Could competition always raise the risk of bank failure?*

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Abstract

The paper examines the relation between the intensity of competition in the loan market and the risk of bank failure, in the context of a simple model where the interplay of systemic and idiosyncratic risks create, through adverse selection and the resulting risk-shifting effect, conditions for non-monotonicity. A distinctive trait of our analysis is the importance given to bank competition proper: the conventional competition-fragility view may be challenged, essentially however at high levels of the loan interest rate, too high to be observable in competitive equilibrium situations. Also, it is shown that, under liquidity shortness, competition in the loan market might principally affect deposit rates, without touching the borrowers’ behavior responsible for the risk shifting effect. Finally, additional scale effects are introduced, if banks’ operating costs net of the income from non-intermediation activities are taken into account which, if negative (resp. positive), strengthen (resp. weaken) the conventional view. However, the randomness of these costs always reinforce this view.

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1 Introduction

The conventional wisdom concerning the relation between bank competition and bank stability is that more of the former undermines the latter. Erosion of market power, with the squeeze of profit margins, reduces banks’ charter values, hence the penalty for failure, inducing riskier choices (Marcus 1984, Keeley 1990, Acharya 1996). Boyd and De Nicoló (2005) challenged this view by taking into account the borrowers’ standpoint. They claim that higher loan rates directly imply a higher risk of borrowers’ bankruptcy and, by moral hazard, further create an incentive for borrowers to make riskier choices, an effect already highlighted by Stiglitz and Weiss (1981). With two opposite effects, the margin and the risk shifting effects, one may wonder whether one of them tends to dominate the other, hence whether the relationship between intensity of competition and risk of bank failure is increasing, decreasing or non-monotonic. Martinez-Miera and Repullo (2010), looking for the relationship between the number of banks (which under Cournot competition is a determinant of the loan interest rate) and the probability of bank failure, find that this relationship is U-shaped.\footnote{For further relevant references on the relation between bank competition and bank stability, see the surveys of the related theoretical and empirical literatures in Berger et al. (2009), Schaeck et al. (2009), Fungáčová and Weill (2013), Jimenez et al. (2013) and Jiang et al. (2017).}

Our purpose is to revisit this debate by extending the framework of the analysis in order to give a more prominent place to competition. In most of the literature devoted to the influence of competition on the risk of bank failure, tougher competition is reduced to the resulting lower loan rates or higher deposit rates, depending on the market in which banks compete. De Nicoló and Lucchetta (2009), for instance, take directly the exogenous deposit interest rate as an index of the bank market power, refraining "from modeling any specific bargaining game generating certain levels of rents" (p.7). In any case, endogenizing interest rates by just making them depend upon the number of banks competing in Cournot markets does not add much to the analysis. By contrast, we want to place bank competition at the core of the analysis. More competition may admittedly result from entry in the banking sector, but also from a switch to tougher regimes of competition, as for example when moving from Cournot to Bertrand competition. The diversity of dimensions of competition, structure (concentration, contestability, diversification, scale economies) but also conduct, has not been stressed enough in
the theoretical literature devoted to the relation between bank competition and bank stability.

The hesitation observed in the empirical literature between alternative ways of measuring the intensity of competition, by referring to Lerner indices and indices of conduct versus concentration indices (see e.g. Bikker and Haaf 2002) is an expression of that diversity. Although no consensus has emerged from empirical studies concerning the impact of bank competition on bank stability, evidence in favor of the competition-fragility view is provided by empirical works using either the Lerner index, like Berger et al. (2009), Turk-Ariss (2010), Fungáková and Weill (2013), Jimenez et al. (2013) or the non-structural H-statistic, like Schaeck et al. (2009). These measures, although not ideal, are certainly more sophisticated than mere concentration indices (e.g. the number of banks, the share of assets held by the k largest banks, or the Herfindahl index).²

Although adding conduct to concentration in symmetric markets, our modeling approach is otherwise close to the one adopted by Boyd and De Nicoló (2005) and Martinez-Miera and Repullo (2010) to challenge the conventional competition-fragility view. The main difference between our work and Boyd and De Nicoló (2005) concerns the very nature of bank failure, identified by them with a representative entrepreneur’s default, since risks are perfectly correlated across the whole set of identical entrepreneurs, a context in which either no entrepreneurial project fails or all projects fail, with complementary probabilities. Then the risk of bank failure unambiguously decreases under more intense competition, leading to lower loan interest rates and, through the risk shifting effect, to less risky entrepreneurial projects chosen under moral hazard.

Still referring to a representative entrepreneurial project, with success probability decreasing in the loan interest rate, Martinez-Miera and Repullo (2010) concentrate on situations where project failures are only partially correlated, according to a uniform parameter ρ taking any value between zero and one. With perfectly correlated risks (ρ = 1), they retrieve the Boyd and De Nicoló result, and with uncorrelated risks (ρ = 0), any risk of bank default vanishes by portfolio diversification. In the intermediate cases where systemic and idiosyncratic risks coexist, the favorable effect on bank stabil-

²Schaeck et al. (2009) find that "competition and concentration capture different characteristics of banking systems, meaning that concentration is an inappropriate proxy for competition" (p. 711). Also, "contestability appears to be more important than market structure in explaining the strength of competition in banking" (OECD 2010, p.21).
ility of stronger competition working through borrowers’ decisions (the risk shifting effect) combines with an unfavorable effect working through lower profit margins (the margin effect). Contrary to the competition-stability view, Martinez-Miera and Repullo show that the margin effect always dominates for a large enough number of banks (Proposition 2). However, they also show by a numerical simulation that the risk shifting effect dominates for a number of banks lower than a ceiling which increases with \( \rho \), so that intensifying competition is favorable to bank stability, at least in highly concentrated loan markets and when borrowers’ risks are highly correlated.

In this paper, we use a simple partial equilibrium model in which a discrete number of identical banks compete more or less softly in the loan market, with a conduct spreading from Cournot to Bertrand. They grant loans to a continuum of heterogeneous entrepreneurs, whose investment projects are subject to both systemic and idiosyncratic risks. While Martinez-Miera and Repullo (2010) represented the two risk factors by standard normal random variables, we assume instead that they are uniformly distributed. We concentrate, to begin with, on the interplay of the opposing margin and risk shifting effects. Our main message is that the risk shifting effect can indeed dominate the margin effect, but only at high levels of the loan interest rate, possibly higher than its collusive value, hence unenforceable as (oligopolistic) competitive equilibrium strategies – an observation which supports the conventional view. We nonetheless illustrate by a computed example the possibility of a stabilizing effect of more intense competition at observable levels of the loan interest rate. We also show that this possibility requires the risk shifting effect to be not only strong but also increasing with systemic risk.

