Abstract

The purpose of this study is to analyze tax interactions among two-tier governments, composed of several regional governments and a federal government. All of them can tax two tax bases, i.e. capital and an energy resource, that are interdependent. We find that the federal trade-off between the taxation of capital versus the taxation of energy only depends on the relative exogenous supply of energy and capital. The trade-off between the taxation of capital versus the taxation of energy is much more complex as it also depends on the comparison between distortive effects of these taxes on both the regional public good and the environmental quality, as well as on the interdependence between the two taxes. We also find a substitutability among federal and regional taxes, whatever the interdependence between the two taxes, implying that the federal government increases a given tax rate in response to a decrease of one of the two regional tax rates.

1 Introduction

It is important to develop efficient environmental policies to mitigate climate change, preserve biodiversity, and reduce water and air pollution. To this end, the public authorities have a wide range of instruments that are generally regulatory (conventional instruments, aimed at constraining agents’ behavior) or economic ones (an incentive to encourage more virtuous behavior). Environmental taxation appears to be a less coercive tool than the norm, and nevertheless efficient when it allows economically to favor for sustainable behavior as compared to harmful behaviors to the environment. Thus, taxation is nowadays recognized as a powerful lever for modifying individual and collective behavior, owing to the financial incentive it addresses to those who support it.

Furthermore, energy taxation is being mutation in sense that fiscal decisions become increasingly centralized. For instance, in a Community framework, the EU has set drastic energy policy targets, including a 20% reduction in greenhouse gas emissions by 2020 and an increase of 20% of the share of renewable energies in gross final energy consumption. However member states remain sovereign in terms of energy resources and way to achieve the objectives set by Europe. For illustration, it is important to emphasize that energy policy for most European countries has at least three distinct levels of decision: the European Union, the State and local authorities. Therefore, while energy transition is increasingly defined at the upper tier (e.g.
European level), the actors involved in its implementation will be increasingly decentralized and close to consumers. In addition, energy resources are various forms and dispersed in space. Hence, their exploitation modifies the geographical distribution of energy potentials.

Related to energy resources, at least two main implications are essentials when implementing energy fiscal policy: there are fiscal and environmental considerations. From fiscal considerations, the concentration of energy resources in certain localities may lead to fiscal disparities between local authorities. Local governments can compete both in terms of fiscal point of view and provision of public services in order to attract mobile production factors. To alleviate these fiscal distortions, central or federal government can levy a federal tax in order to reduce the magnitude of intergovernmental fiscal disparities. Indeed, it is well-known that heterogeneity of energy fiscal endowments can lead to significant fiscal disparities. For illustration, the problematic of fiscal disparities between regions is particularly important in large countries which produce natural resources such as Canada (oil, natural gas, uranium, diamonds, gold, nickel), Australia (coal, uranium, iron, potash, Bauxite) and Russia (natural gas, oil, coal, uranium, aluminum, nickel). In these three federations, the natural resource rent is owned by the regional entities. From an environmental point of view, the exploitation of energy resources can cause significant environmental damage. In this perspective, environmental taxation has attracted increasing attention as taxes can, at least in principle, internalize the external effects of environmental damage. Furthermore, many economists have argued that environmental taxes are an efficient instrument for achieving environmental objectives (see, e.g., [Baumol and Oates (1988) and Pearce and Turner (1990)].

Furthermore, even if high attention has turned to the key feature of the energy transition, however from a normative point of view, there are few contributions on "energy fiscal federalism" defined as the imbrication of different levels of government which possess an autonomous power to tax energy. And yet, there are numerous issues. Should the Member States levy alone the energy tax? What would be the effectiveness of an additional energy tax levied on a larger scale at European level? Is it desirable for fiscal policy to be fully transferred to a central level (Brussels in a Community framework, or the central State in the framework of a country) or contrarily, should be given priority to the decentralization of energy tax policy to sub-national communities in order to better account for heterogeneity among jurisdictions, in particular in terms of energy access? Should the distribution of energy taxation instruments depend on the nature of energies? These questions are then asked to determine which level of government should levy what energy taxation. Decentralization of energy taxation should be analyzed in the light of these mechanisms, taking into account the specificities of the bases of energy resources: the environmental consequences of the exploitation of energy, the more or less renewable nature of energy, their (sometimes immediate) impacts on economic growth, the greater or lesser substitutability between the bases or the globalization of energy markets.

