Taxation and Regulation in a Market of Sin Goods with Persuasive Advertising

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Abstract

We introduce persuasive advertising in a duopoly market of differentiated harmful goods, where firms compete on prices and advertising. Since advertising artificially inflates consumers’ demand, the market allocation is not efficient, with over-consumption of sin goods. We then show that taxation increases aggregate surplus by improving consumers’ welfare and reducing firms’ profits. Adding regulation of advertising increases aggregate surplus further, but by increasing firms’ profits and creating a conflict between consumers not very sensitive to advertising — who experience welfare reductions — and consumers highly sensitive to it — who experience welfare gains.

Keywords: Harmful consumption, Corrective taxation, Regulation of advertising, Taxation under imperfect competition.

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1 Introduction

Product taxation and regulatory interventions by governments are becoming more and more important in those markets where there are public health and social concerns. This is the case of harmful goods, like tobacco, alcoholic drinks and junk food. Although there are many ways in which a public authority may try to shape or limit the consumption of harmful goods (including, for instance, prevention programs), two types of policy instruments that have been widely used are indirect taxes and regulatory restrictions, where the latter can take a variety of forms, depending also on the type of goods under regulation: e.g., restrictions on youth access to the purchase of tobacco or alcoholic drinks, hours limits on the purchase of alcoholic drinks in pubs, and so on.\footnote{Taxation and/or regulatory policies aimed at limiting the consumption of harmful goods have been usually motivated by the argument that there are societal health costs connected with the consumption of harmful goods (see, e.g., Cremer et al., 2012). Other authors employ the intertemporal choice setting under hyperbolic-discounting to show that limiting the consumption of sin goods is justified in terms of social welfare because consumers behave in a time inconsistent manner, because of present-biased preferences (O’Donoghue and Rabin, 2003 and 2006, Gruber and Köszegi, 2004 and 2008).}

Excise taxes are aimed at increasing the price of the good in such a way to decrease the expected consumption. The empirical studies (see, e.g., Clements et al., 1997, and Nelson, 1997) seem to confirm that the price elasticity for alcoholic beverages is significantly large, implying that price increases lead to substantial decreases in consumption, even if there are significant differences between the type of alcoholic beverage and the subgroups of consumers taken into consideration.\footnote{For example, distilled spirits consumption is more responsive to price variations than wine consumption, which in turn is more sensitive to price variations than beer consumption (Leung and Phelps, 1993). On the other hand, the consumption of alcoholic drinks by young people is less responsive to price variations than the consumption by older people (Chaloupka and Wechsler, 1996).} The intensity of public intervention through taxation differs across countries and sectors. As reported by Cnossen and Smart (2005) and by the WHO (2004), the excise tax on a famous brand of tobacco was on average equal to 60\% of the retail price in the EU member states in 2003, whereas it was 19\% for beer, 14\% for wine and 39\% for spirits. Concerning junk food, in 2003 the WHO asked governments to tax them in order to limit obesity and related diseases. In 2013, the Mexican government introduced a tax on sugary drinks of about one peso per liter of soft drinks, and an 8\% sales tax on high-calorie foods.\footnote{The New York Times, *Mexico: Junk Food Tax Is Approved*, October 31, 2013.} In 2010 Australia introduced a 10\% tax on soft drinks, confectionery, biscuits and bakery...
products.\textsuperscript{4}

With regard to regulatory policies, there is a growing use of health warnings against the consumption of tobacco and alcoholic drinks, restrictions on where one can consume the harmful good, limitations around the permissible levels of tar, nicotine or alcohol, and so on. In this paper, we consider a specific regulatory policy, namely limitations to the permitted advertising of sin goods. Indeed, governments are increasingly restricting the possibility for firms to advertise harmful goods. For example, in the US there is a ban for tobacco companies from sponsoring sporting and entertainment events. Similar limitations can be found, among the others, in India, New Zealand and Taiwan. In Thailand, starting from 2013, new regulations require graphic health warnings to cover at least 85\% of cigarettes packages. From 2012, in the UK supermarkets are forced to hide cigarettes under the counter or behind shutters. Australia introduced packaging rules in 2012, imposing a combination of warning photos and no branding or logos. The rationale for such advertisement restrictions is rooted in the idea that advertising tends to artificially increase the demand for harmful goods (Safer and Chaloupka, 2000, Safer and Dave, 2006, Dave and Safer, 2013). Therefore, if governments aim at reducing tobacco or alcoholic drinks consumption, they can reach the goal also by restricting advertising of these goods (Safer and Dave, 2002).

However, perfect implementation of advertising bans is unfeasible, since the increasing reliance of public authorities on regulatory instruments has spurred countervailing measures by the producers of harmful goods. For instance, the Federal Trade Commission reports that in the US the expenditure on advertising by tobacco increased from around $8 billion in 1998 to about $13 billion in 2005 (FTC, 2007), although, as reported above, regulatory measures on advertising were tightened. Producers use also more sophisticated measures to circumvent limitations to advertising. For instance, in Malaysia tobacco firms adopt ‘indirect’ tobacco advertising and trademark diversification, through the establishment of companies marketing non-tobacco products and naming each after a cigarette brand name (Assunta and Chapman, 2014). With regard to the junk food US industry, children’s exposure to advertising declined by 4\% between 2002 and 2008, but adolescent exposure increased by 7\%, hinting that firms

\textsuperscript{4}Other examples can be found in Denmark, France and Hungary. Norway has had high duties on sweetened drinks and chocolate from 1981, and Samoa has taxed sugary drinks since 1984. Peru and Ireland are prompt to add levies on junk foods and there is a consistent debate in the US about this kind of taxation (see, for example, The New York Times, \textit{Bad Food? Tax It, and Subsidize Vegetables}, July 23, 2013).
may react to regulation also by changing the structure of their advertising policies. Yet a different picture emerges from alcohol advertising in US where, following advertising restrictions, it has been observed a significant reduction in youth exposure in magazine advertising between 2001 and 2008, as alcohol advertising has declined by 48% during the same period (CAMY, 2010). In addition to market measures, the producers of sin goods can use their political influence to lobby politicians and obtain more favorable legislative outcomes. For example, political pressures can be directed at resisting to proposals for stricter legislation on advertising restrictions.\(^5\)

In this paper, we address the issues outlined above by developing a theoretical model in which a public authority intervenes in a duopoly market of harmful goods by using an excise tax and regulatory measures on firms’ advertising. Our setup allows for strategic interactions between producers along two dimensions. The first one is the degree of substitutability/complementarity of sin goods. On the one hand, if goods are close substitutes (like, for instance, two brands of beer) then firms have little market power. On the other hand, if goods are complements (like, for instance, tobacco and alcoholic drinks can be for some consumers) then firms have strong market power.\(^6\) The second, and somewhat linked, strategic dimension is advertising. On the one hand, advertising can be predatory, meaning that advertising by one firm increases its own sales at the expense of its rival firm, and vice versa (clearly, this is more likely to be the case when the advertised goods are close substitutes). On the other hand, advertising by one firm can complement advertising by the other firm (like, for instance, when advertising promotes the type of lifestyle associated to a particular consumption habit).

Our theoretical framework builds on the literature on persuasive advertising. In broad terms, advertising can be divided into two categories: informative advertising and persuasive advertising (Dixit and Norman, 1978; see Bagwell, 2007, for a comprehensive review of the literature). The former refers to the case in which advertising informs the consumers about the existence of products or their prices. This paper does not consider this type of advertising.\(^7\) Instead, this paper is about persuasive advertising.\(^8\)

\(^5\)In our theoretical model, we do not consider neither firms’ strategies to limit the efficacy of advertising restrictions, nor lobbying of policy makers. We simply assume that regulation is costly for the government to implement and never perfect.

\(^6\)If sin goods are neither complements nor substitutes we have the special case of two separate monopolies.

\(^7\)A seminal contribution in this area of research is represented by Grossman and Shapiro (1984).

\(^8\)Sometimes the term ‘persuasive advertising’ is used to refer to different marketing practices. For example, it may refer to the firm’s efforts to convince the consumers that what they really want is a
The latter generally refers to marketing policies that enhance the value of a product at the eye of the consumer; that is, they increase the consumers’ willingness to pay but not his or her welfare. This implies that there is overconsumption of harmful goods, which in turn provides a rationale for public intervention to limit their consumption.

Our primary goal is to characterize the optimal structure of policy intervention — taxation and restrictions on advertising of sin goods — assuming that the goal of the public authority is to maximize a social welfare function defined as a weighted sum of consumers’ and producers’ surplus, where the relative weights assigned to consumers’ and producers’ surplus define the preferences of the policy maker. In particular, we allow them to range from equal weights — an efficiency-oriented authority — to zero weight to producers’ surplus — a consumer-oriented authority.

By first focusing only on taxation, our analysis shows that, irrespective of the type social welfare function, the optimal policy increases the welfare of all consumers but reduces firms’ profits. All consumers take advantage from taxation but not all for the same reason. Specifically, taxation in itself harms consumers whose consumption is not highly influenced by advertising, for the simple reason that their consumption is not severely distorted upwards and therefore higher prices because of taxation make them worse off. Overall, they benefit from taxation because the tax revenues they cash (tax revenues are distributed back uniformly on all consumers) more than outweigh the welfare loss for consumption distortion. On the contrary, consumers whose consumption is highly influenced by advertising take advantage both because the tax corrects downward their consumption of sin goods and because they cash tax revenues.

