignore the role that the labor market plays.

Nevertheless, labor is usually assumed to be the only production factor in previous research. International capital flows also play an important role. The fall of the Bretton Woods agreement facilitated capital mobility in 1970. According to the Migration Factbook 2016, foreign direct investment (FDI) has become an important capital resource for many developing and underdeveloped countries, and the trend is still increasing. Taking labor immobility into consideration, the reallocation effect between production factors attracts public attention. Piketty (2014) mentions that capital has gained a great deal of weight in the economy worldwide. Moreover, a report by International
Labor Organization (ILO) points out that the capital share is increasing relative to labor share after 1990. It means that the production output has concentrated in the owners of capital. Thus, we also consider the capital flow in our analysis framework.

Moreover, full employment is usually assumed in the past literatures. In order to acquire a comprehensive understanding of unemployment, we adopt two different methods to generate the unemployment in this research. First, we introduce the search and matching costs into the labor market. Because of information asymmetry, firms cannot hire workers without extra costs. They have to pay for posting vacancies. This is the "friction" of the labor market. After that, the matching between firms and labor is decided by a matching function with exogenous search technology. Once they meet each other, they bargain over the wage. We want to explore how search costs shape the wages and welfare in distinct countries. Next, we incorporate a reference wage mechanism into a general equilibrium heterogeneous firm model to investigate the decision of wages and unemployment rates. Yang and Zeng (2015) consider capital movement, but they do not consider the unemployment issue, which is an essential topic for trade. Finally, evaluating welfare gains from trade is also an important topic for policy makers. After considering the unemployment and capital, we get a modified welfare indicator like that in Arkolakis, et al. (2012, henceforth ACR), and try to disentangle the channels that influence welfare after trade is allowed.

There are several strands of literature related to our research. The first is about the general equilibrium model setting. Melitz (2003) proposes a model with heterogeneous firms. He proves that only much more productive firms engage in exporting, while firms with lower productivity only produce for the domestic market and the remainder leave the market, and hence the aggregate industrial productivity will raise. Yang and Zeng (2015) extend the footloose capital model to incorporate a heterogeneous firms setting as in Melitz (2003). They discover that the equilibrium relative wage is monotonically increasing when the relative exporting and domestic fixed costs are high, that is, the

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wage inequality between two countries increases. If the relative cost is small enough, the relative wage curve will be bell-shaped, that is, the inequality increases first and then converges as the exposure to trade increases. However, these papers do not take the unemployment issue into account.

The second part investigates the impact of trade on unemployment. Search frictions, efficiency wages, fair wage preferences and minimum wages are usually considered when the unemployment is presented. Helpman and Itskhoki (2010) combine the Melitz model with search frictions. They investigate the effects of labor market search frictions on wages, welfare and the magnitude of unemployment. If other things being equal, decreased in domestic search frictions harm the welfare of the other country. However, if the search frictions proportionally decrease in differentiated sectors at the same time, it benefits both countries. After opening to trade, the country with lower relative search frictions may not have a lower unemployment rate. At the same time, Helpman et al. (2010) consider labor heterogeneity and discover that trade enlarges wage inequality. Helpman et al. (2011) consider the policy implication of an unemployment premium and find that it can mitigate the harmful effect of opening to trade to some extent. Itskhoki and Helpman (2015) extend the model further to explore the dynamic process of trade liberalization. They find that firms with the highest productivity expand employment: Some firms may fire workers and stay in the market, others may lay off workers and exit the market gradually, and the others leave the market immediately. High productivity incumbents may pay higher wages and guarantee job security to homogeneous workers.

Other papers adopt other kinds of mechanisms. Egger and Kreickemeier (2009) introduce fair wage preferences into a Melitz model and show that the average profit of active firms, aggregate welfare, and the unemployment rate increase. Egger and Kreickemeier (2012) modify their previous paper and develop a model featuring two types of laborers, that is, workers and managers. The involuntary unemployment rate and the inequality between and within these two groups increase as trade cost falls.
Egger et al. (2012) introduce minimum wages to explain the labor market imperfections and demonstrate that a hike in minimum wages harms domestic workers as well as foreign ones. All these papers only use labor as the sole production factor. Moreover, Zeng et al. (2016) combine the efficiency wage model and footloose capital and provide a rich pattern of unemployment and welfare analysis.

In addition, many empirical papers try to disentangle the relationship between trade and unemployment. Felbermayr et al. (2013) adopt search frictions and a Nash bargaining process to model the labor market. By introducing the concept of labor market spillover, they find that higher search frictions not only raise domestic unemployment, but also the unemployment for the foreign counterpart. Based on a modified Ricardian model, Carrère et al. (2014) use a panel dataset of 97 countries from 1995-2009 to verify that opening to trade raised the unemployment in countries with comparative advantage in sectors with strong labor market frictions, and lower unemployment in countries with comparative advantage in sectors with weak labor market friction. Based on the theoretical model of Helpman et al. (2010), Helpman et al. (2012) estimate effects of opening to trade with Brazilian data and find that trade is critical to increase wage inequality within sectors in both developed and developing countries.

Another stream in the literature is about spatial inequality and wage premium. Martin and Rogers (1995) propose a model with two sectors and two production factors, and conclude that firms tend to move to the country with better domestic infrastructure, and the level of international infrastructure makes this effect even greater. Takahashi et al. (2013) study a model with two production factors and one sector without unemployment. The authors prove the equivalence of spatial inequalities in industrial location and in income. That is, more firms produce in a larger country and workers get a relatively high wage. When the trade cost decreases, the relative wage will increase first and then decrease (in an inverted U shape). That paper also agrees that globalization may benefit both countries.

The last stream of literature is about the evaluation of impacts on welfare. Arkolakis
et al. (2012) (hereafter ACR) conclude that a country’s domestic expenditure share and the elasticity of trade with respect to the variable trade cost are sufficient to estimate the welfare gains from trade if several conditions are satisfied. Yang and Zeng (2015) implement both labor and capital as production factors to relax the trade balance conditions that ACR proposes, and demonstrate that the relative factor price also affects the gains from trade. Heid and Larch (2016) build an Armington model with a search and matching friction to relax the full employment assumption, and estimate the impact of trade agreements and labor market reforms on 28 OECD countries. The gains from trade are larger when unemployment is considered. They also confirm that labor market reforms in one country benefit its trading partners, although the positive effects are small.

In this study, we consider two distinct mechanisms that generate unemployment under the framework of Yang and Zeng (2015) to investigate the effects of trade liberalization on unemployment. One is search friction and the other is the fair wage preferences. The model contains two production factors and one manufacturing sector. The advantage of this model is that we do not need to worry about the unemployment premium because everyone has a unit of capital to achieve the basic living standard. This implies that we do not have to consider other sectors such as governments that engender the allocation problem.

The remainder of this paper is organized as follows. Section 2 outlines the model setting; section 3 studies the unemployment issue with a labor market friction model. Section 4 discuss the unemployment issue based on the fair wages mechanism. Section 5 examines the welfare effects, and finally section 6 concludes.

2 Model Setting

There are two countries, \( i = 1, 2 \), two production factors, and one manufacturing sector in this economy space. Capital is invested as sunk cost to enter the markets,
and workers are employed as the variable costs. Labor is homogeneous in this space. Firms also use capital to finance extra costs such as hiring costs. The total population is \( L = L_1 + L_2 \). Without loss of generality, we assume the everyone has one unit of capital, and the total amount of capital is \( K = L \). The capital flows into the country with higher profits. It is a so-called “footloose capital model.” We employ this model as a workhorse to investigate the unemployment issue generated by search frictions and efficiency wages, separately.