In this study, we consider a two-tier territorial organization with several identical bottom-tier jurisdictions, such as region, and a unique top-tier jurisdiction, such as the federal government. We extend the standard model of fiscal federalism developed by [Keen and Kotsogiannis (2002), Keen and Kotsogiannis (2004)] who consider one tax base, namely the capital, by incorporating an additional tax base, both tax bases being interdependent. The originality is that the tax base is energy, which has an impact on the provision of an environmental public good, namely "environmental quality", which depends negatively to energy consumption. The federal
and regional governments tax the same mobile bases, that are the amounts of capital and energy invested in their territory. As a consequence, both tax bases are co-occupied. Benevolent federal and regional governments use their tax revenues in order to finance public goods that benefit exclusively their immobile inhabitants. Finally, we assume that the federal and regional governments use taxes as their strategy variables in a simultaneous-move Nash game.

Three main results emerge from our paper. We find that the federal trade-off between the taxation of capital versus the taxation of energy only depends on the relative exogenous supply of energy and capital. The regional trade-off between the taxation of capital versus the taxation of energy is much more complex as it also depends on the comparison between the distortive effects of these taxes on both the regional public good and the environmental quality, as well as on the interdependence between the two taxes. We also find a substitutability among federal and regional taxes, whatever the interdependence between the two taxes, implying that the federal government increases a given tax rate in response to a decrease of one of the two regional tax rates.

The present chapter thus contributes to the theoretical literature in two ways. First, it provides additional comprehension elements on the issue of energy fiscal federalism by examining the structure of energy taxes levied by two tiers aimed at internalizing the external effects from environmental damage. Second, it extends earlier analytical work on optimal energy taxation in the presence of other distortionary tax by considering that energy tax is imposed on intermediate inputs for improving environmental quality.

The paper is organized as follows. Section 2 presents a brief literature review. In Section 3, we derive optimal taxes and tax reaction functions in the Nash game played by federal and regional players. Finally, in Section 4, we provide concluding comments.

## 2 Literature review

### 2.1 Fiscal federalism literature

Several studies have emphasized the importance of tax externalities (horizontal and vertical) for the theory of fiscal federalism. Initially developed by Oates Wallace (1972) and formally modelled by Wilson (1986) and Zodrow and Mieszkowski (1986), the existence of tax externalities depends on the mobility of tax base. In particular, in presence of horizontal tax externalities, when a region increases its capital tax rate, some amount of the tax base (usually capital) is reallocated to other regions. This reallocation represents a positive externality, which is not taken into account, implying that taxes and public expenditures are inefficiency low in equilibrium. The literature has paid a recent attention on vertical tax externalities, which arise when two or more different levels of government share the same tax base. Each level of government neglects the adverse effect it has on the other by raising its tax rate. Vertical tax externalities lead to excessively high taxes (Dahlby and Wilson, 2003; Hoyt, 2001; Keen, 1998; Keen and Kotsogiannis, 2002). With horizontal externalities pointing towards regional taxes that are inefficiently low and vertical externalities towards regional taxes that are inefficiently high, it is natural to ask which will dominate. Keen and Kotsogiannis (2002), show that whether equilibrium taxes are too high or too low in equilibrium depends on the elasticities of the demand for capital and the supply of savings.
Consider two interdependent tax bases, which creates indirect horizontal and vertical tax externalities, in addition to standard direct horizontal and vertical tax externalities. We will retain this framework in this chapter.

2.2 Literature about environmental taxation

Generally, an emission tax is used as an environmental policy instrument to reduce local environmental damages. The rents raised from this tax are retained locally, but can be allocated to varying degrees for public finance or for private consumption. In this regard, Alexeev et al. (2016) explore the implications for jurisdictional welfare of sharing environmental rents between private and public consumption. Using three production factor (labor, capital and emissions), they show that jurisdictional welfare increases as environmental rents are initially allocated towards public consumption, yielding a "double dividend", in the sense that welfare of jurisdictional decision-making is highest when all environmental rents are dedicated to public finance. These results illustrate the crucial importance of environmental rent sharing for the efficiency of jurisdictional decision-making.

Linked to the previous point, Kim and Wilson (1997) investigate the possibility of a 'race to the bottom,' under which intergovernmental competition for mobile capital leads to inefficiently lax environmental standards. The term 'race to the bottom' is often used to describe the possibility that intergovernmental competition for mobile capital will lead to inefficiently lax environmental policies. Their model is based on the Bucovetsky-Wilson model of tax competition with multiple tax instruments, extended to include pollution emissions from production activities. Indeed, in addition to capital and labor as input factors, they include emissions in production function. In particular, according to this input factor, they follow Oates and Schwab (1988), by making the emissions-labor ratio, $e = E/L$. They show that decentralized decision-making by independent national governments leads to environmental standards that are inefficiently lax.

According to Wellisch (1995), in the course of jurisdictional competition, jurisdictions set environmental policies efficiently if there are no other market distortions and the environmental policies capture and return environmental rents to local, immobile residents. A similar kind of result is obtained in the classic model of Oates and Schwab (1988). In the presence of other market distortions or if jurisdictions cannot capture the environmental rents, jurisdictional environmental policy-making is not likely to be efficient.