We then take into account the deposit market, where banks, behaving now as perfect competitors, face a continuum of depositors. We show that the possibility of liquidity shortness reinforces the conventional competition-fragility view. Finally, we also take into account the existence for any bank of an operating cost and of an income generated by non-intermediation activities. We thus obtain a net operating cost, possibly negative, that we

\[ \text{\footnotesize 3They further emphasize the weakness of the risk shifting effect, its modulus being constrained from above by the condition that the inverse loan demand should be decreasing.} \]

\[ \text{\footnotesize 4In our framework, with heterogeneous borrowers, the risk shifting effect works through adverse selection, rather than through moral hazard as in Boyd and De Nicoló (2005) or in Martinez-Miera and Repullo (2010), where borrowers are homogeneous.} \]

\[ \text{\footnotesize 5Indeed, bank activities do not reduce to deposit-loan intermediation and are not exclu-} \]
treat as exogenous and stochastic. Although several papers mention the importance of economies of scale in banking, this is, to our knowledge, the first paper to explicitly add the influence of net operating costs to that of the well identified margin and risk shifting effects, when discussing the relation between the intensity of competition and the risk of bank default. Operating costs create the possibility of bank default, even in the context of pure idiosyncratic borrowers’ risks and broad portfolio diversification. Indeed, although the risk of banks’ default certainly depends on the overall riskiness of their more or less diversified loan portfolios, with good loans providing a buffer against bad ones, it also depends on the capacity for these portfolios to generate an aggregate profit covering losses which originate in operating costs and non-intermediation risky activities. Moreover, net operating costs induce scale economies (if positive) or diseconomies (if negative) in the basic banking activity, an effect which adds to the risk shifting effect or to the margin effect, respectively. Their randomness reinforces in any case the conventional competition-fragility view.

The rest of the paper is organized as follows. In section 2, we present the entrepreneurial sector, analyze its demand for loans, and examine how adverse selection influences loan riskiness. In section 3, we present the banking sector, and analyze the relation between competition for loans and the risk of bank failure, the main question addressed by this paper. In section 4, we briefly consider the influence of bank competition in the deposit market. We examine the role of operating costs and of income from non-intermediation banking activities in section 5. Section 6 concludes.

2 Entrepreneurs

There is a continuum of unit mass of entrepreneurs, each entrepreneur $i \in [0,1]$ being endowed with an effort capacity and with an investment project characterized by an index $x_i$ of productivity and an index $\pi_i$ of safeness. This project can be operated in period 0 at a fixed scale (normalized to one) and with an effort $e \in \{0,1\}$, so as to yield in period 1 the output $e + x_i$ if it succeeds or the output $(1 - \gamma)e$ (with a percentage loss $\gamma \in [0,1]$) if it fails. More productive projects are riskier, and we assume, like Stiglitz and

sively fund-based, but extend to several stakeholder and non-traditional fee-based activities (e.g., asset management, securities brokerage, M&A advising). See DeYoung and Torna (2013).
Weiss (1981), that the set of projects satisfies the mean preserving property: 
\[ \pi_i x_i = g \leq 1, \text{ for any } i. \]

2.1 Investment risk

Entrepreneur \( i \)'s project succeeds if \( z_i \equiv (1 - \rho) s_i + \rho S \leq \pi_i \), where \( s_i \) and \( S \) are independent random variables uniformly distributed over \([0, 1]\), describing \textit{idiosyncratic risk} and \textit{systemic risk}, respectively, and where \( \rho \in [0, 1] \) is a measure of the correlation between project failures.

The random variable \( z_i \) is the sum of two independent random variables \((1 - \rho) s_i \) and \( \rho S \), uniformly distributed over \([0, 1] \) and \([0, \rho] \) respectively, so that all the \( z_i \)'s follow the same trapezoidal distribution, with cumulative distribution function\(^6\)

\[
F(z, \rho) = \begin{cases} 
\frac{z^2}{2\rho(1-\rho)} & \text{if } 0 \leq z \leq \min(\rho, 1-\rho) \\
\frac{z - \min(\rho, 1-\rho)/2}{\max(\rho, 1-\rho)} & \text{if } \min(\rho, 1-\rho) \leq z \leq \max(\rho, 1-\rho) \\
1 - \frac{(1-z)^2}{2\rho(1-\rho)} & \text{if } \max(\rho, 1-\rho) \leq z \leq 1 
\end{cases}
\] (1)

and probability density function

\[
f(z, \rho) = \begin{cases} 
\frac{z}{\rho(1-\rho)} & \text{if } 0 \leq z \leq \min(\rho, 1-\rho) \\
\frac{z - \min(\rho, 1-\rho)/2}{\max(\rho, 1-\rho)} & \text{if } \min(\rho, 1-\rho) \leq z \leq \max(\rho, 1-\rho) \\
\frac{1-z}{\rho(1-\rho)} & \text{if } \max(\rho, 1-\rho) \leq z \leq 1
\end{cases}
\] (2)

Notice that the density function \( f(\cdot, \rho) \) is continuous, so that the cumulative distribution function \( F(\cdot, \rho) \) is differentiable. Notice also that the function \( F(z, \cdot) \) is symmetric with respect to 0.5: \( F(z, 1-\rho) = F(z, \rho) \).

2.2 Investment decision

Having no initial capital endowment, the entrepreneur must borrow one unit of capital, at date 0, in order to operate her project. After obtaining a loan from a bank at a non-negative \textit{loan interest rate} \( R \), she owes to that

\(^6\)In order to check the expression for the cumulative distribution function \( F(z, \rho) \), consider the rectangle \( \rho \times (1-\rho) \) in the space \((\rho S, (1-\rho) s) \). Then, as all the points in this rectangle are equiprobable, just compute the areas of the regions defined by \((1-\rho) s + \rho S \leq z \) relative to the total area \( \rho (1-\rho) \) of the rectangle.
bank, at date 1, the principal 1 plus the interest R. However, by limited liability, she will actually pay \(\min\{1 + R, e + x\}\) if the project succeeds, \(\min\{1 + R, (1 - \gamma) e\}\) otherwise. Consequently, the debt is only partially recovered by the bank, in case of entrepreneur’s default.