All individuals have identical preferences. They spend their income in a continuum of differentiated goods. Consumers in country \( i \) enjoy the utility as

\[
U_i \equiv \left\{ \int_{a \in \Omega_i} \left[ q(a) \frac{\sigma}{\sigma-1} \right] da \right\}^{\frac{\sigma}{\sigma-1}}, \quad \text{where } i = 1, 2, \tag{1}
\]

\( U_i \) is the typical CES utility function with elasticity of substitution \( \sigma > 1 \). \( \Omega_i \) denotes the total variety set available in the country, and \( q(a) \) is the demand for a variety \( a \).

The price index is listed as follows:

\[
P_i = \left[ \int_{a \in \Omega_i} p_i(a)^{1-\sigma} da \right]^{\frac{1}{1-\sigma}}, \tag{2}
\]

where \( p_i(a) \) means the price of variety \( a \) in country \( i \). After solving the utility maximization problem of consumers, the demand for goods in country \( i \) is

\[
q_{ij}(a) = \frac{Y_j}{P_j} \left[ \frac{P_i}{p_{ij}(a)} \right]^\sigma, \quad i, j = 1, 2. \tag{3}
\]

where \( p_{ij}(a) \) is the price of goods produced in country \( i \) and consumed in country \( j \), and \( Y_j \) represents the aggregate income in country \( j \).

### 3 Models with Labor Market Frictions

In the first framework, we consider the imperfect labor market, which features search frictions. Firms have to pay a search cost to match with workers, and after they meet each other, they bargain over wages. We set the population \( L \) is large enough to ensure
that some workers cannot successfully reach their counterpart firm. Once they commit to each other, firms cannot lay off workers, and workers do not leave the office. There is no on-the-job training or recruitment. Before the owners of firms enter the market, they have to invest \( f_e \) units of capital to learn their productivity \( \phi \), which is drawn from a Pareto distribution \( G(\phi) = 1 - \phi^{-k} \) with density function \( g(\phi) = k\phi^{-(k+1)} \). Without loss of generality, we set the lower bound of productivity equal to 1 \( (\phi > 1) \). Firms exit the market if the productivity is too low to cover entry costs. If they choose to stay and produce, \( f_d \) and \( f_x \) units of capital are needed to acquire access to the domestic and foreign markets, respectively. The trade costs take the form of an iceberg cost, which means that \( \tau_{ij} > 1 \) unit of goods must be exported in order to supply the foreign market with one unit of manufactured goods. The production technology of a typical firm is \( l_{ij}(\phi) = \tau_{ij}q_{ij}(\phi)/\phi \), where \( l_{ij}(\phi) \) is the labor input needed in country \( i \) to meet the demand \( q_{ij}(\phi) \) of country \( j \). For simplicity, we further assume the iceberg cost is symmetric between countries, that is, \( \tau_{ij} = \tau_{ji} = \tau \) and \( \tau_{ii} = 1 \). The total labor demand of a firm in country \( i \) is

\[
l_i = \frac{q_{ii}(\phi) + I_i(\phi)\tau q_{ij}(\phi)}{\phi}, \quad i \neq j, \tag{4}
\]

where \( I_i(\phi) \) is the indicator of exporting status of a firm. If a firm with productivity \( \phi \) decides to export, then \( I_i(\phi) = 1 \). \( I_i(\phi) = 0 \) for a firm only producing for the domestic market. With the rule of equilibrium price \( p_{ij} = \tau p_{ii} \) for \( i \neq j \), insert the demand function (3) into (4) and rearranging, we get

\[
p_{ii}(\phi) = (\phi l_i)^{-\frac{1}{\sigma}} \left[ Y_i P_i^{\sigma-1} + Y_j P_j^{\sigma-1} (\tau)^{1-\sigma} \right]^{\frac{1}{\sigma}},
\]

The total revenue of a typical firm with productivity \( \phi \) in country \( i \) is

\[
R_i(\phi) = p_{ii}(\phi) q_{ii}(\phi) + I_i(\phi) p_{ij}(\phi) q_{ij}(\phi) = (\phi l_i)^{-\frac{2-\sigma}{\sigma}} \left[ Y_i P_i^{\sigma-1} + I_i(\phi) Y_j P_j^{\sigma-1} (\tau)^{1-\sigma} \right]^{\frac{1}{\sigma}}. \tag{5}
\]

In order to clearly investigate the phenomenon of unemployment, we only consider the manufacturing sector in our model. That is, workers can only choose to work for the
manufacturing sector or not. Besides, we introduce a matching algorithm in this setting.

The firm has to pay $b_i$ units of capital as search costs before they meet a worker. If the firm hires $l_i$ units of workers with capital rent $r_i$, then it has to pay the hiring cost $r_i b_i l_i$. A typical firm takes $b_i$ as given, but it is an endogenous variable which is decided by the general equilibrium. Once they meet each other, workers have the power to negotiate wages with firms, and firms have to give them a reasonable wage or they are not willing to be involved into production process, and also we do not introduce the notion of a labor union into the model. Firms bargain with individuals rather than the labor union over the equilibrium wage. We adopt the model of Stole and Zwiebel (1996a,b) to split the revenue $R_i$ of each firm, and assuming the equal bargaining weight between labor and firms, workers get the $(\sigma - 1) / (2\sigma - 1)$ share of firm revenue\(^2\). This results suggests that the outside option of a worker is unemployment, so she gets zero other than the capital rent once she becomes unemployed. After firms begin to produce, then the workers cannot leave their duty at will. After bargaining, the firm gets $\sigma / (2\sigma - 1)$ of revenue and pays a search cost $r_i b_i l_i$ and fixed cost. Firms in country $i$ face the profit maximization problem as

$$
\pi_i(\phi) = \max_{l_i} \left\{ \frac{\sigma}{2\sigma - 1} (\phi l_i)^{\frac{\sigma - 1}{\sigma}} \left[ Y_i P_i^{\sigma - 1} + I_i(\phi) Y_j P_j^{\sigma - 1} r_j^{1-\sigma} \right]^{\frac{1}{\sigma}} - r_i b_i l_i - r_i [f_d + I_i(\phi) f_x] \right\},
$$

By solving the firm’s maximization problem, we can get the labor demand of a $\phi$-firm as the function of aggregate variables and searching friction:

$$
l_i(\phi) = l_{ii}(\phi) + I_i(\phi) l_{ij}(\phi),
$$

\(^2\)The decision rule can be expressed as

$$
\frac{\partial}{\partial h} (R(h) - w(h) h) = w(h)
$$

This bargaining method takes into account the number of workers, that is, conditional on the number of workers employed by firm now, the marginal revenue of hiring one more worker is equals to the wage, the marginal worker will affect the wage others receive, and the solution to this differential equation is $w(h) = \frac{\sigma - 1}{2\sigma - 1} \frac{R(h)}{h}$. 


where \( l_{ii} (\phi) \) and \( l_{ij} (\phi) \) are the number of workers hired by a \( \phi \)-firm for domestic and foreign markets, respectively, and

\[
\begin{align*}
l_{ii} (\phi) &= \left( \frac{\sigma - 1}{2\sigma - 1} \right) ^{\sigma} (r_{ib})^{\sigma - 1} (r_{ib})^{-\sigma} Y_i P_i^{\sigma - 1}, \\
l_{ij} (\phi) &= \left( \frac{\sigma - 1}{2\sigma - 1} \right) ^{\sigma} (r_{ib})^{\sigma - 1} (r_{ib})^{-\sigma} Y_j P_j^{\sigma - 1},
\end{align*}
\]

and then we have the prices as the constant mark-up over the marginal costs as follows,

\[
p_{ii} (\phi) = \left( \frac{2\sigma - 1}{\sigma - 1} \right) \frac{r_{ib}}{\phi} \quad \text{and} \quad p_{ij} = \tau p_{ii}.
\]

In turn, we can derive the wages paid by a \( \phi \)-firm in country \( i \) as follow:

\[
w_i (\phi) = \left( \frac{\sigma - 1}{2\sigma - 1} \right) \frac{R_i (\phi)}{l_i (\phi)} = r_{ib}.
\]

