INTRA-FAMILIAL TIME TRANSFERS, RETIREMENT AND LABOR SUPPLY*

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Abstract

This paper examines the effects of pay-as-you-go (PAYG) pension system on retirement decision of the old and aggregate labor supply with presence of intra-familial time transfers. To do so, I extend a two-overlapping-generations growth model under the framework of ad hoc altruism. I show that increase in time transfers from the old raises young labor supply. I also obtain that increase in the payroll tax rate incites the old agent to retire earlier. However, such effect on aggregate labor supply is twofold: if time transfer is highly efficient, decrease in old labor supply due to increase of the payroll tax rate can be compensated by increase in young labor force; otherwise, aggregate labor supply declines. Finally, increase in the population growth rate also incites the old to retire earlier if the given level of population growth in the economy is relatively high.

JEL Codes: H55, J22, J26

Keywords: retirement, intergenerational transfers, time transfers, social security, labor supply.

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1 Introduction

This paper examines two important factors of intergenerational transfers that affect retirement and labor supply in the economy: the first one is PAYG pension system, which is one kind of public transfers from current workers to the old and the second one is one kind of private transfers that is intra-familial time transfer from the old to young adults to help them with home production.

On one hand, the effect of intra-familial time transfers on young labor supply is receiving increasing attention with important empirical evidences (Attias-Donfut, 2008, 2009, Arpino et al., 2012). However, this type of transfer has not been much concerned in previous theoretical literature even though it is important in the sense that although decision of retirement of the old agents decreases the old labor supply, the time transfers made by them to help adult children with home production or grandchild care can provide the young agent (especially women) more time for working. Thus, the effects on aggregate labor supply need to be again reexamined because any factor that can change retirement decision of the old agent can indirectly affect decision of labor supply of the young agent as well.

On the other hand, it has been confirmed in economics literature the strong correlation between old age labor force participation or retirement and PAYG pension system (Börsch-Supan, 1998, 2000, Blundell, French and Tetlow, 2016). In particular, when European countries are experiencing population aging, PAYG pension system is an useful public policy tool to achieve influence on retirement incentives, which thus reduces the increasing burden to the pension system. Jackson (2003) and Willmore (2004) stated that PAYG pension system had done a great job in the fall in average age of retirement from 65 to 60 in Germany. Also, French and Jones (2012) concluded that the labor supply of older workers is responsive to changes in retirement incentives while the one of younger workers is less responsive. Hence, lowering taxes on older workers in many developed countries is likely to be explanation of trend towards postponed retirement

Studying intergenerational transfers in European countries, Attias-Donfut and Ogg (2009) confirmed the importance of both private and public transfers as they cannot be replaced but support each other. In this paper, I try to built up a theoretical two-overlapping-generations growth model (OLG model) that includes intra-familial time transfer - a common practice in society - in order to explore how changes in the population growth rate and the payroll tax rate can affect retirement decision of the old agent and how aggregate labor supply of the economy is affected by these changes in interacting with presence of time transfers.

In addition, my study of time transfers is based on the framework of ad hoc altruism or specifically, paternalistic altruism. This kind of altruism focus on the aspect that parents obtain utility from the level of intergenerational transfers they made to their adult children. It’s different from another popular kind one: rational altruism where the utility of the beneficiary (children) is an argument of the utility of the benefactor (parents) (See Michel, Thibault and Vidal (2006) for kinds of altruism).
These two types of altruism have been used widely in studying intra-familial time transfer. Cardia and Michel (2004) incorporated intergenerational transfers of time and bequests into a rational altruistic OLG model. They obtained that bequests increase savings and capital accumulation, while time transfers instead increase capital accumulation by relaxing time constraint of young worker and thus making them working more. They also found that for less developed economies although the degree of altruism is not strong enough to generate bequests, time transfers may still matter. Belan et al. (2010) also study the effect of intergenerational transfers of time on young and old labor supply in the model of ad hoc altruism. The transfers of time in this paper imply the responsibility of grandchild care of old worker, which may affect the labor supply of young worker. In this paper, my model is based mostly on the framework by Michel and Pestieau (2013) in which the effect of PAYG pension system has been studied in an OLG model without intra-familial time transfer.

My results show that intra-familial transfer, PAYG pension system and population growth rate are strongly correlated to determine how each factor impact retirement and aggregate labor supply. Specifically, analyzing equilibrium confirms the positive relationship between retirement length and young labor supply. If the old agent decides to retire earlier, s/he can spend more her/his retirement time helping her/his adult children with home production, especially grandchild care. Thus, the young agent now frees up more time and is able to work more.

I also obtain that increase in the payroll tax rate incites the old agent to retire earlier in which the effect on the price of retirement length dominates the negative effect on income. Nonetheless, impact of the payroll tax rate on aggregate labor supply depends on efficiency of time transfers. Briefly, the payroll tax rate has a positive effect on aggregate labor supply when time transfers are highly efficient. In contrast, when time transfers are less efficient, aggregate labor supply declines for a raise in the payroll tax rate since in this case, increase in the young labor force can not be compensated by decrease in old labor supply. I also found that improvement of population growth rate is more likely to increase the retirement length for a given relatively high level of population growth in the economy.

The remainder of this paper is organized as follows. The literature is briefly reviewed in Section 2. Section 3 sets up the model with two generations: the young and the old, the firm and the pension system. Section 4 analyses the equilibrium. Section 5 studies the effects of the payroll tax rate and the population growth rate on retirement decision and aggregate labor supply. Section 6 concludes.

2 Literature review

Several forms of altruism have been introduced in the literature. Rational altruism as defined by Barro (1974) is one of the main frameworks. Barro’s analysis is based on an assumption that individuals are motivated by dynastic altruism. The transfers can be from parents to children (descending) or from children to parents (ascending). The utility of the beneficiary is an argument
of the utility of the benefactor. This approach by Barro was followed by a strand of research by Weil (1987, 1989) on conditions of positive bequests (see Michel, Thibault and Vidal (2006) for a review). In the Barro’s dynastic model, as soon as family transfers are operative, pay-as-you-go pension system becomes neutral. Stated differently, the private intergenerational transfers from parents to children offset the public intergenerational transfers operated by the pension system.

However, Barro’s result about the neutrality of the pay-as-you-go pension system is obtained in a very particular case. First, rational altruism is not necessarily the only way to represent intra-familial transfer motive. A number of other assumptions have been explored into the literature: *ad hoc* altruism (also called paternalistic altruism or warm-glow: parents obtain utility from the level of the bequest and do not consider utility of the children) or strategic altruism (parents choose the transfers in order to manipulate child’s decisions). With these assumptions, the neutrality of the pay-as-you-go pension system is no longer guaranteed.