We assume the same marginal disutility of effort \(v \in (0, g]\) for all entrepreneurs, and a uniform distribution of the characteristic \(\pi\) of the different projects over the interval \([0, 1]\). The entrepreneur’s expected utility is

\[
U(e, R) = F(\pi, \rho) \max\{e + x - (1 + R), 0\} \quad \text{(3)}
\]

\[+(1 - F(\pi, \rho)) \max\{(1 - \gamma)e - (1 + R), 0\} - v e.\]

Clearly, she chooses to make an effort only if the resulting expected utility is larger than the one obtained with no effort, which imposes the following incentive compatibility constraint on loans: \(U(1, R) \geq U(0, R) = (F(\pi, \rho)/\pi)\max\{g - \pi(1 + R), 0\}\). Besides, no loan will be accepted by the entrepreneur unless the higher expected utility (the one with \(e = 1\)) is non-negative: \(U(1, R) \geq 0\). This results in a participation constraint on loans, which is however implied by the incentive compatibility constraint. We must consequently take into account the unique condition, ensuring participation with effort, that the marginal disutility of effort \(v\) be at most equal to the marginal (gross) expected utility of effort

\[
\frac{F(\pi, \rho)}{\pi} (\max\{g - \pi R, 0\} - \max\{g - \pi(1 + R), 0\}) \quad \text{(4)}
\]

\[
= \begin{cases} 
F(\pi, \rho) & \text{if } \pi \leq \frac{g}{1+R} \\
\frac{F(\pi, \rho)}{\pi} (g - \pi R) & \text{if } \frac{g}{1+R} \leq \pi \leq \min\{\frac{g}{R}, 1\} \\
0 & \text{if } \min\{\frac{g}{R}, 1\} \leq \pi \leq 1
\end{cases}
\]

Clearly, the entrepreneurs endowed with the least productive projects, such that \(x = g/\pi \leq R\) quit the loan market. We may thus distinguish two classes of entrepreneurs wanting to obtain a loan at rate \(R\): those endowed with the most productive projects (class 1), such that \(x \geq 1 + R\), and those endowed with less productive projects (class 2), such that \(R \leq x \leq 1 + R\). The demand \(D\) for loans will result from the behavior of these two classes.

### 2.3 Demand for loans

As to class 1, the ceiling imposed by the marginal expected utility of effort (4) upon \(v\) is the cumulative distribution function \(F(\cdot, \rho)\), hence increasing
in $\pi$. Consequently, we may restrict $\pi$ to the interval $[F^{-1}(v, \rho), g/(1 + R)]$. By uniformity of the distribution of $\pi$ over $[0,1]$, the demand for loans of class 1 is thus $D^1(R, v, \rho) = \max\{g/(1 + R) - F^{-1}(v, \rho), 0\}$. As to class 2, we prove in the Appendix that the ceiling now imposed upon class 1 is thus further restricted to the interval $[\overline{\pi}(R, v, \rho), \min\{g/R, 1\}]$ can be obtained $\rho = 0.5$ and $g = 0.6$ in this example. The marginal utility of effort is $F(\pi, 0.5)$ for class 1, represented by the curve increasing from the origin, and it is $(F(\pi, 0.5)/\pi)(0.6 - \pi R)$ for class 2, represented by three inverse U-shaped curves corresponding to three values of $\pi$. The horizontal line is the marginal disutility of effort $v (v = 0.5$ in this example). The figure displays two distinct cases plus the borderline one. In the first case, $R$ is low and $\pi(R, 0.5, 0.5) = 0.6/(1 + R) = 0.5$ (0.6/(1 + R) corresponding to the rightmost dashed vertical line). The two classes wish to obtain a loan in this case, with $D^1(R, 0.5, 0.5) = 0.6/(1 + R) - F^{-1}(0.5, 0.5)$ and $D^2(R, 0.5, 0.5) = \pi(R, 0.5, 0.5) - 0.6/(1 + R)$. In the second case, $R$ is high and $\pi(R, 0.5, 0.5) > 0.6/(1 + R)$, which is equivalent to $0.6/(1 + R) < F^{-1}(0.5, 0.5) = 0.5$ (0.6/(1 + R) corresponding to the leftmost dashed vertical line). In this case, only class 2 expresses a demand $D^2(\cdot, v, \rho)$ is a decreasing function of $\rho$. The demand functions $D^1(\cdot, v, \rho) and D^2(\cdot, v, \rho)$ are decreasing if positive, since $\pi(\cdot, 0.5, 0.5)$ and $-\overline{\pi}(\cdot, 0.5, 0.5)$ are both decreasing.
We thus obtain the following aggregate demand for loans:

\[
D(R, v, \rho) = \begin{cases} 
\frac{\pi (R, v, \rho) - F^{-1}(v, \rho)}{F^{-1}(v, \rho)} - 1 & \text{if } 0 \leq R \leq \frac{g}{F^{-1}(v, \rho)} - 1 \\
\frac{\pi (R, v, \rho) - \pi (R, v, \rho)}{F^{-1}(v, \rho)} & \text{if } \frac{g}{F^{-1}(v, \rho)} - 1 \leq R \leq \tilde{R}(v, \rho, g) 
\end{cases},
\]

(6)

the function \( D(R, v, \cdot) \) conserving the symmetry property of the distribution function: \( D(R, v, 1 - \rho) = D(R, v, \rho) \). Figure 2 represents the graph of the demand function \( D(\cdot, 0.5, 0.5) \) corresponding to the example illustrated by Figure 1.
For $R \in [0, 0.2]$, both classes express a demand for loans, but $\pi(R, 0.5, 0.5) = 1$ if $R \leq 0.1$, so that demand is fully inelastic, equal to 0.5, in this case, whereas $\pi(R, 0.5, 0.5)$ is decreasing in $R$ if $R > 0.1$, entailing an elastic demand. For $R \in [0.2, 0.265]$, class 2 contributes alone to the demand for loans, and variability of $\pi(\cdot, 0.5, 0.5)$ enhances the demand elasticity. The demand for loans vanishes for $R \geq 0.265$.

### 2.4 Loan riskiness

Since a project characterized by safeness index $\pi$ succeeds if $\pi \geq (1 - \rho) s + \rho S$, that is, if $s \leq (\pi - \rho S) / (1 - \rho)$ and since idiosyncratic risk $s$ is uniformly distributed over $[0, 1]$, each project succeeds, conditionally on the realization $S$ of the systemic risk, with probability $\min \{\max \{(\pi - \rho S) / (1 - \rho), 0\}, 1\}$, which is the probability that $z \leq \pi$, given $S$. Hence, by the law of large numbers, the mass of successful projects conditional on $S$, of course among the projects actually realized (as given by equation (6)), is

$$M(S, R, \rho) = \int_{\max\{a, \rho S\}}^{\min\{b, 1 - \rho + \rho S\}} \frac{\pi - \rho S}{1 - \rho} d\pi + \max\{b - (1 - \rho + \rho S), 0\},$$

with

\[
a = \begin{cases} 
F^{-1}(v, \rho) & \text{if } 0 \leq R \leq \frac{\rho - g}{F^{-1}(v, \rho)} - 1 \\
\pi(R, v, \rho) & \text{if } \frac{\rho - g}{F^{-1}(v, \rho)} - 1 \leq R \leq \frac{g}{R(g, v, \rho)} 
\end{cases}
\]

and

\[
b = \pi(R, v, \rho).
\]

For simplicity of notation, we will from now on omit the reference to $v$ as an argument of the functions involving the entrepreneurs’ behavior.