Bovenberg and De Mooij (1994) aim to explore under which conditions environmental taxes do indeed reduce the efficiency costs of financing public spending in presence of other distortion

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9 The double dividend hypothesis states that it is possible to improve the quality of the environment and the efficiency of the tax system. In other words the double dividend assumption implies that there would be environmental and non-environmental gains in the application of an environmental tax when revenues from the environmental tax are used to reduce a form of taxation that creates more distortions. In the literature, the double dividend is known into two forms: weak and strong forms. According to Goulder et al. (1999), the weak form states that it is cheaper to use green tax revenues to reduce a pre-existing distortionary tax than to redistribute those revenues into lump-sum transfers. The strong form implies that taxing a polluting good and recycling its revenues in order to reduce a representative distortionary tax does not entail any cost, or even brings income to the state.
tax (in particular, labor tax). To this end, they formulate a simple general equilibrium model of a small open economy. Besides labor \( L_1 \) and capital \( K \), they also include in production function a third input \( E \) which causes environmental damage. It is called the "polluting" input and can be thought off as energy. In their model, they formalize an inverse relationship between the quality of the natural environment and the demand for, respectively, polluting consumption commodities and polluting inputs into production. They find that environmental taxes typically render the overall tax system a less efficient instrument to finance public spending. They also find that pre-existing tax distortions in the labor market reduce rather than enhance the attractiveness of environmental policy, in general, and of a heavy reliance on environmental taxes, in particular. The fundamental reason is that the environment is a collective good; all residents benefit - irrespective of the amount of labor they supply.

Bovenberg and Goulder (1996) extend the model in Bovenberg and De Mooij (1994) by incorporating intermediate inputs. They also consider three inputs in production function, namely, labor \( L_1 \), "clean" and "dirty" intermediate goods. They examine how optimal environmental tax rates deviate from rates implied by the Pigovian principle in a second-best setting where other, distortionary taxes are present. They show that, in the presence of distortionary taxes, optimal environmental tax rates are generally below the rates suggested by the Pigovian principle—even when revenues from environmental taxes are used to cut distortionary taxes. As mentioned in Bovenberg and Goulder (1996), economists typically have analyzed environmental taxes without taking into account the presence of other distortionary taxes. The omission is significant because the consequences of environmental taxes depend fundamentally on the levels of other taxes.

3 Model

3.1 Federal and regional governments

We assume a two-tier intergovernmental structure comprised of a federal government and \( n \) regional governments, indexed by \( i = 1, ..., n \). Pure public goods are provided at both federal and regional tiers, with no spillovers and no scale economies. Each regional government \( i \) provides a regional public good in quantity \( g_i \), which is financed by the taxation at the rate \( t_i \) of the amount of capital \( k_i \) invested in the region and the taxation at the rate \( \tau_i \) of the amount of energy input \( e_i \) used in the region. The regional budget constraint is thus given by

\[
g_i = t_i k_i + \tau_i e_i
\]  

The federal government provides publicly provided private goods in quantity \( nG \), which are financed by respectively the taxation at rates \( T_k \) and \( T_e \) of the amount of capital \( \sum_i k_i \) and energy \( \sum_i e_i \) employed in region \( i \). The federal budget constraint is thus given by:

\[
G = \frac{T_k}{n} \sum_i k_i + \frac{T_e}{n} \sum_i e_i
\]  

We assume that both federal and regional governments are utilitarian and benevolent. Each jurisdiction acts so as to maximize the utility of its representative citizen.
3.2 Capital and energy markets

There is an unique immobile firm in each region. The firm located in region $i$ produces an output $f(k_i, e_i)$ from the capital $k_i$ borrowed on the market at the interest rate $r^k_i$ and from the energy $e_i$ borrowed on the market at the price $r^e_i$. The production function, which is identical across regions, is assumed to be monotonously increasing in both production factors with $f'(\cdot) > 0$ and to have diminishing factor productivities $f''(\cdot) < 0$. Capital and energy can be complementary inputs in production ($f_{ke} > 0$ and $f_{ek} > 0$) or competitive inputs ($f_{ke} < 0$ and $f_{ek} < 0$). We make the restrictive assumption that $f''(\cdot) = 0$ to ensure that the demands for capital and energy are linear, which makes it possible to derive closed form solutions. The owner of the firm is the representative citizen of the region, which receives the entire profit, which is standard in this type of model:

$$\pi_i = F(k_i, e_i) - (\rho_{k_i} + t_i + T_k)k_i - (\rho_{e_i} + \tau_i + T_e)e_i.$$

For simplicity, we assume that the profit is not taxed. Since producers act as price takers, firm profit maximizing behavior implies the following first-order conditions system:

$$\begin{align*}
    f_{k_i} &= r^k_i, \\
    f_{e_i} &= r^e_i,
\end{align*}$$

which means that capital and energy are remunerated at their marginal productivities, with $r^k_i = \rho_{k_i} + t_i + T_k$ and $r^e_i = \rho_{e_i} + \tau_i + T_e$. The resulting demands for capital $k(r^k_i, r^e_i)$, energy $e(r^k_i, r^e_i)$ and the expression of the profit $\pi_i(r^k_i, r^e_i)$ are presented in Appendix A.