When regulation of advertising is brought into the picture, we find that the optimal combination of the two instruments (with respect to taxation only) increases producers’ surplus as well as the welfare of consumers that are highly sensitive to advertising, while it reduces that of consumers that are not highly influenced by advertising. The increase in profits is due to the fact that regulation of advertising calls for lower taxation. Lower taxation, combined with regulation, makes also all types of consumers better off, but only if the loss in tax revenues is not accounted for. If the latter are accounted for, we find that consumers that are not very sensitive to advertising would be better off.

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particular variety. That is, advertisement is used to change the ideal product variety. Furthermore, advertisement may be ‘persuasive’ in the sense that it leads consumers to perceive a higher differentiation between products with respect to the real difference between the products. That is, advertisement is used to increase the perceived difference among similar products. For a complete discussion of these issues, see von der Fehr and Stevik (1998).
under a tax-only policy than under a tax-plus-regulatory policy.

The rest of the paper is organized as follows. In Section 2 we characterize consumers’ preferences for sin goods in the presence of advertising and derive the demand functions. In Section 3 we derive the market equilibrium and then examine the impact of policy instruments on prices, advertising and consumption of sin goods. Government intervention in the sin goods market is examined in Section 4. Section 5 concludes.

2 The model

We consider a population of individuals that consume two types of goods: harmful goods (e.g., tobacco, alcoholic drinks, junk food) and ‘standard’ goods. Sin goods are exchanged in a duopoly market, with firms competing in prices and advertising. The markets for standard goods are instead perfectly competitive.

Our setup builds on the theoretical literature examining the impact of persuasive advertising on social welfare (see, e.g., Dixit and Norman, 1978, for an early contribution; see Bagwell, 2007, for a survey). Our novel feature consists in the introduction of public regulation of advertising of harmful goods as an additional instrument that can complement the traditional tax instrument.

In order to sharply focus on public intervention to correct for market distortions induced by advertising, we abstract from other reasons for public intervention in sin goods markets, such as overconsumption due to time inconsistent choices. This motivation for public intervention has been extensively analyzed within the framework of hyperbolic discounting (see, e.g., K˝oszegi 2005, O’Donoghue and Rabin, 2006). This means that, in our setup, consumers behave rationally when comparing the hedonic pleasure with the health harms of sin goods consumption (Becker and Murphy, 1988), while they are induced by advertising to over-consume such goods. As we discuss in Section 5, it is straightforward to extend our model to account for both factors of market failure, without affecting the main conclusions.

2.1 Advertising and consumers’ preferences

Let \( x_i \) denote the quantity consumed of the harmful good sold by firm \( i, i = a, b \), and let \( z \) denote the consumption of standard goods. Consumers’ preferences are represented
by the following utility function:

\[ u(x_a, x_b, z) = \sum_{i \in \{a,b\}} \left( \rho_i - h - \frac{x_i}{2} \right) x_i - \gamma x_a x_b + z. \]  

(1)

The variable \( \rho_i > 0 \) represents the marginal utility of sin good \( i \) at zero consumption level. It is therefore a measure of the intensity of preferences for good \( i \), and we describe below how this term is affected by firms’ advertising. The parameter \( h \geq 0 \) represents the per-unit-of-consumption health harm.\(^9\) Finally, the parameter \( \gamma \) captures the degree of product substitutability/complementarity between goods \( a \) and \( b \); \( \gamma \in (-1,1) \) is required to ensure strict concavity of the utility function in \((x_a, x_b)\). For \( \gamma > 0 \), the larger is \( \gamma \) the higher is the degree of substitutability in consumption, since an increase in consumption of good \( a \) reduces the marginal utility of good \( b \), and vice versa. For \( \gamma < 0 \), the larger is \( \gamma \) in absolute value the higher is the degree of complementarity in consumption, since an increase in consumption of good \( a \) increases the marginal utility of good \( b \), and vice versa. Marginal utilities are instead independent for \( \gamma = 0.\)\(^ {10} \)

As shown by the last term of Eq. (1), the utility of standard consumption goods is linearly increasing in consumption \( z \).

Let the parameter \( \rho > 0 \) represent the pre-advertising marginal utility (at zero consumption levels) of both goods. The post-advertising marginal utilities, \( \rho_i, i = a, b \) (again, at zero consumption levels), are assumed to take the following form, as a function of advertising levels \( \eta_a \) and \( \eta_b \) by firms (for a similar modelling approach of advertising, see Friedman, 1983):

\[ \rho_i = \rho + \varphi(r)s(\eta_i - k\eta_j), \quad i, j = a, b; \ i \neq j. \]  

(2)

Eq. (2) shows how persuasive advertising can impact on the preferences for sin goods. In particular, \( s \geq 0 \) is a parameter capturing to which extent a consumer is

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\(^9\)The assumption of rational consumers à la Becker and Murphy (1988) implies that the entire unit health harms \( h \) are accounted for in the utility function shown in Eq. (1). In the theory of hyperbolic discounting, present-biased preferences are characterized by unit health harms \( \beta h \), where \( \delta \) is the discount factor, while \( \beta \in [0,1] \) is the hyperbolic discount term, leading to over-consumption of sin goods.

\(^{10}\)In the given formulation of the utility function, positive values of the parameter \( \gamma \) represent an inverse measure of the degree of horizontal product differentiation between two types or brands of the same harmful good (e.g., two types or brands of alcoholic drinks). For \( \gamma = 0 \), product differentiation is maximal; for \( \gamma \rightarrow 1 \), goods become homogenous, since they are perfect substitutes in consumption. When \( \gamma < 0 \) (complementarity in consumption), the natural interpretation is that goods \( a \) and \( b \) are of different type; for instance, joint consumption of cigarettes and alcoholic drinks.
influenced by advertising, while the term $\varphi(r) \geq 0$ expresses the impact of government regulation on advertising effectiveness in influencing consumers (regulation is described in more detail below). Eq. (2) then shows that, for given $\varphi(r)s > 0$, preferences for sin good $i$ become more (less) intense with respect to the pre-advertising level if $\eta_i - k\eta_j$ takes positive (negative) values. Note that $\rho_i$ is — as it is natural to assume in the context of persuasive advertising — an increasing function of advertising $\eta_i$ by firm $i$, while it can be either an increasing or a decreasing function of advertising $\eta_j$ by the other firm, depending on the sign taken by the parameter $k \in [-1, 1]$. A positive value of $k$ implies that advertisement is ‘predatory’, since if firm $i$ increases the advertisement of its product the marginal utility of buying the product of the rival firm decreases. In other words, firm $a$ benefits from investing in advertising not only by making its product more valuable, but also by making the product of firm $b$ less valuable, to consumers (and similarly for firm $b$).\textsuperscript{11} In contrast, a negative value of $k$ implies that advertisement is cooperative, since advertisement by one firm makes the products sold by both firms more valuable to consumers. This is the case in which firms’ advertising focuses more on marketing the ‘product’ rather than individual brands, so that each firm’s advertising benefits its product as well as the competing one.\textsuperscript{12}

In interpreting Eq. (2), it is also important to realize the possibility of a link between the values taken by the parameters $k$ and $\gamma$. In fact, if the two harmful products are perceived as weak substitutes by consumers, then advertising by firm $j$ should not impact much on consumers’ preferences for the good produced by firm $i$, and vice versa (i.e., if $\gamma$ is close to zero, then also $k$ should be close to zero). The same for complementary sin goods (i.e., if $\gamma < 0$, then $k$ should be close to zero). On the contrary, if products are perceived as highly substitutable in consumption, then advertising by firm $j$ is likely to have a negative impact on consumers’ preferences for good $i$, and vice versa (i.e., if $\gamma$ is close to one, then $k$ should be positive).

Turning finally to government regulation, the variable $r \geq 0$ appearing in Eq. (2) represents a measure of a regulatory instrument (e.g., a ban on certain types of advertising contents, or a ban on certain types of media) that the government can use to dampen the impact of advertising on consumers’ preferences. We assume that $\varphi(r)$ equals one in the absence of regulation ($r = 0$) and then decreases as $r$ increases; i.e.,

\begin{itemize}
  \item[$\textsuperscript{11}$] A positive value of $k$ encompasses also the case of ‘comparative’ advertising (Chakrabarti and Haller, 2011, Anderson and Renault, 2009, Alipranti et al., 2013).
  \item[$\textsuperscript{12}$] This occurs, for instance, when advertising promotes primarily the kind of life-styles which are usually associated with the consumption of the harmful good.
\end{itemize}
\( \varphi(0) = 1, \varphi' < 0. \) Stricter regulation \( r, \) by reducing \( \varphi(r), \) implies that the firm has to increase its advertising level \( \eta_i \) to obtain the same impact on consumers’ preferences.

### 2.2 The demand for sin goods

The consumer’s budget constraint is

\[
 z \leq I + \ell - \sum_{i \in \{a,b\}} p_i x_i, \tag{3}
\]

where \( p_i \) is the price of the good produced by firm \( i, \) \( I \) is consumer’s income (exogenously given), and \( \ell \) is a lump sum transfer from the government, equal for all consumers, which is defined in Eq. (23) below. The price of good \( z \) is normalized to unity, since we assume that firms selling good \( z \) under perfect competition operate at constant marginal costs, whose level is normalized to unity.