Moreover, pay-as-you-go pension systems have also consequences on labor supply that are neglected in the Barro’s analysis. First, social contributions are paid by workers as a percentage of their wage, introducing distortions in the labor leisure choice. Secondly, the introduction of the PAYG pension system affects retirement behavior. These labor supply effects can also lead to a non-neutrality result.

In this paper, I would like to analyze the non-neutrality of PAYG pension system and this study could be favored through an *ad hoc* altruism framework. I will focus on time transfers since it has not been received much attention in existing literature despite important empirical evidences. Indeed, Wolff and Attias-Donfut (2007) gave evidences of intergenerational transfers across generations in Europe under the forms of financial transfers and time transfers. According to data from *SHARE* survey on people aged 50 and over, nearly three out of ten respondents are involved in financial transfers during the year but more than four out of ten are involved in time transfers. Moreover, while most of the financial transfers go to children themselves, most of time transfers are made to help on grandchild care. Also based on *SHARE* survey, Albertini et al. (2007) confirmed not only the existence of such transfer system but also the fact that transfers from elderly parents to their children are much more frequent and usually more intense than those in the opposite direction. Their findings base on the data in ten Western European countries in which these transfers vary across different welfare regimes. More evidences of the effect of time transfers in form of grandchild care on young labor supply have been found by Attias-Donfut (2008) in his research in European families. Arpino et al. (2012) also focused on time transfers of grandparents to mothers’ labor market participation. Using an Instrumental Variable approach on Italian data, they found that the effect of grand-parental childcare on mothers’ labor supply is positive, statistically significant and economically relevant.

In most countries in Asia, intra-familial time transfer from grandparents is even more common practice because of their cultural context encouraging them doing so. For instance, Feng and Zhang (2018) showed that two-thirds of urban Chinese children under age 6 are being taken care of by at least one grandparent according to China’s Research Center on Aging. Reporting recent
surveys in Myanmar, Thailand and Vietnam, Knodel and Nguyen (2015) pointed out that substantial proportions of persons aged 60 and older co-reside with grandchildren and commonly provide grandparental care. Important evidence of grandparental child care is also found in Australia by Condon et al. (2013).

On theoretical ground, there are some papers that raised the importance of time transfers as part of intergenerational transfers worth to be mentioned. Cardia and Michel (2004) incorporated intergenerational transfers of time and bequests into a rational altruistic OLG model. They obtained that bequests increase savings and capital accumulation, while time transfers instead increase capital accumulation by relaxing time constraint of young worker and thus making them working more. They also found that for less developed economies although the degree of altruism is not strong enough to generate bequests, time transfers may still matter. Belan et al. (2010) also study the effect of intergenerational transfers of time on young and old labor supply in the model of ad hoc altruism. The transfers of time in this paper imply the responsibility of grandchild care of old worker, which may affect the labor supply of young worker.

In this paper, my model is based mostly on the framework by Michel and Pestieau (2013). They studied the effect of the social security system (PAYG) on agent’s decision of retirement and obtain that workers retire earlier at the same time as society becomes more and more indebted through increasing pay-as-you-go pension liabilities; and moreover, mandatory early retirement may be socially desirable in case of under-accumulation. My paper will incorporate time transfers into a model of PAYG system in a small open economy. The objective is to examine the effect of the pension systems on retirement decision and aggregate labor supply in interacting with presence of time transfers.

Previous literature also gives empirical evidence for this effect, in which a noticeable one is the paper by French and Jones (2012). In this paper, they examined how reforms of public pension systems (reductions in the generosity of public pensions for instance) affect labor supply in a life cycle context. They obtained that the labor supply of older workers is responsive to changes in retirement incentives meanwhile, the labor supply of younger workers is less responsive. This result was again confirmed later by Blundell, French and Tetlow (2016). In this research, they review the main factors that have been identified as being important in retirement incentives. One of the noticeable incentives that this research has pointed out is substitution effects. They stated that changes in wage opportunities such as taxes, benefits and pension systems in a country can affect the relative attractiveness of working vs not working. They also give important evidences that public pension systems affect retirement decision of older people across many developed countries like US and European countries.

3 The model

Consider a discrete-time, infinite-horizon, OLG model of a small open economy. In such framework, capital freely moves across countries and labor is a country-specific factor. Then, the gross
interest rate, denoted by \( R \), is equal to the world interest rate and constant over time. The wage, denoted by \( w \) is also constant over time.

### 3.1 The firm

At each date, the representative firm produces a single output \( Y \) by using labor and capital. The production function is denoted by \( Y = F(K, L) \) where \( K \) and \( L \) are aggregate capital and aggregate effective labor, respectively. Assume that \( F \) exhibits the constant return to scale. The profit maximization problem of this firm at time \( t \) is:

\[
\max_{K_t, L_t} \{ F(K_t, L_t) - wL_t - RK_t \}
\]

Let \( k \) be capital per unit of effective labor, that is: \( k = \frac{K}{L} \). Let \( f(k) = F(K/L, 1) = F(k, 1) \). Then, from the firm’s problem:

\[
R = f'(k_t)
\]

which implies that \( k_t = k \) is constant over time. In addition,

\[
w = f(k) - kf'(k)
\]

is satisfied.

### 3.2 The consumer problem

Population grows at the constant rate \( n \) (\( n \geq 0 \)). There are two adult generations: the young and the old, of size \( N_t = (1 + n)N_{t-1} \) and \( N_{t-1} \) respectively.

Each agent is endowed with one unit of time. We allow for time transfers from the old to the young generation. In the first period, the young agent allocates his/her time between a paid activity and home production. The good consumed by the young is produced by using time, \( z_t \), a market good, \( c_t \), and a Cobb-Douglas technology with constant return to scale:

\[
\phi_t = c_t^{1-\alpha_y} z_t^{\alpha_y}
\]

where \( \alpha_y \) is the efficiency of time in home production of the young agent. The time input \( z_t \) combines the time spent by the young and the time spent by the old producing the home good consumed by the young. We assume perfect substitutability between the time of the young and that of the old in producing home good \(^1\):

\[
z_t = 1 - h_t + \frac{\tilde{A}T_t}{1+n}
\]

where \( h_t \) is the labor supply of young adult or the time spent by the young producing the market

\(^1\)Previous papers give different set-ups of time transfers by the old, in which Cardia and Michel (2004) also define the efficiency of time transfers as we do; meanwhile, Cardia and Ng (2003) use a general form: \( z_t = f(1-h_t, \mu_T) \)
good. We impose a restriction on $h_t$: $0 \leq h_t \leq 1$, which implies that the young agent can’t work more than her/his endowment of time. $T_t$ is time transfer given by the parent to his/her $1+n$ children. $\lambda T_t$ represents the total time transfer, in efficiency units, of the old to the young. Therefore, $\lambda T_t/(1+n)$ measures the per-child time transfer (in efficiency units). The old are more efficient than the young for values of $\lambda$ larger than 1.