For each realization $S$ of the systemic risk, and for each value of the interest rate $R$, we can define the proportion of successful projects in the investment portfolio $p(S, R, \rho) = M(S, R, \rho) / D(R, \rho) = M(S, R, \rho) / (b - a)$, with $a$ and $b$ as defined by (8). By performing the integration on the RHS of (7) and using (6), we obtain in the case $b \leq 1 - \rho + \rho S$

\[
p(S, R, \rho) = \begin{cases} 
\frac{1}{1 - \rho} \left( \frac{a + b}{2} - \rho S \right) & \text{if } \rho S \leq a \\
\frac{1}{1 - \rho} \left( \frac{b - \rho S}{2(b - a)} \right) & \text{if } \rho S \geq a 
\end{cases}
\]
and in the case $b \geq 1 - \rho + \rho S$

$$p(S, R, \rho) = \begin{cases} \frac{b-(\rho S+(1-\rho)/2)}{b-a} & \text{if } \rho S \leq a \\ \frac{2(1-\rho)(b-a)}{b-a} (a-\rho S)^2 & \text{if } \rho S \geq a \end{cases}.$$ \hspace{1cm} (10)

The proportion of successful projects conditional on $S$ is naturally always decreasing in $S$, and always increasing in both $a$ and $b$. As $b = \pi(R, \rho)$, it is non-increasing, through $b$, in the loan interest rate $R$. An increase in $R$ either will not have any effect on the demand for loans ($\pi(R, \rho)$ may be equal to 1 for very low values of $R$) or else will eliminate some of the least productive projects, putting more weight on more productive but riskier projects. We thus obtain through adverse selection the risk shifting effect highlighted by Stiglitz and Weiss (1981). This effect is stronger when it is not countervailed by an increase in $a$, responding to an increase in $R$, that is, when $a = F^{-1}(v, \rho)$, for values of $R$ low enough to attract borrowers from class 1.

## 3 Banks

There are $n$ banks endowed with a license to operate in the intermediation activity, financing with their deposits the loans granted to the entrepreneurs.

### 3.1 Expected profit and bank default

The cost of the loan granted to an entrepreneur is the deposit interest rate $r$, which we take provisionally as exogenous. So, the expected profit per loan, conditional on the realization $S$ of systemic risk, is

$$\tilde{\Pi}(S, R, r, \rho) = p(S, R, \rho) (1 + R) + (1 - p(S, R, \rho)) (1 - \gamma) - (1 + r)$$

$$= p(S, R, \rho) (R + \gamma) - (r + \gamma).$$ \hspace{1cm} (11)

The default threshold $S^*(R, r, \rho)$ is the value of the systemic risk that triggers bank default, occurring for any $S > S^*(R, r, \rho)$, so that, since the distribution of $S$ over $[0, 1]$ is uniform, $1 - S^*(R, r, \rho)$ is the probability of bank failure. The default threshold is determined by the zero expected profit condition\footnote{Naturally, we are considering levels of the loan interest rate ensuring profitability but also riskiness of the banking activity ($\Pi(0, R, r, \rho) > 0$ and $\Pi(1, R, r, \rho) < 0$, respectively).}

$$\tilde{\Pi}(S^*, R, r, \rho) = p(S^*, R, \rho) (R + \gamma) - (r + \gamma) = 0.$$ \hspace{1cm} (12)
In order to assess the effects on the default threshold of a variation in \( R \), we use this equation to compute by differentiation the elasticity of \( S^* (R, r, \rho) \) with respect to \( R \):

\[
\epsilon_R S^* (R, r, \rho) = \frac{\text{margin effect}}{R + \gamma} - \frac{\text{risk shifting effect}}{\epsilon_S p (S^*, R, \rho)},
\]

where \( \epsilon_R p (S^*, R, \rho) \) and \( \epsilon_S p (S^*, R, \rho) \) are the elasticities of the proportion of successful projects with respect to \( R \) and \( S \), respectively.

We may observe that the default threshold \( S^* \) depends positively on \( R \) through the margin effect: a higher \( R \) increases the bank profit per loan \( R - r \) when entrepreneurs succeed, providing a buffer to cover the loss per loan \( r + \gamma \) when they do not. It also depends negatively on \( R \) through the risk shifting effect, making \( \epsilon_R p (S^*, R, \rho) < 0 \) when \( \overline{\pi} (R, \rho) < 1 \). The intensity of competition, responsible for lower values of \( R \), is thus seen to have through the margin effect an unfavorable influence on bank failure, increasing the probability of its occurrence, but a favorable effect through the risk shifting effect. Is this effect strong enough to undermine the conventional wisdom? We address this question in subsections 3.3 and 3.4.

### 3.2 Competition for loans

In the context of Cournot competition under perfect product substitutability (the one to which refer Martínez-Miera and Repullo 2010), the Herfindahl index of concentration (the market share \( 1/n \) when strategy profiles are symmetric) may be used as an inverse index of intensity of competition. More generally, intensity of competition encompasses other dimensions, in particular the degree of product substitutability and the degree of competitive toughness, the latter increasing when switching, for instance, from Cournot to Bertrand competition. Here we will focus on this dimension, referring to a homogeneous oligopoly in which competitive toughness varies continually from Cournot to Bertrand (see d’Aspremont and Dos Santos Ferreira 2009).

For that purpose, we assume that bank \( j \) chooses the price-quantity pair \( (R_j, l_j) \in [r, \overline{R} (\rho)] \times [0, \infty) \) so as to maximize its expected profit.
\( l_j \Pi (R_j, r, \rho) \). Taking limited liability into account and using (11), we have:

\[
l_j \Pi (R_j, r, \rho) \equiv l_j \int_0^{S^*(R_j, r, \rho)} \tilde{\Pi} (S, R_j, r, \rho) \, dS =
\]

\[
l_j \int_0^{S^*(R_j, r, \rho)} \frac{1}{S^*(R_j, r, \rho)} \int_0^{S^*(R_j, r, \rho)} p(S, R_j, \rho) \, dS (R_j + \gamma) - (r + \gamma)
\]

where \( P(R_j, r, \rho) \) is the expected proportion of successful borrowers. The maximization of expected profit is supposed to be performed under two constraints:

\[
R_j \leq \min_{j' \neq j} R_{j'} \tag{15}
\]

\[
R_j \leq D^{-1} \left( l_j + \sum_{j' \neq j} l_{j'}, \rho \right). \tag{16}
\]

The first inequality is a competitiveness constraint, imposing a ceiling on the loan interest rate, equal to the minimum of the values set by the other suppliers of the same homogeneous service. The second inequality is the usual Cournot condition, also imposing a ceiling on the loan interest rate, now determined by the inverse demand for loans when each bank takes as given the aggregate volume of loans that its competitors intend to grant.