The first equality comes from the results of the bargaining process: workers get a \((\sigma - 1) / (2\sigma - 1)\) share of revenue. And combined with (6), we can get the second equality. This result is also shown by Helpman and Itskhoki (2010), Helpman et al. (2010) and Carrère et al. (2014). The wage is equal to the value of search costs, which implies that every firm pays the same wages within a country. Rewriting the profit function with (6), then the operating profit of a \( \phi \)-firm in country \( i \) is given by:

\[
\pi_i (\phi) = \pi_{ii} (\phi) + I_i (\phi) \pi_{ij} (\phi),
\]

where \( \pi_{ii} (\phi) \) and \( \pi_{ij} (\phi) \) are the operating profit from domestic country \( i \) and foreign country \( j \), respectively, as follows,

\[
\begin{align*}
\pi_{ii} (\phi) &= \frac{1}{\sigma - 1} \left( \frac{\sigma - 1}{2\sigma - 1} \right) ^{\sigma} (r_{ib})^{1-\sigma} Y_i P_i^{\sigma - 1} (r_{ib})^{-\sigma} - r_{if_d}, \\
\pi_{ij} (\phi) &= \frac{1}{\sigma - 1} \left( \frac{\sigma - 1}{2\sigma - 1} \right) ^{\sigma} (r_{ib})^{1-\sigma} Y_j P_j^{\sigma - 1} (r_{ib})^{-\sigma} - r_{if_x}.
\end{align*}
\]

The domestic and export production thresholds \((\phi_{ii} \text{ and } \phi_{ij})\) that ensure non-negative
profits are defined as follows:

\[
\phi_{ii} = \frac{2\sigma - 1}{\sigma - 1} \left[ \frac{(2\sigma - 1) r_i f_d}{(r_i b_i)^{1-\sigma} Y_i P_{i}^{\sigma-1}} \right]^{\frac{1}{\sigma-1}},
\]

\[
\phi_{ij} = \frac{2\sigma - 1}{\sigma - 1} \left[ \frac{(2\sigma - 1) r_i f_x}{(r_i b_i)^{1-\sigma} Y_j P_{j}^{\sigma-1} r_i^{1-\sigma}} \right]^{\frac{1}{\sigma-1}}.
\]  

(9)

In line with previous literature, we consider the parameters that satisfy the partition condition, that is, \( \phi_{ij} > \phi_{ii} \). This means that firms with higher productivity can afford the extra export costs, and they serve the domestic and export market at the same time. Firms with lower productivity only produce for the domestic market. Besides, \( 1 - G(\phi_{ii}) \) and \( 1 - G(\phi_{ij}) \) are the accumulated probability entering the domestic and foreign market, respectively. \( \pi_{ii}(\tilde{\phi}_{ii}) \) and \( \pi_{ij}(\tilde{\phi}_{ij}) \) are the average operating profit in the domestic market and foreign market, and firms enter into the market only if the expected operating profits can cover the entry cost \( f_e \), that is,

\[
r_{i}f_e = [1 - G(\phi_{ii})] \pi_{ii}(\tilde{\phi}_{ii}) + [1 - G(\phi_{ij})] \pi_{ij}(\tilde{\phi}_{ij}),
\]

\[
r_{j}f_e = [1 - G(\phi_{ji})] \pi_{ji}(\tilde{\phi}_{ji}) + [1 - G(\phi_{jj})] \pi_{jj}(\tilde{\phi}_{jj}).
\]  

(10)

where

\[
\tilde{\phi}_{ii} = \left[ \int_{\phi_{ii}}^{\infty} \phi^{(\sigma-1)} \mu_{ii}(\phi) \, d\phi \right]^{\frac{1}{(\sigma-1)}} = \left[ \frac{k}{k - (\sigma - 1)} \right]^{\frac{1}{(\sigma-1)}} \phi_{ii},
\]

\[
\tilde{\phi}_{ji} = \left[ \int_{\phi_{ji}}^{\infty} \phi^{(\sigma-1)} \mu_{ji}(\phi) \, d\phi \right]^{\frac{1}{(\sigma-1)}} = \left[ \frac{k}{k - (\sigma - 1)} \right]^{\frac{1}{(\sigma-1)}} \phi_{ji}.
\]  

(11)

where

\[
\mu_{ii}(\phi) = \begin{cases} 
\frac{q(\phi)}{1-q(\phi_{ii})} = \frac{k}{\phi} (\frac{\phi_{ii}}{\phi})^{-k}, & \text{if } \phi > \phi_{ii}, \\
0, & \text{otherwise}.
\end{cases}
\]  

(12)

3.1 Labor Market

There is search friction in the labor market, meaning that firms and workers cannot meet each other without costs. Firms spend money on posting vacancies to inform
and hire workers. After they meet each other, search and matching costs result in a bargaining process. Workers bargain with the employers over the "match surplus". If a worker does not accept the offer provided by employer, then the worker is unemployed and earns nothing. Once the worker accepts the offer, she cannot resign. The successful matches are decided by a matching function $H(V, L)$ where $V$ is the aggregate vacancies and $L$ is the amount of population. Follow the setting of Helpman and Itskhoki (2010), we let the matching function have a Cobb-Douglas form as follows,

$$H_i(V_i, L_i) = m_i (L_i)^{\gamma} (V_i)^{1-\gamma},$$  \hspace{1cm} (13)$$

where $H_i$ is the total hiring amount in country $i$, $V_i$ is the number of vacancies, $m_i$ is the efficiency of matching technology and $L_i$ is the total population and the amount of workers that search for jobs in country $i$. The probability of a worker getting a job is equal to $\lambda_i = H_i/L_i$ and the probability of filling a vacancy is $H_i/V_i = (m_i)^{\frac{1}{1-\gamma}} (\lambda_i)^{\frac{\gamma}{1-\gamma}}$. $\gamma \in (0, 1)$ is the elasticity of the matching function with respect to the number of searching workers and the negative elasticity of the probability of filling a vacancy. When $\gamma$ is large, the additional vacancy ruins the probability of filling a vacancy. That is, excess supply in vacancies exists. When $\gamma = 1$, the number of matches is only related to the number of workers, and the labor market reduces to perfect competition.

Because workers are risk neutral, the supply of labor is decided by the expected payoff, which is defined below:

$$\omega_i = \lambda_i w_i = \lambda_i r_i b_i,$$ \hspace{1cm} (14)$$

where $\omega_i$ is the expected wage in country $i$ and the second equality is from (7). Denote the posting cost per vacancy as $\nu_i$ and the hiring cost of labor as $r_i b_i = r_i \nu_i V_i/H_i$ per worker in country $i$. Then we can derive the relationship between search cost and employment rate as

$$b_i = \nu_i (m_i)^{\frac{1}{1-\gamma}} (\lambda_i)^{\frac{\gamma}{1-\gamma}},$$ \hspace{1cm} (15)$$

Combining equations (13), (14) and (15), we can get the employment rate and the
wage as the function of expected wage

\[
\lambda_i = \left( \frac{\omega_i}{r_i \nu_i} \right)^{1-\gamma} \quad m_i < 1,
\]

\[
w_i = r_i b_i = \frac{(\omega_i)^\gamma}{m_i} (r_i \nu_i)^{1-\gamma}.
\] (16)

where we assume \( \nu_i (m_i)^{1-\gamma} > \omega_i / r_i \), so the employment rate \( \lambda_i < 1 \), and the unemployment rate \( u_i = 1 - \lambda_i = 1 - m_i (\omega_i)^{1-\gamma} / (r_i \nu_i)^{1-\gamma} \). The form of search friction is in line with the Diamond-Mortensen-Pissarides (DMP) model (see also Pissarides, 2000).