The old agent also spends a fraction of time working, denoted $\theta_{t+1}$, and then retires. The retirement length is denoted $T_{t+1}$. Moreover, $\mu_{t+1}$ is fraction of retirement length that the old agent spends on time transfers. In other words, $T_{t+1} = \mu_{t+1} T_{t+1}$ is time transfers or the time spent by the old producing home good consumed by the young and $(1-\mu_{t+1}) T_{t+1}$ is time spent producing home good consumed by himself. Thus, the time constraint in the second period is:

$$\theta_{t+1} + T_{t+1} = \theta_{t+1} + \mu_{t+1} T_{t+1} + (1-\mu_{t+1}) T_{t+1} = 1 \quad (3)$$

where $0 \leq T_{t+1} \leq 1$, $0 \leq \theta_{t+1} \leq 1$ and $0 \leq \mu_{t+1} \leq 1$.

The good consumed by the old is also produced using his own time $(1-\mu_{t+1}) T_{t+1}$, a market good, $d_{t+1}$, and a Cobb-Douglas production function with constant return to scale:

$$\psi_{t+1} = d_{t+1}^{1-\alpha_o} [(1-\mu_{t+1}) T_{t+1}]^{\alpha_o} \quad (4)$$

where $\alpha_o$ is the efficiency of time in home production of the old agent.

Now we describe the budget constraints of the consumer in both periods. During the first period, the young agent decides the amount of consumption, $c_t$, and the amount of saving, $s_t$. S/he receives the labor income, $w h_t$ and faces a social security - payroll tax rate on income, $\tau \in (0, 1)$. Thus the budget constraint in the first period is:

$$c_t = (1-\tau) w h_t - s_t \quad (5)$$

In the second period, the agent receives the interest from her/his saving: $R s_t$ where $R$ is the gross interest rate. His/her labor income is $w \theta_{t+1}$ that is also taxed by the payroll rate $\tau$. Once the old agent retires, s/he is eligible to receive the retirement benefits, $b_{t+1}(1-\theta_{t+1})$. The budget constraint in period $t+1$ is:

$$d_{t+1} = R s_t + (1-\tau) w \theta_{t+1} + b_{t+1}(1-\theta_{t+1}) \quad (6)$$

The old agent works when $b_{t+1} < (1-\tau) w$ or when the pension benefits are lower than the income s/he would receive when working.

Combine equations (2), (3), (5) and (6), the agent’s intertemporal budget constraint is:

$$c_t + \frac{d_{t+1}}{R} = (1-\tau) w \left( 1 + \frac{\lambda T_t}{1+n} - z_t \right) + \frac{(1-\tau) w (1-T_{t+1})}{R} + \frac{b_{t+1} T_{t+1}}{R} \quad (7)$$

In this paper, I consider ad hoc altruism. The idea is that parents do not obtain utility from the
well-being of their children, but instead from the act of giving time to them. This is opposite with the model of rational altruism by Barro (1974) where parents care about their children’ welfare by weighting their children’ utility in their own utility function (See Michel et al. (2006) for details on different types of altruism).

Consequently, the intertemporal utility in log forms of the agent living in periods \( t \) and \( t + 1 \) is:

\[
U_t = U_t(\varphi_t, \psi, T_{t+1}, \mu_{t+1}) = \ln \varphi_t + \delta \ln \psi_{t+1} + \beta \ln \mu_{t+1} \ln T_{t+1} \tag{8}
\]

where \( \delta \) is discount factor or time preference and \( \beta \) is the degree of altruism.

This utility function can be rewritten as follows:

\[
U_t = (1 - \alpha^y) \ln c_t + \alpha^y \ln z_t + \delta \left[ (1 - \alpha^e) \ln d_{t+1} + \alpha^e \ln((1 - \mu_{t+1}) T_{t+1}) + \beta \ln(\mu_{t+1} T_{t+1}) \right] \tag{9}
\]

Each agent decides the levels of \( c_t, h_t, d_{t+1}, T_{t+1} \) and \( \mu_{t+1} \) to maximize his utility function (9) under the intertemporal budget constraint (7) and restrictions: \( 0 \leq h_t \leq 1 \), \( 0 \leq T_{t+1} \leq 1 \) and \( 0 \leq \mu_{t+1} \leq 1 \).

Before moving to the first-order conditions, we notice that since the utility is under the log forms, \( T_{t+1} > 0 \) and \( 0 < \mu_{t+1} < 1 \) must hold in the solution of the maximization problem. In addition, I will only consider the equilibrium with labor supply in both periods \( h_t > 0 \) and \( T_{t+1} > 0 \). The implication of this constraint is to avoid solutions where time transfers would have no effect on young labor supply.

The first-order conditions with respect to \( c_t, z_t, d_{t+1}, T_{t+1} \) and \( \mu_{t+1} \) are:

\[
d_{t+1} = \frac{\delta(1 - \alpha^e) R}{1 - \alpha^y} c_t \tag{10}
\]

\[
\frac{\alpha^y}{z_t} \leq \frac{(1 - \alpha^y)(1 - \tau) w}{c_t} \quad \text{with equality if } h_t < 1 \tag{11}
\]

\[
\frac{\delta(\alpha^e + \beta)}{T_{t+1}} \geq \frac{(1 - \alpha^y)((1 - \tau) w - b_{t+1})}{R c_t} \quad \text{with equality if } T_{t+1} < 1 \tag{12}
\]

\[
- \frac{\delta \alpha^e}{1 - \mu_{t+1}} + \frac{\delta \beta}{\mu_{t+1}} = 0 \quad \tag{13}
\]

Depending on parameter values, the young agent will fully work or partially work and the old agent will fully retire or partially retire in equilibrium. In this paper, I consider that all agents are identical that means their decisions are made with the same preferences or there’s no heterogeneity.