An interior solution \((R_j, l_j)\) to bank \(j\)'s problem, with \(R_j > r\) and \(l_j > 0\), will satisfy both constraints as equalities. The corresponding first order conditions, under differentiability of \(\Pi (\cdot, r, \rho)\), can consequently be expressed as

\[
l_j \frac{\partial \Pi (R_j, r, \rho)}{\partial R_j} - \lambda_j - \mu_j = 0 \tag{17}
\]

\[
\Pi (R_j, r, \rho) + \frac{\mu_j}{\partial D (R_j, \rho) / \partial R_j} = 0,
\]

with non-negative Lagrange multipliers \(\lambda_j\) and \(\mu_j\). By confining our analysis to symmetric profiles, we obtain from these two first order conditions and denoting \(R_j = R, l_j = l, \lambda_j = \lambda\) and \(\mu_j = \mu\) for any \(j\) the resulting condition:

\[
1 = \frac{\mu}{\lambda + \mu} \frac{1/n}{-\epsilon_R D (R, \rho)} \epsilon_R \Pi (R, r, \rho), \tag{18}
\]

14
where $\epsilon_R D (R, \rho)$ and $\epsilon_R \Pi (R, r, \rho)$ are the elasticities with respect to $R$ of the demand for loans and of the expected profit per loan, respectively, and where $\theta \equiv \mu / (\lambda + \mu)$ is the conduct parameter of the New Empirical Industrial Organization literature (see Bresnahan 1989, Corts 1999). It increases as the implicit cost $\mu$ imposed by the consensual participation constraint (accommodating the rivals’ loan targets) increases relative to the implicit cost $\lambda$ imposed by the more confrontational competitiveness constraint (reflecting the conflicting interests of all the competitors). We may accordingly see $\theta$ as an index of competitive softness displayed at a particular equilibrium, and taking values between 0 (at Bertrand equilibrium) and 1 (at Cournot equilibrium).\footnote{Competitive softness is endogenous. It parameterizes a particular equilibrium in a large set of (symmetric) oligopolistic equilibria. Referring to a given value of $\theta$ which indexes a specific conduct is however not different in nature from referring to the Cournot or Bertrand regimes, which are just limit cases of our parameterization. Following the convention used in the NEIO literature, we denote by $\theta$ the conduct parameter $\mu / (\lambda + \mu)$, viewed as an index of competitive softness, whereas d’Aspremont and Dos Santos Ferreira (2009) denote instead by $\theta$ the complementary index $\lambda / (\lambda + \mu)$ of competitive toughness.}

The role of $\theta$ appears clearly if we replace $\epsilon_R \Pi (R, r, \rho)$ in the profit maximization condition (18) by its expression computed from (14). That condition appears then as an extended formula for the Lerner index of market power, namely the relative margin of the expected revenue per loan $P (R, r, \rho) (R + \gamma)$ over its marginal cost $r + \gamma$:

$$1 - \frac{r + \gamma}{P (R, r, \rho) (R + \gamma)} = \frac{R/ (R + \gamma) - (\epsilon_R P (R, r, \rho))}{(n/\theta) (\epsilon_R \Pi (R, \rho)) - \epsilon_R S^* (R, r, \rho)}.$$  \hfill (19)

On the right-hand side of this equation, which determines the equilibrium value of $R$, the first term in the denominator is, for $\theta = 1$, the reciprocal of the usual Cournot’s degree of monopoly $(1/n) / (-\epsilon_R D (R, \rho))$. As $\theta$ decreases (as competition becomes tougher), the degree of monopoly diminishes, completely vanishing as $\theta$ tends to zero (the Bertrand case).\footnote{The loan interest rate expected to be realized $P (R, r, \rho)$ is then equal to the expected marginal loan cost $r + (1 - P (R, r, \rho)) \gamma$.} The second term in the denominator expresses the effect on market power induced by limited liability, positive if $S^* (\cdot, r, \rho)$ is increasing (the competition-fragility view), negative if $S^* (\cdot, r, \rho)$ is decreasing (the competition-stability view). The two terms in the numerator express the two effects of a change in the loan interest rate which have already been highlighted, namely the positive margin effect and the negative risk shifting effect. The latter applies however to the
expected proportion of successful projects, not to the proportion conditional on \( S \), as in equation (13).

### 3.3 The intensity of competition and the risk of bank failure

The intensity of competition is often directly identified with the number \( n \) of banks, which makes sense in a symmetric homogeneous oligopoly, with producers operating at constant returns and competing à la Cournot. Keeping all these characteristics but extending beyond Cournot the potential competitive conduct leads us, in accordance with the formula of the Lerner index, to prefer \( n/\theta \) as a synthetic index of intensity of competition. This choice reminds us that competition may well be intensified through conduct (a lower \( \theta \)) even under higher concentration (a lower \( n \)).

The question we want to address concerns the sense of the response of the default threshold \( S^* \) to a change in the intensity of competition \( n/\theta \), through the resulting variation of the loan interest rate \( R \). A negative elasticity \( \epsilon_R S^* (R, r, \rho) \) of \( S^* \) with respect to \( R \) means that a higher intensity of competition, depressing the loan interest rate, leads to a lower probability of bank failure, vindicating the competition-stability view. Now, referring to equations (13) and (19), we see that, since the Lerner index must be positive at an imperfectly competitive equilibrium, \( \epsilon_R S^* (R, r, \rho) < 0 \) implies

\[
-\epsilon_R P (R, r, \rho) < \frac{R}{R + \gamma} < -\epsilon_R P (S^*, R, \rho). \tag{20}
\]

In other words, the response of the probability of success at a high level of systemic risk (its default threshold) must be larger in absolute value than the response of the average probability for all levels of systemic risk (smaller than that threshold, because of limited liability), a necessary condition compatible with our specification (except of course if \( \rho = 0 \)):

**Claim 1** A necessary condition for the competition-stability view to be validated within some interval of equilibrium values of the loan interest rate is that the risk shifting effect be strengthened by a higher systemic risk.