**Lemma 1** The employment rate \( \lambda_i \) and wage \( w_i \) (also the search cost \( b_i \)) are increasing in expected wage \( \omega_i \) in both countries.

Apparently, the wage \( w_i \) and unemployment rate \( u_i \) are not directly related by the trade openness, and this affects the expected wage \( \omega_i \), then the effects pass on to \( w_i \) and \( u_i \).

### 3.2 Market Equilibrium Structure

Now, we move to examine the equilibrium with incomplete specialization. First we derive the average operating profits, then combining them with (9) we get

\[
\pi_{ii} \left( \tilde{\phi}_{ii} \right) = \left[ \int_{\phi_{ii}}^{\infty} \pi_{ii} (\phi) \mu_{ii} (\phi) \, d\phi \right] = \left[ \left( \frac{\phi_{ii}}{\phi_{ii}} \right)^{\sigma-1} - 1 \right] r_i f_d,
\]

\[
\pi_{ij} \left( \tilde{\phi}_{ij} \right) = \left[ \int_{\phi_{ij}}^{\infty} \pi_{ij} (\phi) \mu_{ij} (\phi) \, d\phi \right] = \left[ \left( \frac{\phi_{ij}}{\phi_{ij}} \right)^{\sigma-1} - 1 \right] r_i f_x. \quad (17)
\]

Rewrite the free entry condition (10) as

\[
(\phi_{ii})^{-k} f_d + (\phi_{ij})^{-k} f_x = \frac{k - (\sigma - 1)}{\sigma - 1} f_e,
\]

\[
(\phi_{jj})^{-k} f_d + (\phi_{ji})^{-k} f_x = \frac{k - (\sigma - 1)}{\sigma - 1} f_e. \quad (18)
\]
And by (2), then the aggregate price index is given by

\[ P_i = \frac{2\sigma - 1}{\sigma - 1} \left\{ \frac{k}{k - \sigma + 1} \left[ M_i^e (r_i b_i)^{1-\sigma} (\phi_{ii})^{\sigma - 1 - k} + M_j^e (\tau r_j b_j)^{1-\sigma} (\phi_{ij})^{\sigma - 1 - k} \right] \right\}^{\frac{1}{\sigma - 1}}. \]  

(19)

where \( M_i^e \) is the number of entrants in country \( i \) and \( M_{ii} = M_i^e [1 - G (\phi_{ii})] \) is the number of firms that survive in the domestic market. Next we try to pin down the number of entrants. In the equilibrium, capital rents are identical across countries, that is, \( r_i = r_j = r \). The aggregate income of country \( i \) is \( Y_i = (w_i \lambda_i + r) L_i \). By labor demand (6), and free entry condition (18), we can derive the total payment to workers as

\[ W_i = \sum_j [1 - G (\phi_{ij})] M_i^e \int w_i l_{ij} (\phi) \mu_{ij} (\phi) d\phi = kr M_i^e f_e, \]  

(20)

Combined with (7), the capital market equilibrium is

\[ K = f_e \sum_{i=1}^2 M_i^e + \sum_{i=1}^2 \left\{ M_i^e \left[ (\phi_{ii})^{-k} f_d + (\phi_{ij})^{-k} f_x \right] + \frac{W_i}{r} \right\} = \frac{\sigma}{\sigma - 1} k M^e f_e, \]  

(21)

Thus, we have the number of entrants as

\[ M^e = \frac{\sigma - 1}{\sigma} \frac{K}{k f_e}. \]  

(22)

The relationship between capital share and wage payment is \( r K (\sigma - 1) / \sigma = \sum_{i=1}^2 W_i \), then we can derive the capital payment as

\[ r = \frac{\sigma}{(\sigma - 1) K} \sum_{i=1}^2 (\lambda_i w_i L_i). \]

Rewriting the aggregate income as

\[ Y_i = (w_i \lambda_i + r) L_i = \left[ \frac{\lambda_i w_i (2\sigma - 1) L + \sigma L_j (\lambda_j w_j - \lambda_i w_i)}{(\sigma - 1) L} \right] L_i. \]  

(23)

And the number of firms in country \( i \) is decided by \( W_i = kr M_i^e f_e = w_i \lambda_i L_i \), and

\[ M_i^e = \frac{w_i \lambda_i L_i}{kr f_e}. \]  

(24)
Therefore the share of capital in country $i$ is given by
\[
\frac{M_{ii}}{M_{ii} + M_{jj}} = \frac{[1 - G(\phi_{ii})] M_i^e}{[1 - G(\phi_{ii})] M_i^e + [1 - G(\phi_{jj})] M_j^e} = \frac{\omega_i L_i}{\omega_i L_i + (\phi_{jj})^{-k} L_j}.
\]

Next we move to solve for the equilibrium expected wages. By (9), the relationship between different thresholds is
\[
\phi_{ij} = b \tau \left( \frac{f_x}{f_d} \right)^{\frac{1}{\sigma - 1}} \phi_{jj}, \quad \phi_{ji} = \frac{\tau}{b} \left( \frac{f_x}{f_d} \right)^{\frac{1}{\sigma - 1}} \phi_{ii},
\]
where $b = b_i/b_j$. Combine (25) with (18), then we get
\[
\frac{(\phi_{ii})^k}{(\phi_{jj})^k} = \frac{1 - (b)^k \Lambda}{1 - (b)^{-k} \Lambda}.
\]

where $\Lambda = (\tau)^{-k} (f_x/f_d)^{\frac{\sigma - 1 - k}{\sigma - 1}}$ as the measure of trade freeness. Set $\omega = \omega_i/\omega_j$, and combine the zero profit condition (9), aggregate price (19), number of firms (24) and (26), then we can derive the equilibrium condition for expected wage $\omega$ as
\[
F(\omega) \equiv f_0(\omega) + f_1(\omega)\Lambda + f_2(\omega)\Lambda^2 = 0,
\]
where
\[
\begin{align*}
    f_0(\omega) &= L_j L_i (\omega - 1), \\
    f_1(\omega) &= \sigma L \left[ L_j (T)^{-k} (\omega)^\gamma - L_i (T)^k (\omega)^{1 - \gamma} \right], \\
    f_2(\omega) &= L_i \omega (\sigma L - L_j) - L_j (\sigma L - L_i),
\end{align*}
\]

where $T = m (\nu)^{\gamma - 1}$ is the search technology of country $i$ relative to country $j$, and $m = m_i/m_j$, $\nu = \nu_i/\nu_j$, $i \neq j$. $m$ represents the relative search efficiency level between two countries, and $\nu$ is the relative cost of opening a vacancy. We set $\gamma k \geq 1$ and $\omega_j$ as numeraires, that is $\omega_j = 1$, then $(\nu_j)^{1 - \gamma} > m_j$ must hold. Then the unemployment rate of country $j$ is uniquely decided by the parameters $m_j$ and $\nu_j$. 

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3.3 Benchmark Case: Symmetric Labor Market Equilibrium

The two countries are the same in terms of search technology and posting cost, that is $\nu = m = 1$, but the population is asymmetric, $L_i > L_j$.