Solving the agent’s maximization problem under the intertemporal budget constraint (7), we obtain:

\[
c_t = \frac{1 - \alpha^y}{1 + \delta + \delta \beta} \left( 1 - \tau \right) w \left( 1 + \frac{\lambda \mu_t}{1 + n} + \frac{1}{R} \right) \tag{14}
\]

\[
z_t = \gamma \left( 1 + \frac{\lambda \mu_t}{1 + n} + \frac{1}{R} \right) \quad \text{where } \gamma = \frac{\alpha^y}{1 + \delta + \delta \beta} \tag{15}
\]
\[ T_{t+1} = \frac{\eta R (1 - \tau) w}{(1 - \tau) w - b_{t+1}} \left( 1 + \frac{\lambda \mu T_t}{1 + n} + \frac{1}{R} \right) \]
\[ \eta = \frac{\delta (\alpha^o + \beta)}{1 + \delta + \delta \beta} \] (16)
\[ \mu_{t+1} = \frac{\beta}{\alpha^o + \beta} \] (17)

From equation (17), we can see that the old agent always transfers a fraction of her/his retirement length \( \mu \) to her/his \( 1 + n \) adult children. As long as the old agent retires earlier, her/his adult children will receive higher time transfers proportionally. The fraction of time transfers, however, depends positively on the degree of altruism \( \beta \) and negatively on the efficiency \( \alpha^o \) of time in home production of the old. In other words, the old agent transfers a larger fraction of her/his retirement length to adult children if s/he is more altruistic or if the efficiency of time in her/his own home production is small. Because of the homogeneity assumption for all agents, the value of \( \mu \) is constant over time. To simplify, from now on we denote:

\[ \mu_t = \mu(\beta) = \frac{\beta}{\alpha^o + \beta} \quad \forall t \]

**Proposition 1.** Solution of the consumer problem implies a linear and positive relation between young labor supply \( h_t \) and retirement length \( T_t \):

\[ h_t = \min \{ \max \{ 0; h(T_t) \}; 1 \} \quad \forall t \]

where

\[ h(T) = (1 - \gamma) \frac{\lambda \mu T}{1 + n} + 1 - \gamma - \frac{\gamma}{R} \]

\[ g' > 0 \]

**Proof.** The proof is trivial by combining (2) and solution of \( z_t \) (15) under constraints \( 0 < h_t \leq 1 \) and \( 0 < T_t \leq 1 \). \( \square \)

This proposition gives a positive relationship between \( h_t \) and \( T_t \); when the old agent retires earlier, which is equivalent to the fact that his/her time transfer to the young agent increases, the young agent will be able and have desire to work more. Moreover, if \( \lambda \geq 1/(1 - \gamma) \), when per-child time transfer \( \mu T/(1 + n) \) increases by one unit, young labor supply increases by a value larger than one since the more efficient time transfers from the old, the less the young spends time on home production and hence, s/he is able to give more labor supply. In contrast, the less efficient time transfer, the less its effect on young labor supply.

In the case when the old and the young are equally efficient in home production: \( \lambda = 1 \), if \( n \geq 0 \), an increase by one unit of retirement length would increase young labor supply by a value smaller than one because \( \mu(1 - \gamma)/(1 + n) < 1 \). In other words, the old agent doesn’t transfer all of her retirement length because s/he has preferences on production of home good consumed by her/his own and furthermore, the young agent doesn’t "spend" all of time transfers from her/his parents on working either because s/he also has preferences on producing home good consumed by her/his own.

To simplify, I impose the following assumption:
Assumption 1.

\[ \alpha y < \frac{R(1 + \delta + \delta \beta)}{1 + R} \]

This assumption implies that in the first period, the efficiency of the young in home production is not too large.

Assumption 1 guarantees \( h(T) > 0 \) for any value of \( T \). The relation of \( h_t \) and \( T_t \) becomes:

\[ h_t = \min\{h(T_t); 1\} \quad \forall t \quad (18) \]

Accordingly,

\[ T_{t+1} = \min\left\{ \eta \frac{R (1 - \tau) w}{(1 - \tau) w - b_{t+1}} \left( 1 + \frac{\lambda \mu T_t}{1 + n} + \frac{1}{R} \right); 1 \right\} \]

Again, we recall that \((1 - \tau) w - b_{t+1}\) needs to be positive so that \( T_{t+1} \) is always positive \( \forall t \).

Under assumption 1, we can illustrate in figure 1 the set of possible values of pair \{\( h_t, T_t \)\} fitted in the square \([0, 1] \times [0, 1]\). In this figure, \( h_t > 0 \) for any value of \( T_t \). When \( h_t \) (resp. \( T_t \)) exceeds the bound 1, the young provides full labor supply (resp. the old doesn’t work), that is \( h_t = 1 \) (resp. \( T_t = 1 \)). The relation between \( T_t \) and \( h_t \) in figure 1(a) does not contain the case where \( h_t = 1 \) and \( T_t < 1 \) while figure 1(b) does not allow for \( h_t < 1 \) and \( T_t = 1 \).

4 Equilibrium analysis

4.1 Relation between pension benefits and retirement length

In each period, the government balances the social security so that:

\[ b_t T_t N_{t-1} = \tau w (N_t h_t + N_{t-1} \theta_t) \]

which is equivalent to:

\[ b_t T_t = \tau w [(1 + n) h_t + (1 - T_t)] \quad (19) \]

**Proposition 2.** Under Assumption 1, combining the solution of consumer problem with the balanced budget of the PAYG pension system leads to a negative relation between pension benefit \( b_t \) and retirement length \( T_t \).

**Proof.** The balance of pension system budget, which is the equation (8), can be rewritten:

\[ b_t = \frac{\tau w}{T_t} [(1 + n) h_t + 1 - T_t] \quad (20) \]

Plugging \( h_t \) in equation (18) into equation (20), I obtain:

\[ \Rightarrow b_t = \min \left\{ \frac{B_1}{T_t}; \frac{B_2}{T_t} + C_2 \right\} \quad \forall t \quad (21) \]

\[ \text{Notice that assumption 1 is derived from and equivalent to } \gamma + \frac{\lambda}{R} < 1 \]
Figure 1: Relation between retirement length and young labor supply
where:
\[ B_1 = \tau w \left[ (1 + n) \left( 1 - \gamma - \frac{\gamma}{R} \right) + 1 \right] \]

\[ C_1 = \tau w [\lambda \mu (1 - \gamma) - 1] \]

and
\[ B_2 = \tau w [2 + n] \]

\[ C_2 = -\tau w \]

we have \( B_1 > 0 \) under Assumption 1 and \( B_2 > 0 \) so we conclude the proof.