Capital and energy are both perfectly mobile in perfectly competitive markets. They move across regions until they earn the same net return everywhere, that is respectively, $\rho_k = r^k_i - (t_i + T_k)$ and $\rho_e = r^e_i - (\tau_i + T_e) \forall_i$. The representative citizen of each region $i$ is initially endowed with $\bar{k}$ units of capital and $\bar{e}$ units of energy. The overall supply of capital is $n\bar{k}$ and the overall supply of energy is $n\bar{e}$. Given that $r^k_i = \rho_k + t_i + T_k$ and $r^e_i = \rho_e + \tau_i + T_e \forall_i$, the capital and energy market-clearing conditions are respectively $\sum_{i=1}^n k(\rho_k + t_i + T_k, \rho_e + \tau_i + T_e) = n\bar{k}$ and $\sum_{i=1}^n e(\rho_k + t_i + T_k, \rho_e + \tau_i + T_e) = n\bar{e}$, which implicitly define the equilibrium values of the net returns on capital and energy, $\rho_k(T_k, t_1, \ldots, t_n, \rho_e(T_e, \tau_1, \ldots, \tau_n)$, $\rho_e(T_e, \tau_1, \ldots, \tau_n)$). Differentiating the markets-clearing conditions yields, at the symmetric equilibrium:

$$\begin{align*}
    \frac{\partial \rho_k}{\partial t_i} &= \frac{\partial \rho_e}{\partial t_i} = -\frac{1}{n}, \\
    \frac{\partial \rho_k}{\partial T_k} &= \frac{\partial \rho_e}{\partial T_k} = -1, \\
    \frac{\partial \rho_k}{\partial e_i} &= \frac{\partial \rho_e}{\partial e_i} = \frac{\partial \rho_k}{\partial e_i} = \frac{\partial \rho_e}{\partial e_i} = 0, \\
    \frac{dr^k_i}{dt_i} &= 1 + \frac{\partial \rho_k}{\partial t_i} = n\frac{1}{n}, \quad \frac{dr^e_i}{dt_i} = 1 + \frac{\partial \rho_e}{\partial t_i} = n\frac{1}{n}, \\
    \frac{dr^k_i}{dT_k} &= \frac{dr^e_i}{dT_k} = \frac{dr^k_i}{dt_i} = \frac{dr^e_i}{dt_i} = 0
\end{align*}$$

The mobility of the two tax bases generate a double common-pool problem(?). The choice of a region’s tax rate affects its own tax base, the tax base of other same-tier regions and the tax base of the federation, because of the combination of tax-base sharing and mobility. Due to the interdependence between the two tax bases, the choice of a region’s tax rate on a given tax base also affects its other tax base, the other tax base of other same-tier regions and the other tax base of the federation. These indirect tax effects do not go through the returns as
\[
\frac{d\rho_k}{dt_i} = \frac{d\rho_e}{dt_i} = \frac{d\rho_k}{dT_e} = \frac{d\rho_e}{dT_k} = 0 \text{ but through the demand functions } k_i(t^k_i, r^e_i) \text{ and } e_i(t^k_i, r^e_i). \text{ In addition, it should be noted that contrary to regional tax choices that affect the gross return (} r^k_i \text{ for capital and } r^e_i \text{ for energy), federal choices have no impact on gross returns.}
\]

### 3.3 The representative citizen

Citizens are assumed to be identical and immobile, such that we consider a representative citizen in each jurisdiction. The representative citizen in a region \( i \) obtains utility \( U(C_i, g_i, G, E_i) \) from the consumption of the regional public good \( g_i \), the consumption of a the federal public good \( G \) as well as utility from the consumption of a private good \( C_i \) and an environmental public good, namely "environmental quality" denoted by \( E_i \). Environmental quality \( E_i \) is a regional good, which is defined as follow:

\[
E_i = h(e_i)
\]  

(3)

with \( h_{e_i} < 0 \) which links up to energy resource. This expression formalizes the inverse relationship between the quality of the natural environment and the demand for energy inputs into production. We assume that energy consumption \( (e_i) \) harms environmental quality \( (E_i) \). Indeed, in the process of production, the use of energy input is associated with less environmental quality due to for instance a higher level of emissions. Following Bovenberg and Goulder (1996), we consider that environment quality is a collective good and weakly separable from private goods. Accordingly, households take the quality of the environment as given and they ignore the adverse effect of their demand for energy on the quality of the environment.