Lemma 2 (i) Equilibrium relative expected wage $\omega^* = 1$ for $\Lambda = 0, 1$,
(ii) There exists a solution $\omega^* > 1$ for $\Lambda \in (0, 1)$,
(iii) The relative wage $\omega$ is increasing when $\Lambda = 0$ and then decreasing as $\Lambda = 1$,

Proof. (i). Insert $\Lambda = 0, 1$ into (27), then the results are trivial.
(ii). $F(1) = \sigma \Lambda (1 - \Lambda) (L_j - L_i) < 0$ and $F'(\omega) > 0$ when $\omega \in \left[ 1, \left( \frac{L_j}{L_i} \right)^{\frac{1}{1-2\gamma}} \right]$, and another constraint $\omega < \Lambda^{-\frac{1}{2\gamma}}$ comes from (9). Then we know that there exists a $\omega^* > 1$.
(iii). Rewrite $F(\omega)$ as $F(\Lambda, \omega)$, and we know that

$$\frac{\partial F(\Lambda, \omega^*)}{\partial \Lambda} \bigg|_{\Lambda=0} = \sigma L (L_j - L_i) < 0, \quad \frac{\partial F(\Lambda, \omega^*)}{\partial \omega} \bigg|_{\Lambda=0} = L_j L_i > 0,$$

$$\frac{\partial F(\Lambda, \omega^*)}{\partial \Lambda} \bigg|_{\Lambda=1} = \sigma (L_i - L_j) L > 0, \quad \frac{\partial F(\Lambda, \omega^*)}{\partial \omega} \bigg|_{\Lambda=1} = \gamma k \sigma L^2 > 0,$$

then we can get

$$\frac{\partial \omega}{\partial \Lambda} \bigg|_{\Lambda=0} > 0 \text{ and } \frac{\partial \omega}{\partial \Lambda} \bigg|_{\Lambda=1} < 0.$$

Proposition 1 (1) When the search efficiency and the cost of opening a vacancy are the same, wage premium ($\omega > 1$) ensures a lower unemployment rate in the country with more population;

(2) The unemployment in country $i$ is decreasing first and then increasing after trade liberalization.

Proof. (1). By lemma 1 and lemma 2

$$\frac{\lambda_i}{\lambda_j} = \omega^{1-\gamma} > 1,$$
(2). By lemma 1 we know that the employment rate is increasing in expected wages, and with lemma (2) we prove that the wage is increasing when the trade freeness is low and increasing when free trade is achieved. Then we can infer that the unemployment rate is decreasing first and then increasing in the trade costs.

In this model, the capital share is increasing and then decreasing in country $j$ while the labor share is decreasing and then increasing in country $j$ as the trade freeness becomes larger. Because more population attracts more capital to set up firms in country $i$, it also raises the labor demand. Workers in country $i$ enjoy higher expected wages.

Next we want to see the impact of search technology $T$ on the equilibrium wage

$$\frac{\partial F(\omega^*, T)}{\partial T} = -k\sigma L \left[ L_j (T)^{-k-1} (\omega^*)^{-k} + L_i (T)^{-k-1} (\omega^*)^{1-k} \right] < 0,$$

thus

$$\frac{\partial \omega}{\partial T} > 0.$$

**Lemma 3** The relative expected wage is increasing in the relative search technology.

Better search technology increases the employment rate given wages, thus expected wage is increasing in search technology.

### 3.4 Asymmetric Equilibrium: Better Market Condition

Now we move to examine the asymmetric search technology case. Consider the condition that $T > 1$, that is, the search technology of country $i$ is better than that of country $j$ and $L_i < L_j$. Therefore, the home market effect may be dominated by search technology advantage. That is, the country with less population, which is traditionally called the "small country", could have higher expected wages than its counterpart.

**Lemma 4** Given $T > 1$, there exists a sufficient condition $\bar{L}_i = L_{ij} / (T)^{2k}$ such that $L_i \geq \bar{L}_i$, and the wage equation (27) has a unique solution $\omega^* > 1$. 

Proof. (1)

\[ F(1) = \sigma L \left[ (L_i - L_j) \Lambda + L_j (T)^{-k} - L_i (T)^k \right] \Lambda. \]

We want to find the condition under which \( F(1) < 0 \). When \( T > 1 \) and \( L_i > L_j \), this condition always holds. Now we examine whether the solution exists when \( L_i < L_j \) for all \( \Lambda \in (0, 1) \). By simple calculation, we find that when \( T > 1 \) and \( L_i \geq \bar{L}_i \), \( F(1) < 0 \) still holds. This means that a technological advantage could mitigate the numerical inferiority of a population.

**Proposition 2** The expected wage could be higher in the country with less population, that is \( L_i < L_j \), and the advantage of search technology could sustain the higher expected wage even when there is free trade.

Proposition 2 demonstrates that the home market effect may be dominated by search technology advantage. The country with less population, or "small country," could have higher expected wages than its counterpart. This result is different from that of "home market effects" in the literature which ignore the search technology in the labor market.

**Lemma 5** Equilibrium expected wage \( \omega^* = 1 \) when \( \Lambda = 0 \), and \( \omega^* = (T)^{\frac{1}{T}} > 1 \) depends on the relative posting cost and matching technology when \( \Lambda = 1 \).

Proof. By (27), when \( \Lambda = 1 \), the wage function can be rewritten as \( F(\omega) = \sigma L \left[ 1 - (T)^k (\omega)^{-\gamma^k} \right] \left[ (T)^{-k} (\omega)^\gamma L_j + \omega L_i \right] = 0 \), then the solution is derived from \( (T)^k (\omega)^{-\gamma^k} = 1 \). Rearranging the equality, we can get \( \omega = (T)^{\frac{1}{T}} \neq 1 \) if \( T \neq 1 \), which is decided by the search friction. ■

And the proposition follows up, that is,

**Proposition 3** The search technology can create a relative expected wage premium.
When there is free trade, better search technology in country $i$ ensures higher employment $\lambda = \lambda_i / \lambda_j = (T)^{1/2} > 1$, thus the expected wage is higher in country $i$, and the relative employment is only related to search technology. Under better search technology, the wage may not converge to the same level even when free trade is achieved, with lemmas 3 and 4,

$$\frac{\partial F(\Lambda, \omega^*)}{\partial \Lambda} \bigg|_{\Lambda=0} = \sigma L \left[ L_j (T)^{-k} - L_i (T)^k \right] < 0, \quad \frac{\partial F(\Lambda, \omega^*)}{\partial \omega} \bigg|_{\Lambda=0} = L_j L_i > 0,$$

$$\frac{\partial F(\Lambda, \omega^*)}{\partial \Lambda} \bigg|_{\Lambda=1} = \sigma L (L_i \omega - L_j) + 2L_j L_i (1 - \omega) \geq 0,$$

$$\frac{\partial F(\Lambda, \omega^*)}{\partial \omega} \bigg|_{\Lambda=1} = \sigma L \left( \frac{\gamma k L_j}{\omega} + \gamma k L_i \right) > 0.$$

and

$$\frac{\partial \omega}{\partial \Lambda} \bigg|_{\Lambda=0} > 0 \quad \text{and} \quad \frac{\partial \omega}{\partial \Lambda} \bigg|_{\Lambda=1} \geq 0.$$

After taking the relative search technology $T$ into consideration, expected wages may be still increasing even in free trade (i.e. $\Lambda = 1$), and it means that the employment may be still increasing in country $i$ comparing with the previous symmetric case - the wage and employment is decreasing in country $i$ in the trade cost when the trade freeness is high. We use following figure to demonstrate the results\(^3\).

![Figure 1: The effects of trade cost on expected wages.](image)

\(^3\)In Figure 1 we use the following parameters: $\{\sigma, \gamma, k, L_i/L\} = \{4, 0.25, 4, 0.45\}$. Simonovska and Waugh (2014) estimate the range of $k$ between 2.79 and 4.46, so it is reasonable that we set $k = 4$. Besides, Chen and Peng (2017) also adopt $\sigma = 4$. 

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Figure 1 reveals that the expected wage in small country ($L_i/L = 0.45 < 0.5$) is always lower when the search technology is symmetric ($T = 1$). By contrast, in the case of Takahashi et al. (2013), they find that the wage is always higher in the country with more population without considering the search technology and firm heterogeneity. If we consider the asymmetric search technology, the expected wage $\omega_i$ is increasing in trade freeness when the relative search technology is in median level ($T = 1.05$). This is because the employment rate $\lambda_i$ is increasing and the nominal wage $w_i$ paid by firms do not decrease too much. However, when the search technology is high enough ($T = 1.1$), firms do not have to attract labors by high nominal wage, thus the expected wage is decreasing even if the employment rate is increasing.