According to the balanced budget of PAYG pension system, \( T \) have direct effects on \( b \) through increase of retirement length and lower old labor supply, which decrease pension benefits. However, with higher \( T \), the old agent retires earlier so she can make more time transfers to their children, which may increase young labor supply and hence, pension benefit. The latter indirect effect is dominated by two former direct effects. Thus, we obtain a negative relationship between \( T \) and \( b \).

### 4.2 Dynamics of retirement length

Replacing (20) in the solution of the consumer problem (16) leads to the following dynamics of \( T_t \)

\[
T_{t+1} = \min \left\{ \eta \frac{R (1 - \tau)}{1 - \tau \left( \frac{1}{1 + n} h_{t+1} \right)} \left( 1 + \frac{\lambda \mu T_t}{1 + n} + \frac{1}{R} \right) ; 1 \right\}
\]

where

\[
h_{t+1} = \min \left\{ (1 - \gamma) \left( 1 + \frac{\lambda \mu T_{t+1}}{1 + n} \right) - \frac{\gamma}{R} ; 1 \right\}
\]

(22)

Consider the case: \( h_{t+1} < 1 \) and \( T_{t+1} < 1 \). The dynamical system rewrites

\[
T_{t+1} = ET_t + F
\]

(23)

where

\[ E = \frac{\eta R (1 - \tau) \lambda \mu}{(1 + n)(1 - \tau \lambda \mu (1 - \gamma))} \]

\[ F = \frac{G}{1 - \tau \lambda \mu (1 - \gamma)} \]

\[ G = \eta (1 - \tau) (1 + R) + \tau \left[ (1 + n) \left( 1 - \gamma - \frac{\gamma}{R} \right) + 1 \right] > 0 \] (under Assumption 1)

We impose the following assumption:

**Assumption 2.**

\[ \lambda \mu \tau (1 - \gamma) < 1 \]
Assumption 2 is equivalent to \((1 - \tau) w - b_{t+1}\) to be positive (See Appendix A.1 for more details). Notice that when the old is less efficient in home production than the young: \(\lambda \leq 1\), this assumption is always guaranteed.

The dynamics of retirement length can be one of the two following cases:

- If \(E < 1\), starting from \(T_0 \in [0, 1]\), the sequence \((T_t)_t\) converges monotonically towards \(T^* \in [0, 1]\) (Figure 2.a).

\[
T^* = \frac{\eta(1 - \tau)(1 + R) + \tau + \tau(1 + n)(1 - \gamma - \frac{\gamma}{R})}{1 - \mu\tau\lambda(1 - \gamma) - \frac{\lambda\mu(1 - \tau)}{1 + n}}
\]  

- If \(E > 1\), the dynamics are explosive. After some periods, \(T_t\) reaches its upper bound corresponding to a corner equilibrium (Figure 2.b).

In this paper, I want to focus only on the interior equilibrium since the corner equilibrium does not allow to study the effect of retirement length on young labor supply (because \(h\) or \(T\) is fixed at 1). The interior equilibrium is ensured under the conditions shown in the following proposition.

**Proposition 3.** There exists a globally stable steady state \(T^* \in (0, 1)\) associated with labor supply \(h^* \in (0, 1)\), iff one of the two following conditions are satisfied:

1. \(A \geq 1\) and \(E + F < 1\)

2. \(A < 1\) and \(E + \frac{F}{A} < 1\)

where \(A = (1 + n)\frac{\gamma + \gamma / R}{\lambda\mu(1 - \gamma)}\).

**Proof.** From equation (23), the condition \(T_{t+1} < 1\) is equivalent to

\[
T_t < \frac{1}{E} - \frac{F}{E}
\]

From equations (22) and (23), the corresponding value of \(h_{t+1}\) is:

\[
h_{t+1} = 1 - \gamma - \frac{\gamma}{R} + (1 - \gamma)\frac{\lambda\mu(F + ET_t)}{1 + n}
\]
Then \( h_{t+1} < 1 \) is equivalent to

\[
T_t < \frac{(1+n)(\gamma + \frac{\gamma}{R})}{(1-\gamma)\lambda \mu E} \leq \frac{A}{E} - \frac{F}{E}
\]

**Case 1:** When \((1+n)\frac{\gamma + \gamma/R}{\lambda \mu (1-\gamma)} \geq 1\), the conditions for \( h_{t+1} < 1 \) and \( T_{t+1} < 1 \) can be rewritten as:

\[
T_t < \frac{1}{E} - \frac{F}{E} \leq \frac{A}{E} - \frac{F}{E}
\]

Then an interior solution would be obtained for \( t+1 \) if \( T_t \) satisfies

\[
0 \leq T_t < \frac{1}{E} - \frac{F}{E}
\]

which implies

\[
0 \leq T_{t+1} < 1
\]

**Sufficiency for the existence of a globally stable steady state:** The condition

\[
1 < \frac{1}{E} - \frac{F}{E}
\]

is sufficient to get a steady state \( T^* < 1 \) with \( h^* < 1 \). Indeed, in this case, the sequence \( (T_t) \) is upper-bounded by 1 and monotonous, then it converges towards a limit in \((0,1)\).

**Necessity for the existence of a globally stable steady state:** Since the sequence \( (T_t) \) is monotonous and converges towards \( T^* \), starting with \( T_t \) above \( T^* \) implies that \( T_{t+1} < T_t \). For instance, consider \( T_t = 1 \), then \( E + F < 1 \), which concludes the proof.

Figure 3 illustrates the proof.

**Case 2:** When \((1+n)\frac{\gamma + \gamma/R}{\lambda \mu (1-\gamma)} < 1\), along the dynamics, the condition \( h_{t+1} < 1 \) implies \( T_{t+1} <
Figure 4: Steady state $T^*$ and $h^* \in (0, 1)$ when $A < 1$

Since

$$\frac{A}{E} - \frac{F}{E} < 1 - \frac{F}{E}$$

Then an interior solution would be obtained for $t + 1$ if $T_t$ satisfies

$$0 \leq T_t < \frac{A}{E} - \frac{F}{E}$$

which implies

$$0 \leq T_{t+1} < A(1)$$

**Sufficiency for the existence of a globally stable steady state $T^* \in (0, 1)$ with $h^* < 1$: the condition**

$$A < \frac{A}{E} - \frac{F}{E}$$

is sufficient to get a steady state $T^* < 1$ with $h^* < 1$. Indeed, in this case, the sequence $(T_t)$ is bounded and monotonous, then it converges towards a limit in $(0, A)$.