We assume that the representative citizen in region \( i \) is both the owner of a unique firm located in its jurisdiction and the owner of exogenous amounts \( k \) of capital and \( e \) of energy. The amount of each input \( (k) \) and \( e \) can be invested in a firm in any region to earn the net return on capital (respectively on energy), denoted by \( \rho_{ki} \) (resp. \( \rho_{ei} \)), which is equal to the return after regional and federal taxes. As shown before, at the symmetric equilibrium, \( \rho_{ki} = \rho_k \) and \( \rho_{ei} = \rho_e \).

The private consumption is: \( C_i = \pi_i + \rho_k k + \rho_e e = F(k_i, e_i) - (\rho_k + t_i + T_k)k_i - (\rho_e + \tau_i + T_e)e_i + \rho_k k + \rho_e e \).

In order to derive explicit solutions for equilibrium tax rates, we need specific assumptions on the functional form of the utility function.

### 3.4 The timing of the game

In the benchmark case, the federal government and the \( n \) regional governments simultaneously select their fiscal parameters, taking as given the policy choices of the other players. In other words, federal and regional governments play a Nash game. More specifically, the federal government selects tax rates \( T_k \) and \( T_e \) to maximize the welfare of the representative citizen residing within the federation, taking as given tax rates chosen by the regional governments. Simultaneously, each regional government \( i \) selects its tax rates \( (t_i \) and \( \tau_i \) to maximize the welfare of the representative citizen residing within their jurisdiction, taking as given tax rates chosen by the federation and other regional jurisdictions. Federal and regional public goods are determined as residuals after taxes are collected. Given these policy choices, firms determine capital and

\[^{20}\text{The composite good provided can be used for public consumption (} G \text{ or } g_i \), for household consumption } C_i \text{, or environmental quality good } E_i.\]
energy demands, and then production takes place. Finally, each citizen receives profits from her firm’s activity, enjoys the consumption of private, regional and federal public goods and derives utility from the quality of natural environment. Federal and regional jurisdictions take into account the reaction of the capital and energy demands when determining their tax decisions and citizens’ preferences guide the choices of the federal and regional jurisdictions as they are both benevolent. These results are derived in the Nash game.

4 The Nash game

Before illustrating the subgame-perfect equilibrium, we first present the outcome of the first-best solution which will serve for comparison purposes. In this benchmark case, a benevolent social planner aims to maximize the aggregated welfare $W(t_i, \tau_i, T_k, T_e) = \sum_{i=1}^{n} U(C_i, g_i, G, E_i)$ subject to the budget constraints (1), (2) and (5). Suppose that federal and regional governments play a Nash game. In this situation, the federal government and regional governments simultaneously select their fiscal choices.

4.1 The federal government’s problem

The federal government maximizes the aggregated utility of citizens located in the federation by choosing the tax rates $T_k$ and $T_e$, taking as given the tax choices of regional jurisdictions. It thus solves the problem:

$$\text{Max } W(t_i, \tau_i, T_k, T_e) = \sum_{i=1}^{n} U(C_i, g_i, G, E_i)$$

subject to

$$C_i = \pi_i + \rho_k \bar{k} + \rho_e \bar{e},$$

$$g_i = t_k k_i + \tau_e e_i,$$

$$G = T_k \sum_i k_i + T_e \sum_i e_i.$$ The federal tax policy satisfies the following first-order conditions:

$$\text{FOC} / T_k: \sum_{i=1}^{n} \left[ \frac{\partial U}{\partial C_i} \frac{\partial C_i}{\partial T_k} + \frac{\partial U}{\partial g_i} \frac{\partial g_i}{\partial T_k} + \frac{\partial U}{\partial G} \frac{\partial G}{\partial T_k} + \frac{\partial U}{\partial E_i} \frac{\partial E_i}{\partial T_k} \right] = 0,$$

$$\text{FOC} / T_e: \sum_{i=1}^{n} \left[ \frac{\partial U}{\partial C_i} \frac{\partial C_i}{\partial T_e} + \frac{\partial U}{\partial g_i} \frac{\partial g_i}{\partial T_e} + \frac{\partial U}{\partial G} \frac{\partial G}{\partial T_e} + \frac{\partial U}{\partial E_i} \frac{\partial E_i}{\partial T_e} \right] = 0.$$ Using the expressions derived in the Appendix B, at the symmetric equilibrium, the FOCs boil down to:

$$\frac{\partial U}{\partial C_i} = \frac{\partial U}{\partial G}.$$ Since $\frac{\partial E_i}{\partial T_k} = \frac{\partial E_i}{\partial T_e} = 0$, the federal taxation has no impact on the environmental quality.