4 Models with Fair Wages Mechanism

For the sake of getting deep insight into the effects of trade liberalization on the unemployment issue, we consider another framework with fair wage proposed by Akerlof and Yellen (1990). In their specification, workers will only make an effort to work if the wages they receive are believed to be fair. And they prove firms will also pay the fair wages to extract the workers associated with a normal level of effort. According to Egger and Kreickemeier (2009), we assume that the fair wage is decided by two components, the productivity of a distinct firm, and the expected wage after entering the market, that is,

$$w_i(\phi) = \phi^{\theta} [(1 - U_i) \bar{w}_i]^{1-\theta},$$

(28)

where $1 - U_i$ is the employment rate in country $i$, $\bar{w}_i$ is the average wage of employed workers in country $i$, and $\theta \in (0, 1)$ is the rent sharing parameters. We should emphasize that $w(\tilde{\phi})$ is the wage paid by a firm with an average level of productivity $\tilde{\phi}$; it may not be equal to $\bar{w}$. With the same utility function (1) as in the previous part, the demand function is the same as (3), and the price from country $i$ to country $j$ is,

$$p_{ij}(\phi) = \frac{\sigma}{\sigma - 1} \frac{\tau_{ij}w_i(\phi)}{\phi}.$$
with \( \tau_{ij} = \tau_{ji} = \tau > 1 \) being the iceberg trade costs. Then we can derive for the firms revenue
\[
R_{ij}(\phi) = Y_j P_j^{\sigma - 1} \left( \frac{\sigma - 1}{\sigma} \right)^{(\sigma - 1)} (\tau_{ij})^{1-\sigma} [(1 - U_i) \overline{w}_i]^{-\eta} \phi^\eta,
\]
and profit as
\[
\pi_{ij}(\phi) = \frac{1}{\sigma} \left( \frac{\sigma - 1}{\sigma} \right)^{(\sigma - 1)} Y_j P_j^{\sigma - 1} (\tau_{ij})^{1-\sigma} [(1 - U_i) \overline{w}_i]^{-\eta} \phi^\eta - r_i f_{ij}.
\]
Define \((1 - \theta)(\sigma - 1) = \eta\).

In order to investigate the properties of the equilibrium expected wages, we derive the zero profit condition
\[
(\phi_{ij})^{(1-\theta)(\sigma-1)} = \left( \frac{\sigma}{\sigma - 1} \right)^{(\sigma - 1)} \frac{\sigma r_i f_{ij} [(1 - U_i) \overline{w}_i]^\eta}{Y_j P_j^{\sigma - 1} (\tau_{ij})^{1-\sigma}}.
\]
where \( f_{ij} = f_d \) if \( i = j \), \( f_{ij} = f_x \) if \( i \neq j \); \( \tau_{ij} = \tau \) if \( i \neq j \) and \( \tau_{ij} = 1 \) if \( i = j \). Follow (11), the modified average productivity is
\[
\hat{\phi}_{ji} = \left( \frac{k}{k - \eta} \right)^{(1-\theta)\sigma} \phi_{ji},
\]
and the free entry condition is the same as (18). By (2), then the aggregate price index is
\[
P_i = \frac{\sigma}{\sigma - 1} \left\{ \frac{k}{k - \eta} \left[ M_i^\sigma (\phi_{ii})^{\eta - k} [(1 - U_i) \overline{w}_i]^{-\eta} + M_i^{\sigma - 1} (\phi_{ji})^{\eta - k} [(1 - U_j) \overline{w}_j]^{-\eta} \right] \right\}^{\frac{1}{1-\sigma}}.
\]
And combining the free entry condition with the capital clearing condition,
\[
K = f_e \sum_{i=1}^2 M_i^e + \sum_{i=1}^2 \left\{ M_i^e \left[ (\phi_{ii})^{-k} f_d + (\phi_{ij})^{-k} f_x \right] \right\}.
\]
Thus, we can get the total number of firms \( M^e = \eta K/kf_e \). And we know that the total payment to workers accounts for a \((\sigma - 1) / \sigma \) share of aggregate income \( \sum_i Y_i \), that is, \((\sigma - 1) K r = \sum_i (1 - U_i) \overline{w}_i L_i \), then we can derive the equilibrium capital rent as
\[
r = \frac{\sum_i (1 - U_i) \overline{w}_i L_i}{(\sigma - 1) K}.
\]
in turn, the aggregate income of country $i$ is
\[ Y_i = \sigma \left[ (1-U_i) \bar{w}_i \right] K + \left[ (1-U_j) \bar{w}_j - (1-U_i) \bar{w}_i \right] L_j L_i \frac{(\sigma - 1)}{K}. \]

By the same token, the number of firms in country $i$ is $M_i^e = \eta \left( 1-U_i \right) \bar{w}_i L_i / \left[ (\sigma - 1) \cdot r_{f,k} \right]$. Considering the zero profit condition and free entry condition, we get the ratio between two domestic thresholds as
\[ \frac{(\phi_{ii})^k}{(\phi_{jj})^k} = \frac{1 - (\bar{w})^k \Delta}{1 - (\bar{w})^{-k} \Delta}. \] (29)

where $\Delta = (\tau) \frac{-k(\sigma - 1)}{\eta} \left( f_x / f_d \right)^{\eta - k}$ as the measure of trade freeness. Setting $\bar{w} = (1-U_i) \bar{w}_i / (1-U_j) \bar{w}_j$, $i \neq j$ and combining the zero profit condition (9), aggregate price (19), number of firms (24) and (29),

\[ (f_x / f_d)^{\eta - k} \]

\[ F^{RW} (\bar{w}) \equiv f_0(\omega) + f_1(\omega) \Delta^{FW} + f_2(\bar{w}) \left( \Delta^{FW} \right)^2 = 0, \] (30)

where
\[ f_0^{RW} (\bar{w}) = L_j L_i (\bar{w} - 1), \]
\[ f_1^{RW} (\bar{w}) = \sigma L \left( L_j \bar{w}^k - L_i \bar{w}^{1-k} \right), \]
\[ f_2^{RW} (\bar{w}) = L_i \bar{w} (\sigma L - L_j) - L_j (\sigma L - L_i). \]

By the proof of the previous part, we know there exists an equilibrium expected wage $\bar{w}^* > 1$ that satisfies the wage equation. With (21) and $\phi_{ji} / \phi_{ij} = \phi_{ii} / \bar{w}^2 \phi_{jj}$, we immediately know that $(\phi_{ii}) < (\phi_{jj}) < (\phi_{ji}) < (\phi_{ij})$.

Then we obtain the employment rate as
\[ 1 - U_i = \frac{\eta}{k - \eta + \theta} \left( \frac{(\bar{w})^{1-\theta}}{f_x (\phi_{ii})^{-(\theta + k)} + f_d (\phi_{ij})^{-(\theta + k)}} \right). \]

Lemma 6  (1) $d\phi_{ii} / d\Delta > 0$,
(2) $d\phi_{ij} / d\Delta < 0$,
(3) $d\bar{w} / d\Delta < 0$ when $\Delta = 1$. 

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Proof. (1) By the free entry condition we know
\[
\frac{k - \eta f_e}{\eta f_d} = (\phi_{ii})^{-k} \frac{1 - \Delta^2}{1 - \tilde{w}^{-k} \Delta},
\]
and we can get another equation by the zero profit condition, □
\[
(\phi_{ii})^k = \frac{K f_d (\tilde{w}) L_i + L_j \Delta (\tilde{w})^k \sigma \eta}{L_i f_e \sigma (\tilde{w}) K + L_j - (\tilde{w}) L_j k - \eta}.
\]
This equilibrium threshold is decided by the intersection of these two curves. We can draw these two curves on the $\phi_{ii}$-$\tilde{w}$ plane. These two curves increase in the trade cost, thus we know that the equilibrium $\phi_{ii}$ increases in $\Delta$, then we get $d\phi_{ii}/d\Delta > 0$.