**Necessity for the existence of a globally stable steady state $T^* \in (0, 1)$ with $h^* < 1$:** since the sequence $(T_t)$ is monotonous and converges towards $T^*$, starting with $T_t$ above $T^*$ implies that $T_{t+1} < T_t$. For instance, consider $T_t = A$, then $EA + F < A$, which concludes the proof.

Figure 4 illustrates the proof.

The effects of the payroll tax rate $\tau$ will be discussed in details in the next section. Before that, a graph of degree of altruism $\beta$ and payroll tax rate $\tau$ is useful to analyze the combinations between $\beta$ and $\tau$ to reach each interior and corner equilibrium $(h^*, T^*)$. Choosing $\alpha^y = 0.4$, $\alpha^o = 0.4$, $\delta = 0.98$, $R = 1.03$, $\lambda = 1$ and $n = 0$, we draw figure 5 using Python. In figure 5, the colored
area stands for interior equilibrium while the white one stands for the corner equilibrium. When the degree of altruism $\beta$ is not too high ($A > 1$), the time transfers are relatively small. Then an increase in the payroll tax rate incites the old agent to retire earlier (in the next section, we explain that’s because the negative effect on the price of retirement dominates), which moves the interior equilibrium to the corner one where $T = 1$ and $h \leq 1$. On the other hand, when the degree of altruism is high, increase in the payroll tax rate still incites the old to retire earlier. However in this case, the amount of time transfers is relatively large, which increases young labor supply. Then for a given value of payroll tax rate, the young may reach full labor supply at some stages with increasing value of $\beta$.

5 The effects of the payroll tax rate and population growth rate on retirement decision and labor supply

Consider only the interior equilibrium where $T_t < 1$ and $h_t < 1$. For further discussion, let us recall the consumer’s budget constraint (7) equivalent to:

$$c_t + \frac{d_{t+1}}{R} + (1 - \tau)wz_t + \frac{(1 - \tau)w - b_{t+1}T_{t+1}}{R} = (1 - \tau)w \left( 1 + \frac{\lambda T_t}{1 + n} + \frac{1}{R} \right)$$

and the balance of the pension system:

$$b_{t+1}T_{t+1} = \tau w [(1 + n)h_{t+1} + (1 - T_{t+1})]$$

Moreover, equation (21) from Proposition 2 gives the function of pension benefit $b$ when $h < 1$.
is:

\[ b_{t+1} = \frac{B_1}{T_{t+1}} + C_1 \]

where:

\[ B_1 = \tau w \left[ (1 + n) \left( 1 - \gamma \frac{\gamma}{R} \right) + 1 \right] \]

\[ C_1 = \tau w \left[ \lambda \mu (1 - \gamma) - 1 \right] \]

In words, function of pension benefit writes \( b_{t+1} = b(T_{t+1}, \tau, n) \)

5.1 Effect of the payroll tax rate

According to the intertemporal budget constraint, for the young, an increase in the payroll tax rate decreases the overall income and the price of young labor’s time spent on home production. These two effects cancel out in my model so that the solution of \( z_t \) - the time that the young agent spend on home production (equation (15)) does not depend on \( \tau \). Hence, the payroll tax rate affects young labor supply only through the time transfers that the young agent receives from her parents.

For the old, the payroll tax rate affects the old agent’s retirement decision through effect on income and effect on price of retirement. The income effect states that increase in the payroll tax rate makes the old agent relatively poorer then s/he would postpone retirement and work more. On the other hand, an increase in the payroll tax rate also reduces the rewards of working for the old agent meanwhile s/he is able to receive a larger pension benefit when retires. This means the price of retirement decreases which raises retirement incentive of the old. To be more precise, we consider the value of retirement length at steady state as following:

\[ T^* = \frac{\eta R (1 - \tau)}{(1 - \tau)w - b(T^*, n, \tau)} w \left( 1 + \frac{\lambda \mu T^*}{1 + n} + \frac{1}{R} \right) \]  

(25)

We obtain:

\[ \frac{dT^*}{T^*} = \mathcal{K} \left( \frac{w + b'_\tau}{(1 - \tau)w - b(T^*, n, \tau)} - \frac{1}{1 - \tau} \right) d\tau \]

(26)

where \( \mathcal{K} > 0 \) (See Appendix A.2 for computations in details). The second term in the bracket of equation (26) represents the income effect of the payroll tax rate on retirement length, which is negative. Meanwhile, the first term implies the effect on the price of retirement length. We derive the following proposition:

**Proposition 4.** In an economy with altruism and intra-familial time transfers, increase in the payroll tax rate incites the old agent to retire earlier: \( \frac{dT^*}{d\tau} > 0 \)

**Proof.** The derivative of pension benefit \( b \) with respect to \( \tau \) is:

\[ b'_\tau = w \left[ (1 + n) \left( 1 - \gamma \frac{\gamma}{R} \right) + 1 \right] + \lambda \mu (1 - \gamma) - 1 \]

Since \( T \leq 1 \),
\[ b'_r \geq w \left[ (1 + n) \left( 1 - \gamma - \frac{Y}{R} \right) + \lambda \mu (1 - \gamma) \right] > 0 \]

Hence, \( b'_r > 0 \). Equation (26) is equivalent to:

\[
\frac{dT^*}{T^*} = K \left( \frac{(1 - \tau) b'_r + b(T^*, n, \tau)}{(1 - \tau) [(1 - \tau) w - b(T^*, n, \tau)]} \right) d\tau
\]

The fraction in the bracket is positive. So we conclude the proof.

This proposition implies that the effect of the payroll tax rate on the price of retirement dominates the effect on her/his life-cycle income.

Next, we investigate the effect of the payroll tax rate on the aggregate labor supply, accordingly effect on contribution to the pension system.

The aggregate labor supply is:

\[ L_{t+1} = N_t \left[ (1 + n) h_{t+1} + 1 - T_{t+1} \right] \]

Replace the function of \( h_{t+1} \) on \( T_{t+1} \) into the equation of aggregate labor supply above, we obtain:

\[ L_{t+1} = N_t \left[ \lambda \mu (1 - \gamma) T_{t+1} + (1 + n) (1 - \gamma - \frac{Y}{R}) + 1 - T_{t+1} \right] \]  

(27)

Derivative of aggregate labor supply with respect to \( \tau \) is:

\[ \frac{\partial L}{\partial \tau} = [\lambda \mu (1 - \gamma) - 1] \frac{\partial T}{\partial \tau} \]  

(28)

Equation (28) shows that the payroll tax rate affects aggregate labor supply through effect on retirement length \( -\frac{\partial T}{\partial \tau} \) and effect of time transfers on young labor supply \( \lambda \mu (1 - \gamma) \frac{\partial T}{\partial \tau} \). This leads us to the following proposition.