A consumer type is then identified by a pair \((\rho, s, h)\) of attributes: the pre-advertising preferences for sinful consumption \( \rho, \) the sensitivity to persuasive advertising \( s, \) the per-unit-of-consumption health costs \( h. \) We assume instead that the degree \( \gamma \) of product substitutability/complementarity, as well as the sign and degree \( k \) of advertising spillovers, are identical for all consumers and thus single-valued.

Individuals’ attributes are distributed in the population, whose size is normalized to unity, according to the cumulative distribution \( F(\rho, s, h). \) A ‘bar’ over a parameter or a variable denotes its average or expected value over the distribution \( F(.). \)

By maximizing Eq. (1), subject to Eqs. (2) and (3), and taking prices and advertising levels as given, we obtain the individual demands for the harmful goods by a type \((\rho, s, h)\) consumer, \( x_i(p_a, p_b, \eta_a, \eta_b), \) \( i = a, b.\)\(^{13}\) Aggregate demands are then equal to \( \bar{x}_i(.) = E[x_i(.)], \) \( i = a, b \) (see Appendix A.1 for the analytical details).

### 3 Advertising and price competition

In this section, we first characterize market equilibrium and then illustrate how policy instruments impact on firms’ advertising and prices, and on sin goods consumption.

\(^{13}\)We drop the arguments \((\rho, s, h)\) to simplify notation whenever possible. Also, we assume that \( x_i(.) > 0 \) for all types at all equilibrium prices.
3.1 Market equilibrium

In the market for harmful goods, firms maximize profits by first competing on advertising and then on prices.\footnote{14} Firms’ profits are defined as follows:

\[ \pi_i(p_a, p_b, \eta_a, \eta_b) = (p_i - c - t) \bar{x}_i(p_a, p_b, \eta_a, \eta_b) - \frac{\alpha}{2} \eta_i^2, \quad i = a, b, \]  

(4)

where \( c \geq 0 \) is the marginal (and average) production cost, \( t \) is a per unit tax on sales of harmful goods,\footnote{15} and \( (\alpha/2)\eta_i^2, \alpha > 0, \) is the advertising cost function, convex (quadratic) in the advertising \( \eta_i \) by firm \( i \). Note that \( \eta_i \) represents advertising expressed in efficiency units. Since (i) advertising expenditure, \( E_i = (\alpha/2)\eta_i^2, \) is convex in \( \eta_i \), and (ii) the consumers’ willingness to pay for sin good \( i \) is linearly increasing in \( \eta_i \), it means that for firm \( i \) putting money in advertising shows decreasing returns in terms of influencing consumers’ behavior.\footnote{16}

The market equilibrium is solved by backward induction (see Appendix A.1 for the analytical details). In stage 2, for given \((\eta_a, \eta_b)\), each firm maximizes its profit function (4) with respect to the own price, taking as given the price of the other firm. Denote with \( p_i^{**}(\eta_a, \eta_b), i = a, b \), the (unique) Nash equilibrium at stage 2.\footnote{17} By substituting the equilibrium prices \( p_i^{**} \) into Eq. (4), we then obtain profits as a function of advertising:

\[ \pi_i^{**}(\eta_a, \eta_b) = (p_i^{**} - c - t) \bar{x}_i(p_a^{**}, p_b^{**}, \eta_a, \eta_b) - \frac{\alpha}{2} \eta_i^2, \quad i = a, b. \]  

(5)

In stage 1, each firm maximizes its profit function (5) with respect to own advertising, taking as given advertising by the other firm. The unique, and symmetric, Nash
equilibrium with positive advertising by firms is given by:

\[ \eta^*(t, r) = \frac{2(\bar{p} - \bar{h} - c - t)(2 + k\gamma - \gamma^2)\varphi(r)\bar{s}}{\Psi} \]  

(6)

The condition for \( \eta^*(t, r) > 0 \) is

\[ \bar{p} - \bar{h} - c - t > 0, \]  

(7)

while the condition for a unique and stable Nash equilibrium is (see Appendix A.1)

\[ \Psi \equiv \alpha(2 - \gamma)^2(2 + \gamma)(1 + \gamma) - 2(1 - k)(2 + k\gamma - \gamma^2)[\varphi(r)\bar{s}]^2 > 0. \]  

(8)

By substituting \( \eta^* \) from Eq. (6) into second-stage equilibrium prices \( p_i^*(\eta_a, \eta_b) \), we finally obtain the equilibrium price of sin goods,

\[ p^*(t, r) = c + t + \frac{(1 - \gamma)(\bar{p} - \bar{h} - c - t + \varphi(r)\bar{s}(1 - k)\eta^*(t, r))}{2 - \gamma}, \]  

(9)

which is the same for goods \( a \) and \( b \) by symmetry between firms.

Aggregate consumption of each harmful good is equal to (see Appendix A.1)

\[ \bar{x}^*(t, r) = \frac{\bar{p} - \bar{h} + \varphi(r)\bar{s}(1 - k)\eta^* - p^*}{1 + \gamma}, \]  

(10)

while, after substituting \( \eta^* \) from Eq. (6) and \( \bar{x}^* \) from Eq. (10) into Eq. (5), we get the equilibrium profits of each firm:

\[ \pi^*(t, r) = (p^* - c - t)\bar{x}^* - \frac{\alpha}{2}(\eta^*)^2. \]  

(11)

In Appendix A.1, we show that \( \pi^*(t, r) > 0 \), provided that the profit function \( \pi_i^{**} \) defined in Eq. (5) is strictly concave in own advertising \( \eta_i \).

At this point, it is useful to compare, albeit only informally, the market outcomes under advertising and price competition with those under price competition only, with firms abstaining from advertising their products (see Appendix A.3 for equilibrium prices and quantities in a market without advertising). Advertising, by inflating consumers’ demand, allow firms to increase prices and total sales of sin goods. However, firms’ profits increase only if the increase in revenues outweighs the costs of advertising. Firms are more likely to be engaged in a prisoners’ dilemma, with advertising competition resulting in a reduction of their profits, when advertising is predatory (i.e., values

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\(^{18}\)Whenever equilibrium variables are symmetric in the index \( i \), we suppress it to simplify the notation.
of \( k \) close or equal to one) and/or when firms’ products are close substitutes (i.e., values of \( \gamma \) close to one). Product and advertising complementarity (i.e., negative values of \( \gamma \) and \( k \), respectively) are instead associated to positive profit gains from advertising. Note also that advertising, by increasing the price of sin goods, reduces consumption (and welfare) of individuals that are not influenced, or are not much influenced, by advertising, whereas it increases consumption (and perceived welfare, but not ‘true’ welfare) of those that are highly influenced by it.

### 3.2 Comparative statics on policy instruments

We now conduct some comparative statics analysis to see how taxation and regulation of advertising impact on firms’ advertising, prices and sin goods consumption. All the analytical details are in Appendix A.2.

By differentiating Eq. (6), we see that both higher taxation of harmful consumption and stricter regulation of its advertising reduce the equilibrium level of advertising:

\[
\frac{\partial \eta^*}{\partial t} = -\frac{\eta^*}{\rho - h - c - t} < 0, \tag{12}
\]

\[
\frac{\partial \eta^*}{\partial r} = (1 \frac{\partial \varphi}{\partial r} - \frac{1}{\Psi} \frac{\partial \Psi}{\partial r}) \eta^* < 0, \tag{13}
\]

where it is \( \frac{\partial \Psi}{\partial r} > 0 \).

By differentiating Eq. (9), we see that while stricter regulation always reduces the price of sin goods, the effect of higher taxation is ambiguous:

\[
\frac{\partial p^*}{\partial t} = \frac{1}{2 - \gamma} + \frac{1 - \gamma s(1 - k)}{2 - \gamma} \frac{\partial \eta^*}{\partial t}, \tag{14}
\]

\[
\frac{\partial p^*}{\partial r} = \frac{1 - \gamma s(1 - k)}{2 - \gamma} \left( \eta^* \frac{\partial \varphi}{\partial r} + \varphi \frac{\partial \eta^*}{\partial r} \right) < 0. \tag{15}
\]

Stricter regulation, by reducing advertising and hence consumers’ demand, reduces the market price of sin goods. This effect is also present when taxation is increased. However, higher taxation, by increasing firms’ marginal costs, tends also to determine an increase in prices. The net effect is ambiguous.