Proof. (2) By the free entry condition we know that
\[
d\ln \phi_{ii} = -\frac{(\phi_{ij})^{-k} f_x}{(\phi_{ii})^{-k} f_d} d\ln \phi_{ij},
\]
and we immediately get
\[
\text{sign} \left\{ \frac{d \ln \phi_{ij}}{d \Delta} \right\} = \text{sign} \left\{ -\frac{d \ln \phi_{ii}}{d \Delta} \right\} < 0.
\]
(3) Rewrite $F_{RW}(\tilde{w})$ as $F_{RW}(\Delta, \tilde{w})$, and we know that
\[
\left. \frac{\partial F_{RW}(\Delta, 1)}{\partial \Delta} \right|_{\Lambda=1} = \sigma (L_i - L_j) L > 0,
\]
\[
\left. \frac{\partial F_{RW}(\Delta, 1)}{\partial \tilde{w}} \right|_{\Lambda=1} = \sigma L [L_j k + L_i (k - 1) + 1] > 0,
\]
then we can get
\[
\frac{\partial \omega}{\partial \Delta} \bigg|_{\Delta=0} > 0 \quad \text{and} \quad \frac{\partial \omega}{\partial \Delta} \bigg|_{\Delta=1} < 0.
\]
□

With these lemmas then the proposition comes up.

**Proposition 4** Employment rate is decreasing (unemployment rate is increasing) in trade freeness.
Proof.

\[
\frac{d (1 - U_i)}{d \Delta} \bigg|_{\Delta \to 1} = (1 - \theta)(\tilde{w})^{-\theta} \frac{d \tilde{w}}{d \Delta} \left[ f_d(\phi_{ii})^{-(\theta+k)} + f_x(\phi_{ij})^{-(\theta+k)} \right] \\
+ (\theta + k)(\tilde{w})^{1-\theta} f_x(\phi_{ij})^{\theta} - (\phi_{ii})^{\theta} (\phi_{ij})^{-(k+1+\theta)} \frac{d \phi_{ij}}{d \Delta} < 0.
\]

(31)

When approaching free trade, it lowers the export thresholds for marginal firms, and more firms engage in the foreign market; those firms hire more. And lower relative wages also increases the labor demand. However, domestic firms with lower productivity are forced out of the market. Besides, because of the lower relative wages, the price of goods also decreases, and the number of entrants in country \( i \) also decreases. Equation (31) indicates that the unemployment rate increases. ■

The result is similar as the Proposition 1 in symmetric search technology. When the trade freeness is high, then unemployment is increasing in trade freeness. However, in the asymmetric search fiction, the unemployment rate may be decreasing in trade freeness because of the better search technology. Some papers find that the unemployment rate is increasing after trade liberalization (e.g., Egger and Kreickermier, 2009 & 2012, Dinopoulos and Unel, 2017), and others find that the effects of the opening to trade on unemployment is ambiguous (Helpman et al., 2010).

5 Comparisons and Welfare Analysis

In this section, we derive the welfare equations (welfare per capita) under different model settings and then compare the welfare levels with the specification of unemployment. Defining \( E_i = \Sigma_j E_{ij} \) as the total expenditures of country \( i \), and by aggregate accounting we know that \( E_i = Y_i \). Besides, denoting \( X_{ij} = E_{ij}/E_j \) as country \( j \)'s expenditure share on country \( i \). First, by the mass of firms, expected domestic revenue
and aggregate price level, we calculate the expenditure share with expenditure as

$$X_{ii}^{SF} = \frac{\omega_i L_i}{\omega_i L_i + \omega_j L_j (b)^k \tau^{-k} \left( \frac{f_x}{f_d} \right)^{\frac{\sigma - 1}{\sigma - 1}}},$$  \hspace{1cm} (32)$$

where the superscript "SF" stands for the value obtained from the search friction model. Follow ACR’s formula, we can get the partial elasticity of trade with respect to trade variable cost \(\tau\) conditional on the expected wage as

$$\varepsilon^{SF} = \frac{\partial \ln E_{ji}^{SF}/E_{ji}^{SF}}{\partial \ln \tau} = k.$$  \hspace{1cm} (33)$$

The elasticity is exactly the same as the Melitz model without unemployment, but the welfare channel is totally different. Next, by the zero profit condition, the aggregate price level in country \(i\) is

$$P_i^{SF} = \frac{2\sigma - 1}{\sigma - 1} r^{SF} b_i \left[ (2\sigma - 1) r^{SF} f_d \right]^{\frac{1}{\sigma - 1}},$$  \hspace{1cm} (34)$$

Combining with the aggregate price level, we can derive the total domestic expenditure \(E_{ii}\). With \(E_{i1}^{SC} = X_{i1}^{SC} E_{i1}^{SC}\), rewriting the domestic production threshold as

$$\phi_{ii}^{SF} = \left[ (2\sigma - 1) k \cdot \frac{\lambda_i L_i}{k - \sigma + 1} \left( \frac{X_{i1}^{SC} E_{i1}^{SC}}{k f_e} \right) \right]^{\frac{1}{k}},$$

With the production threshold and aggregate price (34), the welfare per capita \((\omega_i + \tau)/P_i^{SF}\) in country \(i\) is

$$V_i^{SF} = \Psi^{SF} (L_i)^{\frac{1}{\sigma-1}} \left( 1 + \frac{\psi^{SF}}{\omega_i} \right)^{\frac{\lambda_i (X_{ii}^{SF})^{-\frac{1}{\sigma}}}{\psi^{SF}}} \lambda_i \left( \frac{\omega_i}{r^{SF}} \right)^{\frac{1}{\sigma-1}} \frac{1}{k}.$$  \hspace{1cm} (35)$$

where \(\Psi^{SF} = [(\sigma - 1)/(2\sigma - 1)] [(k - \sigma + 1) f_e]^{-\frac{1}{k}} [(2\sigma - 1) f_d]^{\frac{\sigma - 1 - k}{\sigma (\sigma - 1)}}\). By the same token, we can get the expenditure shares from the fair wage framework as follow:

$$X_{ii}^{FW} = \frac{\tilde{w}_i L_i}{\tilde{w}_i L_i + \tilde{w}_j L_j (\tilde{w})^{k} \tau^{-k} \left( \frac{f_x}{f_d} \right)^{\frac{\sigma - 1}{\sigma - 1}}},$$  \hspace{1cm} (36)$$

where the superscript "FW" means the fair wage setting, and the partial trade elasticity is

$$\varepsilon^{FW} = \frac{\partial \ln E_{ij}^{FW}/E_{jj}^{FW}}{\partial \ln \tau} = k \frac{1}{1 - \theta}.$$
The elasticity is larger in the fair wage framework, that is, $\varepsilon^{FW} > \varepsilon^{SF}$. It means that change in $\tau$ will generate larger changes in both extensive and intensive margins. Next, we can derive the per capita welfare in country $i$ as

$$V^{FW}_i = \Psi^{FW}(L_i)^{(\sigma-1)\frac{1}{\sigma-1}} \left(1 + \frac{r^{FW}}{w_i} \right)^{\frac{k\sigma-\eta}{(\sigma-1)\eta}} \left(\frac{\tilde{w}_i}{r^{FW}}\right)^{\frac{1}{(\sigma-1)}} (X^{FW}_{ii})^{\frac{\eta}{k(\sigma-1)}}. \quad (37)$$

where $\Psi^{FW} = (\sigma - 1/\sigma)(1 - \theta)^{\frac{\eta}{(\sigma-1)}} [(k - \eta) f_e, \overline{ \omega (\sigma-1)} (\sigma f_d)^{\frac{\eta-k}{k(\sigma-1)}}]$. Finally, in order to compare the welfare between the different settings, then we calculate the expenditure share $X^{FE}_{ii}$ without unemployment as