By taking the average probability of success as defined in (14) and com-
puting its elasticity with respect to $R$, we can give a slightly different formulation of this necessary condition. Indeed, by using equations (12), (13) and (19), we obtain:

\[
\begin{align*}
&> 0 \\
&= -\epsilon SP(S^* (R, r, \rho), R, \rho)\epsilon_R S^* (R, r, \rho) \\
&\quad \left(1 - \frac{r + \gamma}{P(R, r, \rho)(R + \gamma)}(n/\theta) (-\epsilon_R D(R, \rho))\right) \\
&\quad + \int_0^{S^*(R, r, \rho)} p(S, R, \rho) (-\epsilon_R p (S, R, \rho)) dS \\
&\quad \int_0^{S^*(R, r, \rho)} p(S, R, \rho) dS - (-\epsilon_R p(S^* (R, r, \rho), R, \rho)).
\end{align*}
\]

Hence, a necessary condition for $\epsilon_R S^* (R, r, \rho)$ to be negative is that the response of the probability of success at the default threshold of systemic risk be larger in absolute value than its average response. If $-\epsilon_R p(\cdot, R, \rho)$ is monotone increasing, limited liability makes it easier to be satisfied, as it truncates the average response to the values of $S$ smaller than $S^*$:

**Claim 2** *If the risk shifting effect is monotonically strengthened by a higher systemic risk, limited liability favors its possible dominance over the margin effect.*

Now, can the risk shifting effect dominate the margin effect whatever the intensity of competition? If we let the intensity of competition become arbitrarily large ($n/\theta \to \infty$), the Lerner index vanishes ($(r + \gamma) / (R + \gamma) \to P(R, r, \rho)$), so that, by the zero expected profit condition (12), $p(S^*, R, \rho) \to P(R, r, \rho) = \frac{1}{S^*} \int_0^{S^*} p(S, R, \rho) dS$. Since, by (9) and (10), the derivative of $p(\cdot, R, \rho)$ remains bounded away from zero as $R$ decreases, it must be the case that $S^* \to 0$, hence that increasing the intensity of competition eventually

\[\epsilon_R p(R, r, \rho) = -\epsilon_R S^* (R, r, \rho) \left(1 - \frac{p(S^* (R, r, \rho), R, \rho)}{S^*(R, r, \rho)} \int_0^{S^*(R, r, \rho)} p(S, R, \rho) dS\right)
\]

\[-\int_0^{S^*(R, r, \rho)} p(S, R, \rho) (-\epsilon_R p(S, R, \rho)) dS \\
\int_0^{S^*(R, r, \rho)} p(S, R, \rho) dS.
\]
increases the risk of bank failure (cf. Martinez-Miera and Repullo 2010, Proposition 2):

**Claim 3** As the intensity of competition increases, a further increase in this intensity eventually amplifies the risk of bank failure.

Thus, when looking for situations vindicating the competition-stability view we expect them to be confined to cases of low intensity of competition and corresponding high loan interest rates. Notice however that the intensity of competition is lower bounded by monopoly, and that no bank will ever choose a loan interest rate higher than the collusive one, so that too high interest rates are actually not observable in equilibrium. In spite of this crucial remark, it cannot be excluded that a decrease in the loan interest rate, at a level compatible with equilibrium, may lead to a lower risk of bank default. In the following subsection, we present an example of such a situation.

### 3.4 An example supporting the competition-stability view

Let us illustrate by an example the case for a stabilizing effect of intensified bank competition. This example continues the one in subsection 2.3, with \( \rho = v = 0.5 \) and \( g = 0.6 \). Demand for loans vanishes if \( R \geq 0.265 \) and supply of loans requires \( R \geq r \), so that \( R \) must be taken in the interval \([r, 0.265]\). Then, by equations (9) and (10),

\[
p(S, R, 0.5) = \begin{cases} 
    \frac{a + \pi - S}{\pi - a} & \text{if } \frac{\pi - \frac{1}{2}S}{\pi - a} \\
    \frac{\pi - 0.5(S + 0.5) - 0.25(2a - S)^2}{\pi - a} & \text{if } \frac{\pi - \frac{1}{2}S}{\pi - a}
\end{cases}
\]

so that, by the zero expected profit condition (12),

\[
S^*(R) = \begin{cases} 
    2a - 1 + \pi(R) - a + \frac{R - r}{R + \gamma} & \text{if } \frac{\pi(R) - a}{\pi(R) - a} \\
    2a - 1 + 2\sqrt{\pi(R) - a} & \text{if } \frac{\pi(R) - a}{\pi(R) - a}
\end{cases}
\]

with \( a = 0.5 \) if \( R \leq 0.2 \) and \( a = \pi \) if \( R \geq 0.2 \). We recall that \( \pi(R) - a \) is the demand for loans and observe that \( (R - r) / (R + \gamma) \) may be interpreted as the relative margin, to be equalized to the proportion of unsuccessful projects by the zero expected profit condition (12).
We represent in Figure 3 the default threshold $S^*$ as a function of the loan interest rate $R$, for $r = 0.07$ and $\gamma = 0.08$.

![Figure 3: The default threshold $S^*$ as a function of the loan interest rate $R$](image)

For $R \in [0.07, 0.1]$, $\pi (R) = 1$ and $a = 0.5$, so that the demand for loans $D (R, 0.5, 0.5) = 0.05$ is rigid and the risk shifting effect vanishes, reducing $S^*$ to $\sqrt{2(R - r)/(R + \gamma)}$, which increases in $R$ under the margin effect. For $R \in [0.1, 0.2]$ and for $R \in [0.2, 0.265]$, the margin effect is first dominating but ends up being superseded by the risk shifting effect (for $R > 0.191$ and for $R > 0.24$, respectively), so that we get in both intervals inverse U-shaped curves for the graph of $S^*(R)$, as clearly shown by the zoomed-in representation on the right part of the figure. However, the second decreasing segment, corresponding to the higher values of $R$, will never be observed in equilibrium. Indeed, the maximum of the monopoly expected profit (represented, with an adjusted scale, as the thin curve on the left part of the figure) is attained at $R = 0.21$, and no competitive equilibrium interest rate can be higher than this collusive value.

We are thus left with the interval $[0.191, 0.2]$ in which $S^*$ is decreasing and the monopoly expected profit is increasing. Does this interval contain a competitive equilibrium loan interest rate? The highest possible equilibrium rate is the one corresponding to the minimum number of competitors, namely $n = 2$, and to the softest competitive conduct, namely $\theta = 1$ (Cournot). At $R = 0.191$, a symmetric duopoly configuration would lead for each bank to a demand for loans equal to $(\pi (0.191) - 0.5) / 2 = 0.16625$. The Cournot duopolist $j$, if deviating from strategy $(R_j, l_j) = (0.191, 0.16625)$, would obtain at rate $R$ the expected profit $P(R)$ of $(R + 0.08) (D(R) - 0.15)$ $(D(R) - 0.16625)$ represented by the lower curve in Figure 4.
We see that this expected profit is increasing at \( R = 0.191 \). Hence, there is an incentive for the Cournot duopolist to deviate from the strategy \((0.191, 0.16625)\) by decreasing the loan supply and increasing the loan interest rate. The Cournot duopoly equilibrium rate must consequently be higher than 0.191. By a symmetric argument, \( P(R)((R + 0.08) - 0.15)(D(R) - 0.15693) \) is the expected profit accruing to the Cournot duopolist \( j \), when deviating from the strategy \((R_j, l_j) = (0.2, 0.15693)\), where \( 0.15693 = (\pi(0.2) - 0.5)/2 \). This expected profit is represented by the upper curve in Figure 4, which is decreasing at \( R = 0.2 \), so that there is an incentive to increase the loan supply and decrease the loan rate. The Cournot duopoly equilibrium rate must consequently belong to the interval \((0.191, 0.2)\), where \( S^* \) is decreasing in \( R \). A slight intensification of competition (for instance through a decrease of \( \theta \)) would consequently diminish the loan rate and increase \( S^* \). Thus, the competition-stability view is vindicated in this example, although in a quite constrained way.