**Proposition 5.** In an economy with altruism and intra-familial time transfers. The effect of the payroll tax rate on aggregate labor supply is:

1. \( \frac{\partial L^*}{\partial \tau} > 0 \) if \( \lambda > \frac{1}{\mu (1 - \gamma)} \).
2. \( \frac{\partial L^*}{\partial \tau} < 0 \) if \( \lambda < \frac{1}{\mu (1 - \gamma)} \).

Proposition 5 is simply derived from equation (28). This proposition implies:

1. Because of high level of efficiency of time transfers, a decrease in old labor supply can be compensated by increase in young labor force. Hence, aggregate labor supply increases, which in turn increases the contribution to the pension system. In this case with an increase in the payroll tax rate, the old agent strongly desires to retire earlier because his rewards for working decrease meanwhile he can receive higher pension benefit thanks to higher tax and higher contribution to the pension system from young labor force. In this case, the time transfer has eliminated the negative effect of payroll tax rate on labor supply.

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2. Because of lower efficiency of intergenerational time transfers, increase of young labor supply thanks to increase in time transfers is not enough to compensate decrease of old labor force. Hence, aggregate labor supply decreases with an increase of the payroll tax rate. Therefore in this case, a policy that increases the payroll tax rate in order to finance the pension budget causes a reduction of aggregate labor supply if time transfer is relatively less efficient. Yet the negative effect of the payroll tax rate has been softened by intra-familial time transfers.

5.2 Effect of population growth rate

This section analyses the effect of population growth rate on retirement decision of the old in an economy with altruism and intra-familial transfers. Thanks to the intertemporal budget constraint, it’s easy to see that population growth rate affects retirement length in two directions. On one hand, an increase in the number of children or the population growth rate \( n \) in the first period (with the payroll tax rate keeping unchanged) will increase the young labor force in the second period. So the resource of the pension system increases, which increases the pension benefit. According to the intertemporal budget constraint above, increase in the pension benefit reduces the price of retirement. The old agent doesn’t need to work longer to earn the same income. So with unchanged resources, the retirement length increases. In words, an increase in the number of children will incite old agent to retire earlier. On the other hand, an increase in population growth rate will reduce the per-child efficiency of the time transfers from the old which decreases the life-cycle income. Therefore, the retirement length might decline because of income effect.

To be precise, we again consider the value of retirement length at steady state (25), then obtain:

\[
\frac{dT^*}{T^*} = \frac{H}{(1 - \gamma)w} \left( \frac{b'_n(T^*, n, \tau)}{nR(1 - \tau)w} - \frac{\mu\lambda}{(1 + n)^2} \right) dn
\]

(29)

where \( H > 0 \) (see Appendix A.3 for computations in details) and the derivative of pension benefit \( b \) with respect to \( n \) is:

\[
b'_n = \frac{\tau w (1 - \gamma - \gamma / R)}{T^*} > 0
\]

We can easily see that \( \frac{dT^*}{dn} < 0 \) in the interior equilibrium when the PAYG social security system is not operative \( \tau = b = 0 \). The value of retirement length at equilibrium becomes:

\[
T^* = \frac{\eta(1 + R)}{1 - \frac{\lambda n k}{1 + n}}
\]

that depends on the population growth \( n \) only through income effect because each child receives less time transfers. However, when PAYG pension system is present, the impact of population growth rate on retirement length needs to be reexamined since as discussed above, it can go in either positive or negative way. Specifically, according to equation (29), besides the negative effect because of the reduction of per-child efficiency in the second term, the first term of this equation implies the positive impact of population growth on retirement benefits (equivalently negative ef-
fect on the price of retirement length). Therefore, in an economy with altruism and intra-familial time transfers, PAYG pension system reduces the negative effect of population growth on retirement length through intra-familial transfers. In other words, the introduction of PAYG pension system decreases the incentive for working of the old in an economy with increasing population growth rate and increases her/his incentive for working under a decreasing (aging) population growth.

Reconsidering equations (29), the sign of $\frac{\partial T^*}{\partial n}$ depends on the sign of:

$$g(n) = \frac{\tau w (1 - \gamma - \gamma/R)}{\eta R (1 - \tau)w} - \frac{\mu \lambda T^*}{(1 + n)^2}$$

We deduce the following proposition.

**Proposition 6.** In an economy with altruism and intra-familial time transfers, with presence of PAYG pension system, the effect of the population growth rate on retirement length is:

- If $g(0) \geq 0$ then $\frac{\partial T^*}{\partial n} > 0 \forall n \geq 0$.
- If $g(0) < 0$ then there exists $\bar{n} > 0$ such that $\frac{\partial T^*}{\partial n} > 0$ for $n > \bar{n}$ and $\frac{\partial T^*}{\partial n} < 0$ for $n < \bar{n}$.

**Proof.** Using equation (24), we have:

$$T^* \frac{1}{(1 + n)^2} = \frac{1}{1 + n} \frac{n(1-\tau)(1+R)+\tau}{\lambda \mu R (1 - \gamma)} (1 - \gamma - \gamma/R)$$

which is a decreasing function of $n$. Thus, $g(n)$ is increasing in $n$ and we deduce the proposition.

Since we don’t consider negative population growth, this proposition implies that an increase in the population growth can incite the old agent to retire earlier in two situations. The first situation is when the retirement length is increasing at $n = 0$: $\frac{\partial T}{\partial n} \bigg|_{n=0} \geq 0$. The second one is when the retirement length is increasing at $n = 0$: $\frac{\partial T}{\partial n} \bigg|_{n=0} < 0$ with a sufficiently large value of $n$. In the latter case, if the population growth rate is relatively low (aging population), improvement in the population growth rate incites the old agent to postpone retirement. Therefore, given a relatively high population growth in the economy, when PAYG pension system is operative, retirement length is more likely increasing with the population growth rate because the positive effect of contribution to the pension system, which raises her/his pension benefit, dominates the negative effect on her/his life-cycle income.