$$X^{FE}_{ii} = \frac{w_i^{FE} L_i}{w_i^{FE} L_i + w_j^{FE} L_j (w_j^{FE})^k (\tau)^{-k} \left(\frac{f_x}{f_d}\right)^{\sigma-1} X^{FE}_{ii}} \bigg\rvert_{\sigma-1-k}. \quad (38)$$

where the superscript "FE" means full-employment. The partial trade elasticity $\varepsilon^{FE} = k$ is equal to the search friction case. Let $V^{FE}_i$ be the welfare per capita with perfect labor market

$$V^{FE}_i = \Psi^{FE}(L_i)^{(\sigma-1)\frac{1}{\sigma-1}} \left(1 + \frac{r^{FE}}{w^{FE}_i} \right)^{\frac{k\sigma-\eta+1}{(\sigma-1)(\sigma-1)}} \left(\frac{w_i^{FE}}{r^{FE}}\right)^{\frac{1}{(\sigma-1)}} (X^{FE}_{ii})^{-\frac{1}{k}}. \quad (39)$$

where $\Psi^{FE} = (\sigma - 1/\sigma) [(k - \sigma + 1) f_e, \frac{1}{(\sigma f_d)^{\sigma-1}}]$. The magnitude of three domestic expenditure shares depends on the parameter set $\{L_i, L, \sigma, k, \gamma, f_x, f_d, f_e, m_1, m_2, \nu_1, \nu_2\}$ and the trade cost $\tau$. Next we try to compare these domestic expenditure shares to get some economic implications. By (38) and (32), we obtain two equations as

$$\text{sign} \left\{ X^{FE}_{ii} - X^{SF}_{ii} \right\} = \text{sign} \left\{ (T)^{-k} (\omega)^{\gamma k-1} - (w^{FE})^{k-1} \right\},$$

$$\text{sign} \left\{ X^{FW}_{ii} - X^{SF}_{ii} \right\} = \text{sign} \left\{ (T)^{-k} (\omega)^{\gamma k-1} \left(\frac{f_x}{f_d}\right)^{\frac{\eta-k}{(\sigma-1)}} - (\tilde{w})^{k-1} \right\},$$

where both $\omega$ and $w^{FE} > 1$. We can infer that when the relative trade cost $T$ is large enough and $\gamma k$ is small enough, $X^{SF}_{ii}$ may be the largest expenditure share. This is because that the number of firms in the country with higher search technology increases,
the domestic expenditure share will become larger. Next, we summarize the welfare of country $i$ under different setting as follow:

$$V_{i}^{SF} = \Psi_{i}^{SF} \left( L_{i} \right)^{\frac{1}{\sigma - 1}} \times \Phi_{i}^{SF} \times \lambda_{i} \times \left( X_{ii}^{SF} \right)^{-\frac{1}{\sigma}},$$

$$V_{i}^{FW} = \Psi_{i}^{FW} \left( L_{i} \right)^{\frac{1}{\sigma - 1}} \times \Phi_{i}^{FW} \times \left( \tilde{w}_{i} \right)^{\theta} \times \left( X_{ii}^{FW} \right)^{-\frac{1}{\sigma - 1}},$$

$$V_{i}^{FE} = \Psi_{i}^{FE} \left( L_{i} \right)^{\frac{1}{\sigma - 1}} \times \Phi_{i}^{FE} \times \left( X_{ii}^{FE} \right)^{-\frac{1}{\sigma}}. \quad (40)$$

where

$$\Phi_{i}^{SF} = \left( 1 + \frac{r_{i}^{SF}}{\omega_{i}} \right)^{\frac{k(\sigma - 1)}{(\sigma - 1)k}} \times \left( \frac{\omega_{i}}{r_{i}^{SF}} \right)^{\frac{1}{\sigma - 1}},$$

$$\Phi_{i}^{FW} = \left( 1 + \frac{r_{i}^{FW}}{\tilde{w}_{i}} \right)^{\frac{k(\sigma - 1)}{(\sigma - 1)k}} \times \left( \frac{\tilde{w}_{i}}{r_{i}^{FW}} \right)^{\frac{1}{\sigma - 1}},$$

$$\Phi_{i}^{FE} = \left( 1 + \frac{r_{i}^{FE}}{w_{i}^{FE}} \right)^{\frac{k(\sigma - 1)}{(\sigma - 1)k}} \times \left( \frac{w_{i}^{FE}}{r_{i}^{FE}} \right)^{\frac{1}{\sigma - 1}}.$$

$\Phi_{i}^{SF}$, $\Phi_{i}^{FW}$ and $\Phi_{i}^{FE}$ stands for extra effects of relaxing the trade balance condition of ACR. In the footloose capital framework, the relative factor price is important in determining welfare level. Besides, employment rate $\lambda_{i}$ directly affects the welfare level $V_{i}^{SF}$. Higher employment rate implies higher welfare condition on the wage, capital rent and domestic expenditure share. Moreover, the extra term $\left( \tilde{w}_{i} \right)^{\theta}$ in $V_{i}^{FW}$ reflects the weight of workers. If $\theta$ is higher, then workers’ wages is much more firm-specific, that is, firms with higher productivity pay more to their homogeneous workers (irrelevant to the worker’s ability).

In the final step, we use simulation to compare the magnitude of welfare function. The horizontal line of Figure 2 is the trade freeness and the vertical line denotes the welfare level under different frameworks. When $\theta \rightarrow 0$, then firms pay identical wages to their workers as in Melitz (2003), the wage function (30) in fair wage setting is reduced to the full employment setting, thus we know that equilibrium expected wage in fair wage setting is equal to wage in full employment setting. Besides, expenditure shares

\[ \{L_{i}, L, \sigma, \theta, \gamma, f_{x}, f_{d}, f_{e}, m_{1}, m_{2}, \nu_{1}, \nu_{2}\} = \{6, 10, 4, 4, 0.5, 2.5, 2, 1, 1.2, 1, 1.2, 1\}. \]
and welfare level are the same in these two setting. So the relationship between welfare is $V_{FE}^i = V_{FW}^i > V_{SF}^i$. By observing the figures, there are some interesting facts. First, the welfare level is lower in either one scheme with unemployment comparing with that of full employment framework. Second, we can discover that the welfare in fair wage setting is decreasing in $\theta$, that is, when the rent sharing weight of labor becomes larger, the welfare goes down conditional on the the trade freeness. The last one is that the welfare is increasing in trade freeness, that is, opening to trade improves the welfare level even if we consider the unemployment.

![Diagram](image)

Figure 2: Welfare level w.r.t. $\theta$.

6 Conclusion

Unemployment is an critical issue that have not been fully discussed in the trade literature, in order to get further insights into this topic, we investigate unemployment issue through two different frameworks under footloose capital model: one is the search
friction mechanism and the other is the fair wage system. In the first scenario, we have proved that a country with less population can have higher expected wage if it is equipped with higher search technology. And the search technology can result in the higher expected wage in the smaller country when there is free trade. Progress in search technology increases the expected wage; however, employment also increase and it damages the nominal wages worker receive. Besides, the expected wage is increasing and then decreasing as the trade freeness increases. However, if we consider the search technology advantage, the expected wage may not be decreasing when there is free trade. We also find that the unemployment rate increases when free trade is achieved in both scenarios.

In the final part, we rely on numerical simulation to compare the welfare level between different frameworks. The welfare is the lowest in the search friction scenario and the welfare in full employment setting is the highest. Better search technology allows firm with lower productivity to survive in domestic market, hence the domestic expenditure share is also the highest. Besides, the unemployment further damages the welfare. It is also important to implement the trade data to test the validity of these models. In the further study, we will try to estimate the effects of unemployment and evaluate the welfare gains from trade.
References