Both higher taxation and stricter regulation reduce aggregate consumption of harmful goods:

\[
\frac{\partial \bar{x}^*}{\partial t} = \frac{1}{1 + \gamma} \left( \varphi s(1 - k) \frac{\partial \eta^*}{\partial t} - \frac{\partial p^*}{\partial t} \right) = -\frac{\alpha (4 - \gamma^2)}{\Psi} < 0, \tag{16}
\]

\[
\frac{\partial \bar{x}^*}{\partial r} = \frac{1}{1 + \gamma} \left[ s(1 - k) \left( \eta^* \frac{\partial \varphi}{\partial r} + \varphi \frac{\partial \eta^*}{\partial r} \right) - \frac{\partial p^*}{\partial r} \right] = -\frac{1}{\Psi} \frac{\partial \Psi}{\partial r} \bar{x}^* < 0. \tag{17}
\]
As for individual consumption, by substituting the equilibrium prices and advertising levels, defined respectively in Eqs. (9) and (6), into the individual demand functions, defined in Eq. (A.1), we obtain the equilibrium consumption levels of the harmful goods by a type \((\rho, s, h)\) consumer:

\[
x^*(t, r) = \frac{\rho - h + \varphi(r)s(1 - k)\eta^* - p^*}{1 + \gamma}.
\]

By differentiation of Eq. (18), and by using Eqs. (14)-(15), the impact of policy instruments on individual consumption levels can be expressed as follows:

\[
\frac{\partial x^*}{\partial t} = -\frac{1}{(1 + \gamma)(2 - \gamma)} + \frac{\varphi(r)(1 - k)}{1 + \gamma} (s - \tilde{s}) \frac{\partial \eta^*}{\partial t},
\]

\[
\frac{\partial x^*}{\partial r} = \frac{1 - k}{1 + \gamma} (s - \tilde{s}) \left( \eta^* \frac{\partial \varphi}{\partial r} + \varphi \frac{\partial \eta^*}{\partial r} \right) \leq 0 \quad \text{iff} \quad s \geq \tilde{s},
\]

where

\[
\tilde{s} = \frac{1 - \gamma}{2 - \gamma} \hat{s} < \tilde{s}.
\]

In the case of regulation, Eq. (20) shows that individuals characterized by a value of \(s\) above the threshold \(\hat{s}\) reduce consumption in response to stricter regulation, while those with a value below the threshold increase consumption. The explanation is simple. Stricter regulation reduces firms’ advertising and therefore aggregate consumption of harmful goods. However, by reducing their price, it also stimulates individual consumption. For consumers that are not very sensitive to advertising (i.e., those with \(s < \hat{s}\)), the latter effect prevails over the former, while the converse holds true for consumers that are highly influenced by advertising. The same channel operates in the case of taxation, which also reduces firms’ advertising, as it is shown by the second term on the right hand side of Eq. (19). However, since taxation tends also to increase the price of harmful goods, it bears a negative impact on the consumption of all individual types, as shown by the first term in Eq. (19). Increased consumption by consumers that are not very sensitive to advertising is therefore possible also after a tax increase. Note finally that increased individual consumption by consumers that are not very sensitive to advertising, as a result of stricter regulation or higher taxation, is possible only when firms have market power. Under perfect competition \((\gamma \rightarrow 1)\), all consumers reduce their sin goods consumption when either \(r\) or \(t\) are increased. The reason is that when firms price at marginal cost regulation does not impact on market price and taxation is always shifted 100% on consumers price.
4 Government intervention

We now examine government policies aimed at limiting sin goods consumption. We first define the social welfare functional representing the objective function of the public authority. To set the benchmark, we then derive the optimal tax policy in a market in which firms do not advertise sin goods. Finally, we characterize the optimal tax and regulation policy in the presence of advertising.

4.1 Social welfare

To define social welfare, we appeal to the literature on the normative analysis of persuasive advertising (e.g., Dixit and Norman, 1978), which takes the pre-advertising consumers’ preferences as an unbiased measure of the utility enjoyed by consumers of sin goods. Welfare of a type \((\rho, s, h)\) consumer is thus defined as follows:

\[
\omega^{\ast}(t, r) = 2\left(\rho - h - \frac{x^{\ast}}{2}\right)x^{\ast} - \gamma(x^{\ast})^{2} - 2p^{\ast}x^{\ast} + \ell^{\ast} + I, \tag{22}
\]

where \(\ell^{\ast}\) — the per capita lump sum subsidy that the government distributes to consumers — is equal to average tax revenues, \(2t\bar{x}^{\ast}\), minus the administration costs, \(\xi(r)\), that the government sustains to enforce the regulatory measures on advertising, with \(\xi(.)\) a convex function:

\[
\ell^{\ast}(t, r) = 2t\bar{x}^{\ast}(t, r) - \xi(r) \geq 0. \tag{23}
\]

Note that we impose the constraint that net tax revenues must be non-negative, which implies that sin goods cannot be subsidized. The reason for imposing such a constraint, however, is not an \textit{ad hoc} restriction on the tax instrument. The reason is that, while positive revenues can be distributed lump sum to consumers (in the form, for instance, of public transfers or public goods uniformly distributed), negative revenues requires lump sum taxation of consumers. And lump sum taxes, although fully efficient, are however unfeasible because of imperfect information.

With equilibrium profits defined in Eq. (11), aggregate social welfare can be defined as:

\[
\Omega^{\ast}(t, r) = W^{\ast}(t, r) + \theta \Pi^{\ast}(t, r), \tag{24}
\]

\footnote{In line with most of the literature on optimal taxation, it is instead assumed that taxation is costless to administer.}
where

\[ W^* = E_F [w^*], \quad \Pi^* = 2\pi^*; \]

are aggregate consumers’ welfare and aggregate firms’ profits, respectively. The parameter \( \theta \in [0, 1] \) represents the relative weight the public authority assigns to producers’ surplus with respect to consumers’ surplus. At one extreme, if \( \theta = 1 \), then \( \Omega^* \) is a measure of the aggregate surplus generated by the market, which means that the goal of the public authority is market efficiency. At the other extreme, if \( \theta = 0 \), only consumers’ surplus and tax revenues — which are distributed to consumers — count in social welfare.\(^{20}\)

### 4.2 Optimal tax policy in a market without advertising

In order to highlight how advertising calls for public intervention, it is useful to set the benchmark by characterizing the optimal tax policy in a market without advertising (clearly, in such a situation, regulation of advertising is redundant).

In a market without advertising, a public authority taking Eq. (24) as its objective function, would set the tax rate on sales of sin goods equal to (see Appendix A.3):

\[
t_{\text{noadv}} = \frac{(1 - 2\theta)(1 - \gamma)(\rho - h - c)}{3 - 2\theta - 2\gamma(1 - \theta)}. \tag{25}
\]

If the goal of public policy is efficiency (\( \theta = 1 \)), then it is optimal to subsidize sin goods with a negative tax rate,

\[
t_{\theta=1}^{\text{noadv}} = -(1 - \gamma)(\rho - h - c), \tag{26}
\]

in order to correct for firms’ with market power pricing above marginal cost. In fact, the subsidy \( t_{\theta=1}^{\text{noadv}} \) would drive the market price at the marginal production cost \( c \), and the aggregate consumption of sin goods, below its efficient level because of imperfect competition, would be driven to its efficient level. Since the only market distortion is due to the price-cost mark-up (recall that consumers are rational, so they fully account for the health costs of sin goods consumption), taxation targets this market failure by subsidizing firms. However, since the subsidy would be paid by consumers, the above

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\(^{20}\)The public authority may want to exclude, or give little weight, to firms profits for various reasons. For instance, when firms are foreign multinationals whose profits are not distributed to resident consumers, or when profits are distributed to resident individuals but are highly concentrated on few wealthy shareholders, or simply for ideological motivations.
policy would maximize the aggregate surplus of the economy by reducing consumers’ surplus and by augmenting producers’ surplus. Moreover, it would require lump sum taxation of consumers, a situation we have ruled out above by setting the restriction that net revenues must be non-negative.

Which are then the feasible tax policies in the absence of advertising? From Eq. (25), it is immediate to see that $t^{\text{noadv}} \leq 0$ if $\theta \geq \frac{1}{2}$. The optimal tax is zero if $\theta \geq \frac{1}{2}$, while it is positive if $\theta < \frac{1}{2}$. For instance, if $\theta = 0$, then:

$$t_{\theta=0}^{\text{noadv}} = \frac{1 - \gamma}{3 - 2\gamma}(\rho - h - c).$$

Under this policy, although consumers are worse off because taxation reduces their consumption of sin goods (which was already below to its efficient level because of prices above marginal cost), overall they end up being better off because they cash tax revenues. Firms are instead worse off, since their profits decrease.

### 4.3 Optimal tax policy in a market with advertising

In this section, we characterize the optimal tax policy in the presence of persuasive advertising, taking as given the level $r$ of regulation of advertising. The optimal degree of regulation is examined in the next section.

By differentiating the social welfare function, defined in Eq. (24), with respect to the tax rate, we obtain (see Appendix A.4):

$$\frac{1}{2} \frac{\partial \Omega^*}{\partial t} = -\varphi(r)(1 - k)\eta^*E\left[s\frac{\partial x^*}{\partial t}\right] +$$

$$+ \theta(p^* - c)\frac{\partial x^*}{\partial t} - \theta\alpha \eta^*\frac{\partial \eta^*}{\partial t} +$$

$$+ (1 - \theta)\frac{1}{2}\frac{\partial \ell^*}{\partial t} - (1 - \theta)x^*\frac{\partial p^*}{\partial t}. \tag{28}$$

This derivative shows the various types of marginal benefits and marginal costs of a small increase in the tax rate $t$, for given regulation $r$.

The term in the first row is positive and represents the marginal welfare gains accruing to consumers because taxation reduces over-consumption of sin goods whose demand is inflated by firms’ advertising. Note that these marginal benefits depend both on taxation impacting on the price $p^*$ and on taxation impacting on advertising $\eta^*$ (see the expression for $\partial x^*/\partial t$ in Eq. 19). The latter effect cancels out by the envelope theorem in the consumers’ maximization problem (see Appendix A.4). Note also that these marginal welfare gains do not depend on the type of social welfare
function, since consumers’ welfare have a fixed unit weight in Eq. (24). All the other elements appearing in Eq. (28) depend instead on the type of social welfare function.