In a Nash game, the federal government thus chooses the vector of optimal taxes $(\hat{T}_k$ and $\hat{T}_e$) to equalize the marginal desutility from a reduction in private consumption to the marginal utility from an increase in the federal public good provision. Eq. (4) determines the federal govern-
ment’s reaction function $T_k(t_1, ..., t_i, ..., t_n, \tau_1, ..., \tau_i, ..., \tau_n, T_e)$ and $T_e(t_1, ..., t_i, ..., t_n, \tau_1, ..., \tau_i, ..., \tau_n, T_k)$.

Differentiating w.r.t. federal tax rates and regional tax rates, we obtain that
\[
\frac{\partial^2 U}{\partial C_i \partial C_j} \left( \frac{\partial C_i}{\partial T_k} dT_k + \frac{\partial C_i}{\partial T_e} dT_e + \frac{\partial C_i}{\partial t_i} dt_i + \frac{\partial C_i}{\partial \tau_i} d\tau_i \right) = \frac{\partial^2 U}{\partial C_i^2} \left( \frac{\partial C_i}{\partial T_k} dT_k + \frac{\partial C_i}{\partial T_e} dT_e + \frac{\partial C_i}{\partial t_i} dt_i + \frac{\partial C_i}{\partial \tau_i} d\tau_i \right),
\]
which gives
\[
\begin{align*}
\frac{dT_k}{dT_e} &= \frac{dT_k}{dt_i} = \frac{\bar{\epsilon}}{k} \\
\frac{dT_k}{dt_i} &= \frac{dT_e}{dt_i} = -1 \\
\frac{dT_e}{dT_k} &= \frac{dT_e}{dt_i} = -\frac{\bar{k}}{\bar{\epsilon}}
\end{align*}
\]
at the symmetric equilibrium. The federal trade-off between the taxation of capital versus the taxation of energy depends on the relative endowment $\frac{\bar{k}}{\bar{\epsilon}}$. For $\bar{\epsilon} = \bar{k}$, $dT_k + dT_e = 0$. We also find a substitutability among federal and regional taxes, whatever the interdependence between the two taxes.

### 4.2 The regional government’s problem

Each regional government $i$ chooses its tax rates $t_i$ and $\tau_i$ so as to maximize the utility of its representative citizen subject to the governmental budget constraints and the decentralized optimizing behavior of firms and households, taking as given the tax choices of federal and other jurisdictions. It thus solves the problem:

\[
\text{Max } W(t_i, \tau_i) = U(C_i, g_i, G, E_i)
\]
subject to
\[
C_i = \pi_i + \rho_k \bar{k} + \rho_e \bar{\epsilon},
\]
\[
g_i = t_i k_i + \tau_i e_i,
\]
\[
G = \frac{\bar{\epsilon}}{n} \sum_{i} k_i + \frac{\bar{k}}{n} \sum_{i} e_i.
\]
The first-order conditions are:
\[
\begin{align*}
\text{FOC } / t_i : \quad & \frac{\partial U}{\partial C_i} \frac{\partial C_i}{\partial t_i} + \frac{\partial U}{\partial g_i} \frac{\partial g_i}{\partial t_i} + \frac{\partial U}{\partial G} \frac{\partial G}{\partial t_i} + \frac{\partial U}{\partial E_i} \frac{\partial E_i}{\partial t_i} = 0 \\
\text{FOC } / \tau_i : \quad & \frac{\partial U}{\partial C_i} \frac{\partial C_i}{\partial \tau_i} + \frac{\partial U}{\partial g_i} \frac{\partial g_i}{\partial \tau_i} + \frac{\partial U}{\partial G} \frac{\partial G}{\partial \tau_i} + \frac{\partial U}{\partial E_i} \frac{\partial E_i}{\partial \tau_i} = 0.
\end{align*}
\]

Using the expressions derived in the Appendix B, at the symmetric equilibrium, the FOCs boil down to:
\[
\begin{align*}
\left\{ -\frac{\partial U}{\partial C_i} \bar{k} + \frac{\partial U}{\partial g_i} (k_i + t_i k_i \frac{\partial g_i}{\partial t_i} + \tau_i e_i \frac{\partial g_i}{\partial \tau_i}) + \frac{\partial U}{\partial G} \frac{\partial G}{\partial t_i} \frac{\partial G}{\partial \tau_i} = 0 \\
-\frac{\partial U}{\partial C_i} \bar{\epsilon} + \frac{\partial U}{\partial g_i} (e_i + t_i k_i \frac{\partial g_i}{\partial t_i} + \tau_i e_i \frac{\partial g_i}{\partial \tau_i}) + \frac{\partial U}{\partial E_i} \frac{\partial E_i}{\partial t_i} \frac{\partial E_i}{\partial \tau_i} = 0
\right\} (5)
\]