### 4 Depositors

Each depositor is endowed with one unit of money, which he can deposit in some bank so as to be able to spend, in period 1, \( 1 + r \) money units \((r \geq 0)\). We suppose that more and more potential depositors actually deposit their money endowments as the deposit interest rate \( r \) increases, resulting in an increasing deposit supply function \( L : [0, \overline{r}] \to [0, \overline{L}] \). As a limit case, this supply function may be rigid, with \( L(r) = 0 \) for any \( r < \overline{r} \) and \( L(\overline{r}) = \overline{L} \) (with possibly \( \overline{r} = 0 \)). We assume perfect competition in the deposit market, so that the deposit rate \( r \) is adjusted parametrically, so as to balance supply
and demand \( \sum_{j=1}^{n} l_j \). Recall that the deposit interest rate is restricted to belong to the interval \([0, R]\) because for \( r < 0 \) the deposit supply would be zero, and for \( r > R \) no bank would be able to remunerate depositors.

In the case of a rigid deposit supply function, the analysis developed in the preceding section applies immediately, with \( r = \bar{r} \), as long as \( D(R) = nl \leq \bar{L} \). Otherwise, \( r \) and \( R \) must be adjusted upwards so as to ensure the equalities of loan demand and supply and deposit demand and supply: \( D(R) = nl = \bar{L} \). In such a situation of liquidity shortness and rigid deposit supply, more intense competition in the loan market, be it through lower concentration (a higher \( n \)) or through lower competitive softness (a lower \( \theta \)), still decreases market power (according to equation (19)), but in this context through a heightening effect on \( r \), rather than through a depressing effect on \( R \), which is pegged by the equality \( D(R) = \bar{L} \). In any case, the profit margin is squeezed and the risk of bank failure always aggravated, since the risk shifting effect is absent.

In the case of an increasing deposit supply function, the analysis is modified only if \( D(R) = L(r) < \bar{L} \), so that \( r \) cannot be treated as exogenous anymore in the context of the last subsection, but instead as function of \( R \): \( r = L^{-1} \circ D(R) \). A change in the intensity of competition for loans consequently modifies both interest rates, decreasing \( R \) and increasing \( r \). As a consequence, the margin effect is amplified. A simple computation shows indeed that, in the expression (13) for the elasticity \( \epsilon_R S^*(R, r, \rho) \), the margin effect is now

\[
\frac{R}{R + \gamma} + \frac{r}{r + \gamma} \frac{-\epsilon D(R)}{\epsilon L(r)}.
\]

The case analyzed in the previous subsection is, implicitly, the one of perfectly elastic supply of deposits (\( \epsilon L(\bar{r}) = \infty \), with \( D(R) < \bar{L} \)). As soon as the supply of deposits becomes imperfectly elastic, one can however not neglect the second component of the margin effect, operating through the deposit market. Clearly, liquidity shortness reinforces the conventional competition-fragility view.

5 Operating costs and non-intermediation banking activities

We have up to now exclusively considered the fundamental financial intermediation activity of the banking system. This activity is however comple-
mented by other non-traditional activities which generate an income, here taken as exogenous and treated as a negative component of bank j’s net fixed operating cost \( \phi_j \), which can consequently take positive or negative values. The net operating cost \( \phi_j \) is a random value, with the distribution function \( G \) defined on the interval \( \Phi \subset \mathbb{R} \). We assume that the risk associated with the net operating cost is independent across banks and independent of the systemic and idiosyncratic entrepreneurial project risks. For simplicity, we will ignore the possibility of liquidity shortness in this section, taking again \( r \) as exogenous.

The default threshold is now defined, for a symmetric loan profile, by the zero expected profit condition (given the realization \( \phi \) of the net operating cost):\(^{11}\)

\[
\tilde{\Pi} \left( S^*, R, r, \rho, \phi \right) \equiv \left[ p \left( S^*, R, \rho, \phi \right) \left( R + \gamma \right) - \left( r + \gamma \right) \right] \frac{D \left( R, \rho \right)}{n} = \phi. \tag{24}
\]

As \( p \left( \cdot, R, \rho \right) \) is a decreasing function, so is the function \( S^* \left( R, r, \rho, \cdot \right) \): a higher net operating cost leads to a higher risk of bank failure. This is intuitive, but the relevant question is how net operating costs affect the way this risk depends upon the intensity of competition. To answer to that question, we compute the elasticity of \( S^* \left( R, r, \rho, \phi \right) \) with respect to \( R \):

\[
\epsilon_{RS^*} \left( R, r, \rho, \phi \right) = \frac{\text{margin effect}}{R + \gamma} - \frac{\text{risk shifting effect}}{n\phi + \left( r + \gamma \right) D \left( R, \rho \right)} - \frac{\text{scale effect}}{\left( -\epsilon_{R} D \left( R, \rho \right) \right)}.
\tag{25}
\]

In addition to the opposite margin and risk shifting effects, we find now a scale effect operating through the demand for loans, because of the increasing (resp. decreasing) returns induced by a positive (resp. negative) net operating cost \( \phi \).

The scale effect works in conjunction with the margin effect when the income from non-intermediation activities is larger than the gross operating cost \( \left( \phi < 0 \right) \), and in conjunction with the risk shifting effect otherwise. An increase in the number of banks magnifies the scale effect in both cases, independently of its impact on the intensity of competition, and hence on \( R \).

\(^{11}\)Again, we consider levels of the loan interest rate \( R \) and now a domain of the random variable \( \phi \) such that the profitability and the riskiness of the bank intermediation activity are warranted (\( \Pi \left( 0, R, r, \rho, \phi \right) > 0 \) and \( \Pi \left( 1, R, r, \rho, \phi \right) < 0 \), respectively).
As to an increase in the net operating cost $\phi$, it has an indeterminate indirect influence on $\epsilon_R S^* (R, r, \rho, \phi)$ through $p(\cdot, R, \rho)$ plus a direct negative influence through the scale effect, hence an influence favorable to the competition-stability view. This influence is naturally also that of the mean of the random net operating costs.