The first term in the second row of Eq. (28) is negative and represents a marginal cost of increasing taxation, namely the fact that higher taxation increases the price-cost margin $p^* - c$, which is already positive because of imperfect competition. This term thus calls for a subsidy component in the determination of the optimal tax, as already discussed in the previous subsection. The second term is instead positive and represents the marginal benefits brought about by taxation by reducing firms’ costs on advertising, which are a waste of resources for society. Note finally that the marginal cost and the marginal benefit in the second row are proportional to $\theta$, the weight given to profits in social welfare.

While the three elements in the first and second row of Eq. (28) illustrated above are linked to efficiency (aggregate surplus) the two terms in the second row are linked to the distribution of the surplus between consumers and firms. Indeed, both factors have weight $1 - \theta$. The first term represents the net marginal benefits of consumers cashing tax revenues and firms paying for them. For $\theta = 1$, the transfer of resources from firms to consumers through tax revenues becomes irrelevant for social welfare. Finally, the second term in the third row of Eq. (28) represents, provided that taxation increases the price of sin goods, the marginal welfare cost for consumers, net of the marginal benefits for firms, of taxation increasing the price of sin goods.

We next use the derivative shown in Eq. (28) to obtain formulae for the optimal tax rate in the two extreme cases in which firms’ profits have the same weight as consumers’ welfare ($\theta = 1$) and firms’ profits have zero weight ($\theta = 0$).

Setting $\theta = 1$, from the first order condition obtained from setting Eq. (28) equal to zero, we get the following (implicit) expression for the optimal tax rate (see Appendix A.4):

$$ t(r)\big|_{\theta=1} = \varphi(r)s(1-k)\eta^* + (2 - \gamma)\varphi(r)(1-k)\eta^* \frac{\text{cov} [s, \partial x^*/\partial t]}{\partial x^*/\partial t} + (2 - \gamma)\alpha\eta^* \frac{\partial \eta^*}{\partial t} - (1 - \gamma)(\bar{p} - \bar{h} - c). \quad (29) $$

Eq. (29) shows that the optimal tax is made up of three positive components and a negative one (the last one). The first term of the optimal tax rate is equal to the average bias in consumers marginal utilities for sin goods (at zero consumption levels) that are due to advertising. Note that this term is zero if $k = 1$, i.e., if advertising by the two competing firms is ‘fully predatory’, so that they neutralize each other. Since
the covariance term is negative, also the the second component of the optimal tax is positive. That $\text{cov}[s, \partial x^*/\partial t] < 0$ can be immediately checked from Eq. (19): the more a consumer is influenced by advertising, the more she will respond to taxation by reducing her consumption, since taxation reduces advertising. The larger the covariance term in absolute value, the larger is therefore the optimal tax rate, since taxation targets more intensively the consumers that more need to be discouraged from consuming sin goods.

The first term in the second row is positive and depends on the marginal cost of advertising $\alpha$ by firms. As already illustrated above, taxation is useful for correcting for the wasteful costs sustained by firms for advertising. Note, indeed, that this term of the optimal tax is present also if $k = 1$. That is, even if advertising does not distort preferences, taxation is useful as a corrective device for the waste of resources that firms devote to it. The last term, which is negative, is the subsidy component of the optimal tax, already discussed at length above.

Next, by setting $\theta = 0$, we obtain the following expression (again, an implicit equation) for the optimal tax rate (see Appendix A.4):

$$t(r)|_{\theta=0} = \varphi(r)\bar{s}(1-k)\eta^* + \varphi(r)(1-k)\eta^* \frac{\text{cov}[s, \partial x^*/\partial t]}{\partial x^*/\partial t} - \frac{1 - \partial p^*/\partial t}{\partial x^*/\partial t}. \tag{30}$$

In this expression for the optimal tax rate for $\theta = 0$, the first term is identical to the first term in the expression for $\theta = 1$, while the second term is also identical but for the factor $(2 - \gamma)$, which is present in Eq. (29) but not in Eq. (30). The third and last term is instead different and it is positive. This term represents the marginal benefits accruing to consumers when a higher tax increases tax revenues (which are cashed by consumers), net of the loss due to the increase in the price of sin goods (provided, of course, that the price is increasing with taxation). The term $1 - \partial p^*/\partial t$ is always positive even if $\partial p^*/\partial t > 0$, since with linear demand taxation never causes price overshifting and thus $\partial p^*/\partial t < 1$.

Having characterized the optimal tax for given level of regulation of advertising, we not let the public authority to use also this instrument.
4.4 Optimal regulation of advertising

By differentiating the social welfare function with respect to \( r \), we obtain (see Appendix A.5):

\[
\frac{1}{2} \frac{\partial \Omega^*}{\partial r} = -\varphi(r)(1-k)\eta^* \text{E} \left[ s \frac{\partial x^*}{\partial r} \right] - \frac{1}{2} \frac{\partial \xi}{\partial r} + \\
+ \theta(p^* - c) \frac{\partial \bar{x}^*}{\partial r} - \theta \alpha \eta^* \frac{\partial \eta^*}{\partial r} + \\
+ (1-\theta) t \frac{\partial x^*}{\partial r} - (1-\theta) \bar{x}^* \frac{\partial p^*}{\partial r}. \tag{31}
\]

All terms appearing in Eq. (31), but the second one in the first row — representing the marginal cost to enforce stricter regulation of advertising, are similar to the terms appearing in Eq. (28), with derivatives with respect to \( r \) instead of \( t \). Their interpretation is similar and hence is omitted. It is instead interesting to work out in more detail Eq. (31) in the two specific instances of social welfare function considered above for taxation.

By setting \( \theta = 1 \), and setting the tax rate optimally as a function of \( r \) according to Eq. (29), the derivative in Eq. (31) can be written as:

\[
\frac{1}{2} \frac{\partial \Omega^*}{\partial r} \bigg|_{t=t(r) \theta=1} = -\varphi(r)(1-k)\eta^* \left\{ \text{cov} \left[ s, \frac{\partial x^*}{\partial t} \right] - \frac{\partial \text{E}^*}{\partial r} \right\} \frac{\partial \bar{x}^*}{\partial r} + \\
+ \alpha \eta^* \left\{ \frac{\partial x^*}{\partial t} - \frac{\partial \eta^*}{\partial r} \right\} \frac{\partial x^*}{\partial r} - \frac{1}{2} \frac{\partial \xi}{\partial r}. \tag{32}
\]

The derivative in Eq. (32) consists of three terms, the last one of which represents the marginal costs of enforcing stricter regulation. Hence, for regulation of advertising to be desirable, it must be that at least one of the first two terms in Eq. (32) is positive and greater than marginal costs. Let us then examine these two terms.

The first term in Eq. (32) refers to regulation reducing sin goods consumption. Since both covariances are negative (see Eqs. 19 and 20), this term is positive if and only if:

\[
\text{cov} \left[ s, \frac{\partial x^*}{\partial t} \right] \frac{\partial \bar{x}^*}{\partial r} > \text{cov} \left[ s, \frac{\partial x^*}{\partial t} \right]. \tag{33}
\]

If this is the case, it means that stricter regulation of advertising is more effective than higher taxation in determining diversified reductions in sin goods consumption between individuals with different sensitivities to advertising. Then, even if taxation is already optimal set, introducing regulation of advertising improves on social welfare.
A similar argument goes for the second term in Eq. (32, which refers to the reduction in advertising costs sustained by firms. This term is positive if and only if:

\[
\frac{\partial \eta^s}{\partial \pi^t} \frac{\partial \pi^t}{\partial \pi^t} > 0.
\]

That is, if regulation is relatively more effective than taxation at curbing advertising.

If \( \theta = 0 \), and setting the tax rate optimally as a function of \( r \) according to Eq. (30), the derivative in Eq. (31) can be written as:

\[
\frac{1}{2} \left. \frac{\partial \Omega^*}{\partial r} \right|_{t=t(r) | \theta=0} = \frac{\varphi(r)(1-k)}{\eta^*} \left\{ \frac{\text{cov} [s, \partial \pi^t/\partial t]}{\partial \pi^t/\partial t} - \frac{\text{cov} [s, \partial \pi^t/\partial r]}{\partial \pi^t/\partial r} \right\} \frac{\partial \pi^t}{\partial r} + x^s \left\{ \frac{\partial p^s/\partial t}{\partial \pi^t/\partial t} - \frac{\partial p^s/\partial r}{\partial \pi^t/\partial r} \right\} \frac{\partial \pi^t}{\partial r} - \frac{1}{2} \frac{\partial \xi}{\partial r}.
\]

The first and the last term are identical to the first and the third one in Eq. (32). The second term in Eq. (35) is positive if and only if:

\[
\frac{\partial p^s/\partial t}{\partial \pi^t/\partial t} - \frac{\partial p^s/\partial r}{\partial \pi^t/\partial r} < 0.
\]

Since \( \partial p^s/\partial r < 0 \), \( \partial \pi^t/\partial r \) and \( \partial \pi^t/\partial t \), a sufficient condition for inequality in Eq. (36) to hold true is that \( \partial p^s/\partial t > 0 \). If taxation increases the consumers’ price, then regulation is beneficial, since it instead reduces the consumers’ price.