When we consider effect of population growth rate on aggregate labor supply. Derivative of aggregate labor supply with respect to $n$ is:

$$\frac{\partial L_{t+1}}{\partial n} = N_t \left[ (1 - \gamma - \gamma/R) + (\lambda \mu (1 - \gamma) - 1) \frac{\partial T_{t+1}}{\partial n} \right]$$

Equation (30) shows that the population growth rate affects aggregate labor supply through three channels: the first channel is an increase of young labor force which is $1 - \gamma - \gamma/R$; secondly
the effect of change in old labor supply represented by \(-\frac{\partial T_{t+1}}{\partial n}\), and finally the effect of change in young labor supply because of change in time transfers from elderly parents, represented by \(\lambda\mu(1-\gamma)\frac{\partial T_{t+1}}{\partial n}\). Notice that when time transfers are not operative, the effect of population growth on retirement length is positive due to increase in young labor force and then pension benefits. In fact, when \(\mu = 0\), the sign of \(\frac{\partial T}{\partial n}\) is the sign of \(g(n)\) which is always positive. In this case, the effect of population growth rate on aggregate labor supply is twofold: increase in young labor force and decrease in old labor supply. Therefore, the introduction of time transfers provides an additional positive effect of population growth rate on labor supply since the young can dispose more time to work because s/he has her/his parents helping on doing household works.
6 Conclusion

This paper studies an overlapping-two generations model with pay-as-you-go social security system and intra-familial time transfers. We characterized an equilibrium in which old workers are altruistic: they transfer a fraction of retirement length to their adult children to help them producing consumed goods. The positive relationship between time transfers and young labor supply has been derived. When the old agent decides to retire earlier, s/he spends her/his retirement time helping her/his adult children on home production including grandchild care. Thus, the young agent can have more time and is able to work more.

The effects of the payroll tax rate and the population growth rate on retirement decision of the old agent and aggregate labor supply are studied in the last section. We obtain that the payroll tax rate has a positive effect on retirement length because the old agent receives a higher pension benefit which compensates her/him for reduction of income due to higher tax imposed. However, the effect on aggregate labor supply depends furthermore on the efficiency of time transfer. This effect is positive only if time transfer is relatively efficient. In addition, the introduction of time transfers weakens the positive effect of population growth rate on retirement length. Also, it gives an additional positive effect to overall effects of the payroll tax rate or the population growth rate on labor supply.

On the whole, intra-familial time transfers are beneficial in the economy, especially when they are highly efficient. We can evaluate the efficiency of intra-familial time transfers through many factors such as the distance between living places of grandparents and adult children or cultural contexts of different countries. In fact, time transfers are efficient in Asian countries such as China, Japan and Southeast Asian countries as it’s a common practice and their cultures encourage them doing so. Opposing to United States, time transfers seem to be less efficient because of far distance in living places between family generations.
References


A Appendix

A.1 Implication of Assumption 2

Using the budget constraint of the pension system (19), we obtain:

$$\left(1 - \tau\right)w - b_{t+1} > 0$$

$$\iff 1 - \frac{\tau(1 + n) h_{t+1} + 1}{T_{t+1}} > 0$$

Plugging $$h_{t+1} = (1 - \gamma) \left(1 + \frac{\lambda u T_{t+1}}{1 + \mu}ight) - \frac{\gamma}{R} < 1$$, the above inequality writes:

$$\frac{\tau(1 + n) \left[(1 - \gamma) \left(1 + \frac{\lambda u T_{t+1}}{1 + \mu}ight) - \frac{\gamma}{R}\right] + \tau}{T_{t+1}} < 1$$

$$\iff \tau \mu \lambda (1 - \gamma) + \frac{\tau(1 + n)(1 - \gamma - \frac{\gamma}{R}) + \tau}{T_{t+1}} < 1$$

The assumption $$\tau \mu \lambda (1 - \gamma) < 1$$ guarantees that $$T_{t+1}$$ is positive when $$h_{t+1} < 1$$ satisfying $$(1 - \tau)w - b_{t+1} > 0$$.

A.2 Effect of the payroll tax rate on retirement length

$$T^* = \frac{\eta R (1 - \tau)}{(1 - \tau) w - b(T^*, n)} \left(1 + \frac{\lambda u T^*}{1 + n} + \frac{1}{R}\right)$$

$$\Rightarrow \frac{dT^*}{T^*} = \frac{d\tau}{1 - \tau} + \frac{1}{1 + \frac{\lambda u T^*}{1 + \mu} + \frac{1}{R}} dT^* + \frac{b_T'}{(1 - \tau) w - b(T^*, n, \tau)} dT^* + \frac{w + b_T'}{(1 - \tau) w - b(T^*, n, \tau)} d\tau$$

$$\Rightarrow \frac{dT^*}{T^*} \left(1 - \frac{\lambda u T^*}{1 + \frac{\lambda u T^*}{1 + \mu} + \frac{1}{R}} - \frac{b_T' T^*}{(1 - \tau) w - b(T^*, n, \tau)}\right) = \left(\frac{w + b_T'}{(1 - \tau) w - b(T^*, n, \tau)} - \frac{1}{1 - \tau}\right) d\tau$$

Since the derivative of pension benefit $$b$$ with respect to $$T^*$$ is negative: $$b_T' < 0$$ according to the proposition 2 and $$\frac{\lambda u T^*}{1 + \frac{\lambda u T^*}{1 + \mu} + \frac{1}{R}} < 1$$, the above equation can be rewritten as equation (26).

A.3 Effect of the population growth rate on retirement length

$$T^* = \frac{\eta R (1 - \tau)}{(1 - \tau) w - b(T^*, n)} \left(1 + \lambda u T^* + \frac{1}{R}\right)$$

$$\Rightarrow \frac{dT^*}{T^*} = \frac{b_T'}{(1 - \tau) w - b(T^*, n, \tau)} dn + \frac{b_T'}{(1 - \tau) w - b(T^*, n, \tau)} dT^* + \frac{\lambda u}{1 + \frac{\lambda u T^*}{1 + \mu} + \frac{1}{R}} dT^* - \frac{\lambda u T^*}{(1 + n)^2} dT^* - \frac{\lambda u T^*}{(1 + n)^2} dn$$

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\[ d \frac{T^*}{T^*} \left( 1 - \frac{\lambda T^*}{1 + \frac{\lambda T^*}{1 + \pi} + \frac{1}{R}} - \frac{b'_n T^*}{(1 - \tau)w - b(T^*, n, \tau)} \right) = \frac{T^*}{1 + \frac{\lambda T^*}{1 + \pi} + \frac{1}{R}} \left( \frac{b'_n (T^*, n, \tau)}{\eta R (1 - \tau)w} - \frac{\mu \lambda}{(1 + n)^2} \right) dn \]

Similarly to A.2, the above equation can be rewritten as equation (29).