Since the distribution of systemic risk is uniform, the probability of bank failure is now $1 - S (R, r, \rho)$, with $S (R, r, \rho) \equiv \int_\phi S^* (R, r, \rho, \phi) \, dG (\phi)$, the mathematical expectation of the default threshold. The elasticity with respect to $R$ of this mathematical expectation is

$$
\epsilon_R S (R, r, \rho) = \int_\phi \frac{S^* (R, r, \rho, \phi)}{S (R, r, \rho)} \epsilon_R S^* (R, r, \rho, \phi) \, dG (\phi), \quad (26)
$$

a weighted mean of the elasticities $\epsilon_R S^* (R, r, \rho, \phi)$ for different values of $\phi$, with a weight higher and higher as the scale effect works more and more in conjunction with the margin effect (if $\phi < 0$) and less and less in conjunction with the risk shifting effect (if $\phi > 0$). As a consequence,

**Claim 4** A higher variance of random net operating costs is unfavorable to the dominance of the risk shifting over the margin effect that would validate the competition-stability view.

The presence of net operating costs does not essentially modify the results established in their absence as concerns the relation between the intensity of competition and the risk of bank failure. To get an intuition of this simple extension, notice first that maximization in $(R_j, l_j)$ by bank $j$ of its expected net profit $l_j \Pi (R_j, r, \rho) - \mathbb{E} (\phi)$ is equivalent to the maximization of the expected gross profit, since the expected net operating cost does not depend upon the bank’s choice. And the expected gross profit per loan $\Pi (R_j, r, \rho)$ is defined as above, only with the default threshold $S^* (R_j, r, \rho, 0)$ associated with $\phi = 0$ replaced by its mathematical expectation $S (R_j, r, \rho)$ and the expected proportion of successful borrowers taking now the form

$$
P (R_j, r, \rho) = \int_\phi \frac{S^* (R_j, r, \rho, \phi_j)}{S (R_j, r, \rho)} \left( \frac{1}{S^* (R_j, r, \rho, \phi_j)} \int_0^{S^* (R_j, r, \rho, \phi_j)} p (S, R_j, \rho) \, dS \right) \, dG (\phi_j),
$$

still an average of the probabilities of success $p (S, R_j, \rho)$.
Randomness of net operating costs introduces the possibility of bank failure in the absence of systemic risk \((\rho = 0)\), otherwise impossible if bank portfolios are large and well diversified, because of the law of large numbers. However, if \(\rho = 0\), the proportion \(p(S,R,0)\) of successful projects ceases to depend upon \(S\), making \(P(R,r,0)\) coincide with \(p(S,R,0)\). The left-hand side of the zero profit condition (24) is then equal to the expected profit per bank, to be maximized in \(R\) under full collusion, \(\Pi(R,r,0)D(R,0)/n\). Observable (equilibrium) values of \(R\) will necessarily be smaller than its collusive value, and belong to an interval where the expected profit per bank is increasing in \(R\). Hence, the probability of bank failure \(1 - G(\Pi(R,r,0)D(R,0)/n)\) is always decreasing in \(R\), validating the conventional view that more intensive competition increases the risk of bank failure:

**Claim 5** Under random net operating costs, even in the absence of systemic risk and in spite of portfolio diversification, the risk of bank failure is not excluded, but it is necessarily increasing with the intensity of competition for equilibrium values of the loan interest rate.

The argument leading to this claim is a good illustration of the fact that the strength of the risk shifting effect relative to the margin effect cannot be assessed independently of the (oligopolistic) competitive equilibrium conditions. Also, the claim itself implies that systemic risk remains in the present context a necessary condition for the competition-stability view to be validated. Moreover, since Claim 1 can be straightforwardly extended to the case of random net operating costs, a stronger necessary condition required by the competition-stability view is that the risk shifting effect be strengthened by a higher systemic risk.

### 6 Conclusion

We have discussed in the context of a simple model the relation between the intensity of competition in the loan market and the risk of bank failure, a relation conventionally viewed as increasing. Pushing to the fore the first term of this relation, we did not reduce the intensity of competition, as often done, to its main structural component, concentration, but we took also into account the more or less aggressive competitors’ conduct. More importantly, we emphasized the fact that more competition tends to diminish the probability of bank default (contrary to what convention claims), but only
at high levels of the loan interest rate, possibly too high to be compatible with an oligopolistic competitive equilibrium, as they would incite banks to deviate to lower rates. We have shown by an example that the unconventional competition-stability hypothesis cannot be excluded, but our position remains rather in favor of the conventional view.

This position is reinforced when enlarging the analysis, first by bringing in the deposit market, and second by taking into account non-intermediation banking activities. Under conditions of liquidity shortness and deposit supply inelasticity, the effect of more intense competition in the loan market is at least partly dissipated as an increase of deposit rates instead of a decrease of loan rates. This reduces the amplitude of the risk shifting effect, ascribable to borrowers’ behavior, which is the basis of the competition-stability view. As to the extra income derived from non-intermediation banking activities, it adds a negative scale effect to the margin effect supporting the conventional view. It is true that this effect may be weakened by the presence of banks’ operating costs, or even reversed if these costs are higher than the extra income. However, the simple randomness of this effect, whether negative or positive, is in any case unfavorable to the competition-stability view.

References


A  Proof of strict quasi-concavity of function

\[ \Psi'(\pi) \equiv \frac{F(\pi, \rho)}{\pi} (g - \pi R) \]

Since the cumulative distribution function \( F(\cdot, \rho) \) is differentiable, as already noticed, the function \( \Psi \) is differentiable over \((0, 1)\). Thus, the proof amounts to show that \( \Psi \) is strictly concave at any interior critical point, in other words, that

\[ \Psi'(\pi) = \frac{f(\pi, \rho) (g - \pi R) \pi - F(\pi, \rho) g}{\pi^2} = 0 \]

implies

\[ \Psi''(\pi) = \frac{\partial f(\pi, \rho)}{\partial \pi} (g - \pi R) \pi^2 - 2 \left[ \frac{f(\pi, \rho) (g - \pi R) \pi - F(\pi, \rho) g + f(\pi, \rho) \pi^2 R}{\pi^3} \right] \]

\[ = \frac{\partial f(\pi, \rho)}{\partial \pi} \frac{(g - \pi R) - 2 f(\pi, \rho) R}{\pi} < 0. \]

As \( f(\pi, \rho) > 0 \) (because \( 0 < \pi < 1 \)) and \( g - \pi R > 0 \) (because \( \Psi'(\pi) = 0 \)), \( \Psi''(\pi) \) is indeed negative provided \( \partial f(\pi, \rho) / \partial \pi \leq 0 \), that is, by (2), provided \( \pi > \min(\rho, 1 - \rho) \). In the complementary case, if \( \pi \leq \min(\rho, 1 - \rho) \), we have, by (1) and (2), that

\[ \Psi'(\pi) = \frac{g - 2\pi R}{2\rho(1 - \rho)} = 0, \]

implying \( \pi = g/2R \) and \( \Psi''(\pi) = -R/\rho (1 - \rho) < 0. \)