Finally, the third term in Eq. (35) is negative, and represents the marginal loss in tax revenues cashed by consumers when regulation is made stricter.

### 4.5 Optimal policies: a numerical simulation

The analysis of the derivatives of the social welfare function with respect to the policy instruments conducted above highlights which are the driving forces behind public intervention in the sin goods market. However, they do not allow a clear characterization of the optimal structure of government policy. To this end, we present, in Table 1, some numerical simulations (for the details, see Appendix A.6).

The table is divided in three parts. In the first (rows 0a–5a), the demands for sin goods \( a \) and \( b \) are independent (\( \gamma = 0 \)) and there are no advertising spillovers (\( k = 0 \)). In other terms, the market for sin goods consists of two separated monopolies, not interacting with each other. In the second part of the table (rows 0b–5b), sin goods are substitutes in consumption (\( \gamma = 0.5 \)) and advertising is predatory, since demand spillovers are negative (\( k \) positive, equal to 0.25). Finally, in the third part (rows 0c–5c) sin goods are complements in consumption (\( \gamma = -0.5 \)), as it is advertising by firms,
since demand spillovers are positive \( (k \text{ negative, equal to } -0.25) \). This because we want
to see whether and how public policy is affected by the type of strategic interactions
occurring between firms in the sin goods market.

Consider the two-monopoly case. Row 0a shows the equilibrium prices \((p)\), quantities \((x, x_j)\), consumers’ welfare \((W, w_j)\) and firms’ profits \((\Pi)\) in markets without
advertising. The simulation considers four consumers types, indexed by \( j = 1, 2, 3, 4, \)
of increasing sensitivity to advertising (the parameter \( s \) in the model). In particular,
type 1 does not care about advertising, while type 4 and 3 are three and two times more
sensitive to advertising than type 2. Apart from this, consumers are identical. Clearly,
in the absence of advertising, all types of individuals consume the same amount of sin
goods and enjoy the same level of welfare, the value of which has been normalized to
zero to serve as a reference point.

Row 1a shows the equilibrium with advertising \((\eta)\) but without government inter-
vention. As expected, with firms’ advertising, the market price increases, as well as
aggregate consumption and — in this case — firms’ profits. For types 2, 3 and 4, ad-
vertising causes a welfare loss, because it distorts their demand upwards. Also type 1
experiences a welfare loss, but for a different reason: since her demand is not distorted
by advertising, it is the price increase due to advertising that makes her worse-off.

The remaining four rows report the optimal policies, first for an efficiency based
social welfare function \((\theta = 1)\), rows 2a and 3a), then for a consumer-based social welfare
function \((\theta = 0)\), rows 4a and 5a). In both cases, it is first computed the optimal tax
rate with no regulation of advertising, and then the optimal mix of policy instruments.

The results can be summarized as follows. If the public authority uses only taxation,
then the optimal tax makes all types of consumers better off and firms worse off, no
matter the type of social welfare function and no matter the type of market structure
(for \( \theta = 1 \), compare row 1 with 2; for \( \theta = 1 \), compare row 1 with 4; in both cases, for
a, b, c). Column \( t \) reports the unit tax rate, while column \( t\% \) reports the tax \( t \) as a
percentage of producers’ price \((p - t)\). Different consumers benefit from taxation for
different reasons. Taxation in itself has a negative impact on consumers that do not
respond, or respond weakly, to advertising, since they are not overconsuming sin goods
and therefore the price increase caused by taxation makes them worse off. They benefit
from taxation only once the tax revenue they cash is accounted for. Consumers that
are very sensitive to advertising, instead, benefit for two reasons: first because taxation
reduces overconsumption of sin goods; second because they cash tax revenues.
Table 1: Optimal policies in markets with persuasive advertising.

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<td>Sin goods are neither substitutes nor complements (γ = 0) and advertising by firms are independent (k = 0)</td>
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<td>Sin goods are substitutes (γ = 0.5) and advertising by firms is predatory (k = 0.25)</td>
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Values for θ, t, r, p, η, x, x1, x2, x3, x4, W, w1, w2, w3, w4, and Π represent the optimal policies in markets with persuasive advertising. The table provides a comparison of policies under different conditions, including varying levels of advertising and economic parameters.
The picture is different if also regulation of advertising is employed. By comparing row 2 with 3 (for $\theta = 1$), or row 4 with 5 (for $\theta = 0$), we see that adding regulation to taxation increases firms’ profits and the welfare of consumers that are highly sensitive to advertising, whereas it reduces the welfare of consumers that are not highly influenced by advertising. Profits increase because introducing regulation reduces the optimal tax rate: dramatically, when the goal of the authority is efficiency ($\theta = 1$), not so much if the goal is consumers’ welfare ($\theta = 0$). Actually, the combined tax-regulation policy makes in itself all types of consumers better off. It is only because of the drop in tax revenues that the consumers not very sensitive to advertising end up being worse off.

The conclusion is thus that, excluding the welfare changes due to the distribution of tax revenues, adding regulation of advertising to taxation makes both consumers and firms better off. The inclusion of tax revenues, however, shows that regulation determines a conflict of interest between consumers that do not respond to advertising and consumers that they do.

5 Concluding remarks

In this paper we examine how a public authority can intervene in sin goods markets to correct for overconsumption induced by persuasive advertising by firms. In particular, we show how regulation of advertising can complement the traditional instrument of taxing sin goods. The main result is that by introducing regulation of advertising, and accordingly adjust the tax rate optimally, brings benefits not only to the consumers that are more sensitive to advertising — because they are those that more overconsume sin goods — but also firms, that see their profits to increase — because regulation calls for lower taxation. Consumers that are not influenced by advertising, or that are not very sensitive to it, are instead damaged by regulation of advertising — because the benefits of a lower tax on their consumption choices are more than offset by the loss of tax revenues.

As anticipated in Section 2, our results are robust to the relaxation of the simplifying assumption that of fully rational consumers with respect to the health harms caused by sin goods consumption. If, as in O’Donoghue and Rabin (2006), we consider consumers that have present biased preferences, with $\beta \in [0, 1]$ the hyperbolic discounting parameter expressing the degree of the bias (with $\beta = 1$ there is no bias), then our results carry through with the addition of the term $\hat{h} - \hat{\beta}h$ to the formula of
the optimal tax rate, representing the so-called (aggregate) marginal ‘internality’; that is, the share of health harms that consumers fail to internalize in their consumption decisions because of a bias for present sin goods consumption with respect to the health damages they suffer in the future.

Given the wide array of policies that governments employ to cope with unhealthy consumption habits, our work can be extended in several directions. Among these, two are more directly linked to the issue addressed in this paper. The first is to consider alternative, or complementary, policy instruments to impact on the advertising of sin goods. Two means of making more costly for firms to advertise sin goods are partial deductibility of expenditures on advertising to compute profits, or taxation of advertising services. The second extension is to add to the picture ‘advertising’ by the government, in the form of public campaigns to inform the population about the hazards of consuming sin goods. Again, the key research question is to investigate how the various policy instruments interact to fulfill the goals of the public authority.
References

Aliprantis, M, E Mitrokostas and E Petrakis, 2010, Comparative versus informative advertising in oligopolistic markets, working paper, University of Crete.


Center on Alcohol Marketing and Youth (CAMY), 2010 Report.


Mathematical Appendix

A.1 Derivation of market equilibrium

By maximizing Eq. (1), subject to Eqs. (2) and (3), we obtain the individual demands for the harmful goods by a type \((\rho, s, h)\) consumer,

\[
x_i(p_a, p_b, \eta_a, \eta_b) = \frac{\rho - h}{1 + \gamma} + \varphi(r)s \left[ (1 + k\gamma)\eta_i - (k + \gamma)\eta_j \right] + \gamma p_j - p_i,
\]

\[i, j = a, b; \ i \neq j. \quad (A.1)
\]

Aggregate demands are then equal to:

\[
\bar{x}_i(p_a, p_b, \eta_a, \eta_b) = \frac{\rho - h}{1 + \gamma} + \varphi(r)s \left[ (1 + k\gamma)\eta_i - (k + \gamma)\eta_j \right] + \gamma p_j - p_i,
\]

\[i, j = a, b; \ i \neq j. \quad (A.2)
\]

From the first order conditions for maximizing the profit functions \(\pi_i\) defined in Eq. (4) with respect to the own price \(p_i\), we obtain the unique Nash equilibrium:

\[
p_i^{**}(\eta_a, \eta_b) = c + t + \frac{(1 - \gamma) \left[ \tilde{\rho} - \tilde{h} - c - t + \varphi(r)s(\eta_i - k\eta_j) \right]}{2 - \gamma} + \gamma \varphi(r)s(1 + k)(\eta_i - \eta_j), \quad i, j = a, b; \ i \neq j. \quad (A.3)
\]

The second order conditions for a maximum are satisfied, since the profit functions \(\pi_i\) are strictly concave in own prices, \(\partial^2 \pi_i / \partial p_i^2 = -2/(1 - \gamma^2) < 0\). Moreover, the Nash equilibrium is stable, since \(1 - (\partial \tilde{p}_a / \partial p_b)(\partial \tilde{p}_b / \partial p_a) = 1 - \gamma^2/4 > 0\), where \(\tilde{p}_i\) is the best-response function of firm \(i\) to price \(p_j\) set by the other firm.

By substituting the equilibrium prices \(p_i^{**}\) into the profit functions (4), we obtain profits \(\pi_i^{**}(\eta_a, \eta_b)\), defined in Eq. (5), as a function of advertising by firms. We assume that the profit function \(\pi_i^{**}(.)\) is strictly concave in advertising \(\eta_i\). That is, we assume that

\[
\frac{\partial^2 \pi_i^{**}}{\partial \eta_i^2} = \frac{2(2 + k\gamma - \gamma^2)^2 [\varphi(r)s]^2}{(2 - \gamma)^2(2 + \gamma)^2(1 - \gamma^2)} - \alpha < 0, \quad i = a, b. \quad (A.4)
\]

Since \(2 + k\gamma - \gamma^2 > 0\) for \(\gamma \in (-1, 1)\), \(k \in [-1, 1]\), the first term in Eq. (A.4) is positive and reflects the fact that profits, gross of expenditure on advertising, are increasing and convex in \(\eta_i\), as advertising augments consumers’ willingness to pay for sin goods. The second term is negative and reflects the degree of convexity of the expenditure on advertising. Hence, for the profit function to be concave, it is necessary that the parameter \(\alpha\) is sufficiently large.

The condition (A.4) for concavity of the profit function \(\pi_i^{**}(\eta_a, \eta_b)\) in \(\eta_i\) can be expressed as follows:

\[
\Delta \equiv \alpha(2 - \gamma)^2(2 + \gamma)^2(1 - \gamma^2) - 2(2 + k\gamma - \gamma^2)^2 [\varphi(r)s]^2 > 0. \quad (A.5)
\]
From the first order conditions for maximizing the profit functions $\pi^+_i(\eta_a, \eta_b)$ with respect to own advertising, we obtain the best-response $\tilde{\eta}_i$ by firm $i$ as a linear function of advertising $\eta_j$ by firm $j$ (the best response $\tilde{\eta}_j(\eta_i)$ is defined in a similar way):

$$
\tilde{\eta}_i(\eta_j) = \frac{2[(2 + \gamma)(1 - \gamma)(\bar{\rho} - \bar{h} - c - t) - (\gamma + 2k - k\gamma^2)\varphi(r)\bar{s}h]}{\Delta},
$$

$i, j = a, b; i \neq j$ \hspace{1cm} \text{(A.6)}

where $\Delta > 0$ from condition (A.5).21

We assume $\bar{\rho} - \bar{h} - c - t > 0$ (see Eq. 7 in the text), so that the intercept $\tilde{\eta}_i(0)$ of the best response function shown in Eq. (A.6) is positive. Moreover, we assume that its slope is less than unity, i.e.,

$$
\frac{\partial \tilde{\eta}_i}{\partial \eta_j} = -\frac{2(\gamma + 2k - k\gamma^2)(2 + k\gamma - \gamma^2)[\varphi(r)s]^2}{\Delta} < 1, \quad i, j = a, b; i \neq j, \hspace{1cm} \text{(A.7)}
$$

which implies that the condition $\Psi > 0$ defined in Eq. (8) in the text holds true.

$$
\Psi \equiv \alpha(2 - \gamma)^2(2 + \gamma)(1 + \gamma) - 2(1 - k)(2 + k\gamma - \gamma^2)[\varphi(r)s]^2 > 0. \hspace{1cm} \text{(A.8)}
$$

By linearity of the best response functions shown in Eq. (A.6), under conditions shown in Eqs. (A.5), (8) and (7), there exists a unique, stable, and symmetric Nash equilibrium, with positive advertising levels, defined in Eq. (6) in the text.

In terms of the model’s parameters, equilibrium prices, quantities and profits are expressed as follows:

$$
p^*(t, r) = c + t + \frac{\alpha(\bar{\rho} - \bar{h} - c - t)(4 - \gamma^2)(1 - \gamma^2)}{\Psi}, \hspace{1cm} \text{(A.9)}
$$

$$
x^*(t, r) = \frac{\alpha(\bar{\rho} - \bar{h} - c - t)(4 - \gamma^2)}{\Psi}, \hspace{1cm} \text{(A.10)}
$$

$$
\pi^*(t, r) = \frac{\alpha(\bar{\rho} - \bar{h} - c - t)^2\Delta}{\Psi^2}. \hspace{1cm} \text{(A.11)}
$$

### A.2 Taxation and regulation: comparative statics

By differentiating Eq. (6), we get the expressions shown in Eqs. (12) and (13):

$$
\frac{\partial \eta^*_i}{\partial t} = -\frac{2(2 + k\gamma - \gamma^2)\varphi(r)s}{\Psi} = -\frac{\eta^*}{\bar{\rho} - \bar{h} - c - t} < 0,
$$

$$
\frac{\partial \eta^*_i}{\partial r} = \frac{2(\bar{\rho} - \bar{h} - c - t)(2 + k\gamma - \gamma^2)s}{\Psi^2} \left(\Psi \frac{\partial \varphi}{\partial r} - \varphi \frac{\partial \Psi}{\partial r}\right) = \left(\frac{1}{\varphi} \frac{\partial \varphi}{\partial r} - \frac{1}{\Psi} \frac{\partial \Psi}{\partial r}\right) \eta^* < 0,
$$

where, by differentiating Eq. (8), it is

$$
\frac{\partial \Psi}{\partial r} = -4(1 - k)(2 + k\gamma - \gamma^2)s\varphi(\varphi)\frac{\partial \varphi}{\partial r} > 0.
$$

21If $\gamma + 2k - k\gamma > 0$ ($< 0$), then $\partial \tilde{\eta}_i/\partial \eta_j > 0$ ($< 0$), and competition in advertising shows strategic complementarity. (substitutability).
By differentiating Eq. (A.9), we get:

\[
\frac{\partial p^*}{\partial t} = 1 - \frac{\alpha(4 - \gamma^2)(1 - \gamma^2)}{\Psi},
\]
\[
\frac{\partial p^*}{\partial r} = -\frac{\alpha(\bar{\rho} - \bar{h} - c - t)(4 - \gamma^2)(1 - \gamma^2)}{\Psi^2} \frac{\partial \Psi}{\partial r} = -(p^* - c - t) \frac{1}{\Psi} \frac{\partial \Psi}{\partial r} < 0.
\]

By differentiating Eq. (A.10), we get:

\[
\frac{\partial x^*}{\partial t} = -\frac{\alpha(4 - \gamma^2)}{\Psi} < 0,
\]
\[
\frac{\partial x^*}{\partial r} = -\frac{\alpha(\bar{\rho} - \bar{h} - c - t)(4 - \gamma^2)}{\Psi^2} \frac{\partial \Psi}{\partial r} = -\frac{1}{\Psi} \frac{\partial \Psi}{\partial r} \bar{x}^* < 0.
\]

A.3 Optimal tax in a market without advertising

In the absence of advertising, firms compete only on prices. Equilibrium prices and quantities are:

\[
p_{\text{noadv}} = c + t + \frac{(1 - \gamma)(\bar{\rho} - \bar{h} - c - t)}{2 - \gamma}, \tag{A.12}
\]
\[
x_{\text{noadv}} = \frac{\bar{\rho} - \bar{h} - c - t}{(2 - \gamma)(1 + \gamma)} \tag{A.13}
\]

By substituting Eqs. (A.12) and (A.13) into Eq. (11), (22) and (23), and then maximizing the social welfare function defined in Eq. (24), one gets the optimal tax rate shown in Eq. (25).

A.4 Optimal tax in a market with advertising

By adding and subtracting \(2\varphi(r)\eta s(1 - k)\eta x^*\) to the expression, defined in Eq. (22), of welfare of an individual consumer, the latter can be written as:

\[
\begin{align*}
    w^*(t, r) &= u^*(t, r) - 2\varphi(r)\eta s(1 - k)\eta x^*, \\
    u^*(t, r) &= 2 \left( \rho + \varphi(r)s(1 - k)\eta x^* - h - \frac{x^*}{2} \right) x^* - \gamma(x^*)^2 - 2p^*x^* + \bar{\ell}^* + I, \tag{A.14}
\end{align*}
\]

where

\[
    u^*(t, r) = 2 \left( \rho + \varphi(r)s(1 - k)\eta x^* - h - \frac{x^*}{2} \right) x^* - \gamma(x^*)^2 - 2p^*x^* + \bar{\ell}^* + I, \tag{A.15}
\]

is the indirect utility function.

By differentiating Eq. (A.14) with respect to \(t\) (and applying the envelope theorem when differentiating \(u^*\)), we get:

\[
\frac{\partial w^*}{\partial t} = -2x^* \frac{\partial p^*}{\partial t} + \frac{\partial \bar{\ell}^*}{\partial t} - 2\varphi(r)\eta s(1 - k)\eta \frac{\partial x^*}{\partial t}. \tag{A.16}
\]

Aggregating over consumers:

\[
\frac{\partial W^*}{\partial t} = E \left[ \frac{\partial w^*}{\partial t} \right] = -2x^* \frac{\partial p^*}{\partial t} + \frac{\partial \bar{\ell}^*}{\partial t} - 2\varphi(r)(1 - k)\eta^2 E \left[ s \frac{\partial x^*}{\partial t} \right]. \tag{A.17}
\]
As for the other component of social welfare, aggregate profits, by differentiating Eq. (11) with respect to $t$, it is:

$$\frac{\partial \Pi^*}{\partial t} = 2 \frac{\partial \pi^*}{\partial t} = 2(p^* - c) \frac{\partial \bar{x}^*}{\partial t} + 2 \bar{x}^* \frac{\partial p^*}{\partial t} - \frac{\partial \ell^*}{\partial t} - 2 \alpha \eta^* \frac{\partial \eta^*}{\partial t}. \quad (A.18)$$

By substituting for Eqs. (A.17) and (A.18) into the derivative

$$\frac{\partial \Omega^*}{\partial t} = \frac{\partial W^*}{\partial t} + \theta \frac{\partial \Pi^*}{\partial t},$$

we get the derivative shown in Eq. (28).

Let $\theta = 1$. From Eq. (28), the first order necessary condition for maximizing social welfare can be written as:

$$\frac{1}{2} \frac{\partial \Omega^*}{\partial t} \bigg|_{\theta = 1} = -\varphi(r)(1-k)\eta^* E \left[ s \frac{\partial \bar{x}^*}{\partial t} \right] + (p^* - c) \frac{\partial \bar{x}^*}{\partial t} - \alpha \eta^* \frac{\partial \eta^*}{\partial t} = 0. \quad (A.19)$$

By substituting for

$$p^* - c = \frac{t}{2-\gamma} + \frac{1-\gamma}{2-\gamma} [\bar{p} - \bar{h} - c + \varphi(r)\bar{s}(1-k)\eta^*]$$

into Eq. (A.19) and rearranging, we get:

$$\frac{t}{2-\gamma} \frac{\partial \bar{x}^*}{\partial t} = \varphi(r)(1-k)\eta^* E \left[ s \frac{\partial \bar{x}^*}{\partial t} \right] - \frac{1-\gamma}{2-\gamma} [\bar{p} - \bar{h} - c + \varphi(r)\bar{s}(1-k)\eta^*] \frac{\partial \bar{x}^*}{\partial t} + \alpha \eta^* \frac{\partial \eta^*}{\partial t},$$

that can finally be written as:

$$t = (2-\gamma)\varphi(r)(1-k)\eta^* E \left[ s \frac{\partial \bar{x}^*}{\partial t} \right] - (1-\gamma) [\bar{p} - \bar{h} - c + \varphi(r)\bar{s}(1-k)\eta^*] + (2-\gamma)\alpha \eta^* \frac{\partial \eta^*}{\partial t} + \frac{\partial \ell^*}{\partial t} + \frac{\partial \bar{x}^*}{\partial t}(2-\gamma) \frac{\partial \bar{x}^*}{\partial t}.$$

Since

$$E \left[ s \frac{\partial \bar{x}^*}{\partial t} \right] = \bar{s} \frac{\partial \bar{x}^*}{\partial t} + \text{cov} \left[ s, \frac{\partial \bar{x}^*}{\partial t} \right], \quad (A.20)$$

the expression for the optimal tax rate can be written as in Eq. (29).

Let $\theta = 0$. From Eq. (28), the first order necessary condition for maximizing social welfare can be written as:

$$\frac{1}{2} \frac{\partial \Omega^*}{\partial t} \bigg|_{\theta = 0} = -\varphi(r)(1-k)\eta^* E \left[ s \frac{\partial \bar{x}^*}{\partial t} \right] + \frac{1}{2} \frac{\partial \ell^*}{\partial t} + \bar{x}^* \frac{\partial p^*}{\partial t} = 0. \quad (A.21)$$

Substituting for

$$\frac{\partial \ell^*}{\partial t} = 2\bar{x}^* + 2t \frac{\partial \bar{x}^*}{\partial t},$$

Eq. (A.21) can be written as:

$$-\varphi(r)(1-k)\eta^* E \left[ s \frac{\partial \bar{x}^*}{\partial t} \right] + \bar{x}^* \left( 1 - \frac{\partial p^*}{\partial t} \right) + t \frac{\partial \bar{x}^*}{\partial t} = 0,$$

and this latter equation, using Eq. (A.20), can be finally written as in Eq. (30).
A.5 Optimal regulation policy

By differentiating Eq. (A.14) with respect to \( r \) (and applying the envelope theorem when differentiating \( u^* \)), we get:

\[
\frac{\partial w^*}{\partial r} = -2x^* \frac{\partial p^*}{\partial r} + \frac{\partial e^*}{\partial r} - 2\varphi(r)(1-k)\eta^* \frac{\partial x^*}{\partial r}.
\]  

(A.22)

Substituting for \( \frac{\partial e^*}{\partial r} = 2\varphi(r)(1-k)\eta^* \frac{\partial x^*}{\partial r} \), and aggregating over consumers, we get:

\[
\frac{\partial W^*}{\partial r} = E \left[ \frac{\partial w^*}{\partial r} \right] = -2x^* \frac{\partial p^*}{\partial r} + 2t \frac{\partial x^*}{\partial r} - \frac{\partial x^*}{\partial r} - 2\varphi(r)(1-k)\eta^* E \left[ s \frac{\partial x^*}{\partial r} \right].
\]  

(A.23)

By differentiating aggregate profits with respect to \( r \), we get:

\[
\frac{\partial \Pi^*}{\partial r} = 2\frac{\partial p^*}{\partial r} = 2(p^* - c)\frac{\partial e^*}{\partial r} + 2x^* \frac{\partial p^*}{\partial r} - 2t \frac{\partial x^*}{\partial r} - 2\alpha \eta^* \frac{\partial \eta^*}{\partial r}.
\]  

(A.24)

By substituting for Eqs. (A.23) and (A.24) into the derivative \( \frac{\partial \Omega^*}{\partial r} \), we get the derivative shown in Eq. (31).

Let \( \theta = 1 \). The derivative in Eq. (31) then becomes:

\[
\frac{1}{2} \frac{\partial \Omega^*}{\partial r} \bigg|_{\theta=1} = -\varphi(r)(1-k)\eta^* E \left[ \frac{\partial x^*}{\partial r} \right] + \frac{1}{2} \frac{\partial x^*}{\partial r} + (p^* - c)\frac{\partial e^*}{\partial r} - \alpha \eta^* \frac{\partial \eta^*}{\partial r}.
\]  

Substituting for \( E[s(\partial x^*/\partial r)] = s(\partial x^*/\partial r) + \text{cov}[s, (\partial x^*/\partial r)] \), the above derivative can be written as:

\[
\frac{1}{2} \frac{\partial \Omega^*}{\partial r} \bigg|_{\theta=1} = \{p^* - c - \varphi(r)s(1-k)\eta^* \} \frac{\partial x^*}{\partial r} - \varphi(r)(1-k)\eta^* \text{cov} \left[ s, \frac{\partial x^*}{\partial r} \right] - \frac{1}{2} \frac{\partial x^*}{\partial r} - \alpha \eta^* \frac{\partial \eta^*}{\partial r}.
\]  

(A.25)

The first order condition for \( t \) in Eq. (A.19) can be manipulated in a similar way to obtain:

\[
\frac{1}{2} \frac{\partial \Omega^*}{\partial t} \bigg|_{\theta=1} = \{p^* - c - \varphi(r)s(1-k)\eta^* \} \frac{\partial x^*}{\partial t} - \varphi(r)(1-k)\eta^* \text{cov} \left[ s, \frac{\partial x^*}{\partial t} \right] - \alpha \eta^* \frac{\partial \eta^*}{\partial t} = 0.
\]  

(A.26)

Finally, by solving for \( \{p^* - c - \varphi(r)s(1-k)\eta^* \} \) from Eq. (A.26), substituting into Eq. (A.25) and rearranging, we get the derivative shown in Eq. (32).

The analytical steps are similar for \( \theta = 0 \). The derivative in Eq. (31) is:

\[
\frac{1}{2} \frac{\partial \Omega^*}{\partial r} \bigg|_{\theta=0} = -\varphi(r)(1-k)\eta^* E \left[ \frac{\partial x^*}{\partial r} \right] - \frac{1}{2} \frac{\partial x^*}{\partial r} + t \frac{\partial x^*}{\partial r} - \bar{x} \frac{\partial p^*}{\partial r}.
\]

and it can be written as:

\[
\frac{1}{2} \frac{\partial \Omega^*}{\partial r} \bigg|_{\theta=0} = \{t - \varphi(r)s(1-k)\eta^* \} \frac{\partial x^*}{\partial r} - \varphi(r)(1-k)\eta^* \text{cov} \left[ s, \frac{\partial x^*}{\partial r} \right] - \frac{1}{2} \frac{\partial x^*}{\partial r} - \bar{x} \frac{\partial p^*}{\partial r}.
\]  

(A.27)

The first order condition for \( t \) in Eq. (A.19) can be expressed as:

\[
\frac{1}{2} \frac{\partial \Omega^*}{\partial t} \bigg|_{\theta=0} = \{t - \varphi(r)s(1-k)\eta^* \} \frac{\partial x^*}{\partial t} - \varphi(r)(1-k)\eta^* \text{cov} \left[ s, \frac{\partial x^*}{\partial t} \right] - \bar{x} \frac{\partial p^*}{\partial r} + \bar{x} = 0.
\]  

(A.28)

By solving for \( \{t - \varphi(r)s(1-k)\eta^* \} \) from Eq. (A.28), substituting into Eq. (A.27) and rearranging, we get the derivative shown in Eq. (35).
A.6 Numerical simulation

to be written