The system (5) determines the regional government’s reaction functions $t_i(T_k, T_e, t_1, ..., t_{i-1}, t_{i+1}, ..., t_n, \tau_1, ..., \tau_{i-1}, \tau_{i+1}, ..., \tau_n)$ and $\tau_i(T_k, T_e, t_1, ..., t_{i-1}, t_{i+1}, ..., t_n, \tau_1, ..., \tau_{i-1}, \tau_{i+1}, ..., \tau_n)$. 

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Differentiating (5) w.r.t. federal tax rates and regional tax rates, and using the expressions derived in the Appendix B, we obtain at the symmetric equilibrium that:

\[
\begin{aligned}
\left\{ \begin{array}{l}
-\frac{\partial^2 U}{\partial C^2} \left( -k_d T_k - c_d T_c - k_d t_i - c_d r_i \right) + \frac{\partial^2 U}{\partial g_e^2} \left( \frac{\partial g_e}{\partial C} dt_i + \frac{\partial g_e}{\partial r_i} dr_i \right) + \frac{\partial^2 U}{\partial E^2} \left( \frac{\partial E}{\partial C} dt_i + \frac{\partial E}{\partial r_i} dr_i \right) + \\
\frac{\partial^2 U}{\partial E^2} \left( \frac{\partial E}{\partial C} dt_i + \frac{\partial E}{\partial r_i} dr_i \right) + \frac{\partial^2 U}{\partial g_e \partial r_i} \left( \frac{\partial g_e}{\partial r_i} dt_i + \frac{\partial g_e}{\partial C} dr_i \right) + \frac{\partial^2 U}{\partial g_e \partial C} \left( \frac{\partial g_e}{\partial C} dt_i + \frac{\partial g_e}{\partial r_i} dr_i \right) = 0 \\
-\frac{\partial^2 U}{\partial C^2} \left( -k_d T_k - c_d T_c - k_d t_i - c_d r_i \right) + \frac{\partial^2 U}{\partial g_e^2} \left( \frac{\partial g_e}{\partial C} dt_i + \frac{\partial g_e}{\partial r_i} dr_i \right) + \frac{\partial^2 U}{\partial E^2} \left( \frac{\partial E}{\partial C} dt_i + \frac{\partial E}{\partial r_i} dr_i \right) + \\
\frac{\partial^2 U}{\partial E^2} \left( \frac{\partial E}{\partial C} dt_i + \frac{\partial E}{\partial r_i} dr_i \right) + \frac{\partial^2 U}{\partial g_e \partial r_i} \left( \frac{\partial g_e}{\partial r_i} dt_i + \frac{\partial g_e}{\partial C} dr_i \right) + \frac{\partial^2 U}{\partial g_e \partial C} \left( \frac{\partial g_e}{\partial C} dt_i + \frac{\partial g_e}{\partial r_i} dr_i \right) = 0
\end{array} \right.
\end{aligned}
\]

Solving the system

\[
\begin{aligned}
\begin{bmatrix}
\frac{\partial^2 U}{\partial C^2} \left( \frac{\partial g_e}{\partial C} dt_i + \frac{\partial g_e}{\partial r_i} dr_i \right) + \frac{\partial^2 U}{\partial E^2} \left( \frac{\partial E}{\partial C} dt_i + \frac{\partial E}{\partial r_i} dr_i \right) = 0 \\
\frac{\partial^2 U}{\partial g_e \partial C} \left( \frac{\partial g_e}{\partial C} dt_i + \frac{\partial g_e}{\partial r_i} dr_i \right) + \frac{\partial^2 U}{\partial g_e \partial r_i} \left( \frac{\partial g_e}{\partial r_i} dt_i + \frac{\partial g_e}{\partial C} dr_i \right) = 0 \\
\frac{\partial^2 U}{\partial E^2} \left( \frac{\partial E}{\partial C} dt_i + \frac{\partial E}{\partial r_i} dr_i \right) + \frac{\partial^2 U}{\partial E^2} \left( \frac{\partial E}{\partial C} dt_i + \frac{\partial E}{\partial r_i} dr_i \right) = 0
\end{bmatrix}
\end{aligned}
\]

using a Cramer rule, we get:

\[
\begin{aligned}
dt_i \overline{dT_k} &= \frac{1}{\det A} \left[ -\frac{\partial^2 U}{\partial C^2} \left( \frac{\partial g_e}{\partial C} dt_i + \frac{\partial g_e}{\partial r_i} dr_i \right)^2 + \frac{\partial^2 U}{\partial g_e \partial C} \left( \frac{\partial g_e}{\partial C} dt_i + \frac{\partial g_e}{\partial r_i} dr_i \right)^2 + \frac{\partial^2 U}{\partial E^2} \left( \frac{\partial E}{\partial C} dt_i + \frac{\partial E}{\partial r_i} dr_i \right)^2 + \frac{\partial^2 U}{\partial g_e \partial E} \left( \frac{\partial g_e}{\partial C} dt_i + \frac{\partial g_e}{\partial r_i} dr_i \right)^2 \right] \\
&+ \frac{1}{\det A} \left[ \frac{\partial^2 U}{\partial g_e \partial C} \left( \frac{\partial g_e}{\partial C} dt_i + \frac{\partial g_e}{\partial r_i} dr_i \right)^2 + \frac{\partial^2 U}{\partial E^2} \left( \frac{\partial E}{\partial C} dt_i + \frac{\partial E}{\partial r_i} dr_i \right)^2 + \frac{\partial^2 U}{\partial g_e \partial E} \left( \frac{\partial g_e}{\partial C} dt_i + \frac{\partial g_e}{\partial r_i} dr_i \right)^2 \right]
\end{aligned}
\]

\[
\begin{aligned}
dt_i \overline{dT_c} &= \frac{1}{\det A} \left[ -\frac{\partial^2 U}{\partial C^2} \left( \frac{\partial g_e}{\partial C} dt_i + \frac{\partial g_e}{\partial r_i} dr_i \right)^2 + \frac{\partial^2 U}{\partial g_e \partial C} \left( \frac{\partial g_e}{\partial C} dt_i + \frac{\partial g_e}{\partial r_i} dr_i \right)^2 + \frac{\partial^2 U}{\partial E^2} \left( \frac{\partial E}{\partial C} dt_i + \frac{\partial E}{\partial r_i} dr_i \right)^2 + \frac{\partial^2 U}{\partial g_e \partial E} \left( \frac{\partial g_e}{\partial C} dt_i + \frac{\partial g_e}{\partial r_i} dr_i \right)^2 \right] \\
&+ \frac{1}{\det A} \left[ \frac{\partial^2 U}{\partial g_e \partial C} \left( \frac{\partial g_e}{\partial C} dt_i + \frac{\partial g_e}{\partial r_i} dr_i \right)^2 + \frac{\partial^2 U}{\partial E^2} \left( \frac{\partial E}{\partial C} dt_i + \frac{\partial E}{\partial r_i} dr_i \right)^2 + \frac{\partial^2 U}{\partial g_e \partial E} \left( \frac{\partial g_e}{\partial C} dt_i + \frac{\partial g_e}{\partial r_i} dr_i \right)^2 \right]
\end{aligned}
\]

\[
\begin{aligned}
dt_i \overline{d\tau_k} &= \frac{1}{\det A} \left[ -\frac{\partial^2 U}{\partial C^2} \left( \frac{\partial g_e}{\partial C} dt_i + \frac{\partial g_e}{\partial r_i} dr_i \right)^2 + \frac{\partial^2 U}{\partial g_e \partial C} \left( \frac{\partial g_e}{\partial C} dt_i + \frac{\partial g_e}{\partial r_i} dr_i \right)^2 + \frac{\partial^2 U}{\partial E^2} \left( \frac{\partial E}{\partial C} dt_i + \frac{\partial E}{\partial r_i} dr_i \right)^2 + \frac{\partial^2 U}{\partial g_e \partial E} \left( \frac{\partial g_e}{\partial C} dt_i + \frac{\partial g_e}{\partial r_i} dr_i \right)^2 \right] \\
&+ \frac{1}{\det A} \left[ \frac{\partial^2 U}{\partial g_e \partial C} \left( \frac{\partial g_e}{\partial C} dt_i + \frac{\partial g_e}{\partial r_i} dr_i \right)^2 + \frac{\partial^2 U}{\partial E^2} \left( \frac{\partial E}{\partial C} dt_i + \frac{\partial E}{\partial r_i} dr_i \right)^2 + \frac{\partial^2 U}{\partial g_e \partial E} \left( \frac{\partial g_e}{\partial C} dt_i + \frac{\partial g_e}{\partial r_i} dr_i \right)^2 \right]
\end{aligned}
\]
The trade-off between the two regional taxes depends on the relative size of the endowments \( \bar{k} \) and \( \bar{\tau} \), and on the comparison between the distortive effects of these taxes on both the regional public good and the environmental quality, as well as on the interdependence between the two taxes.
5 Conclusion

This chapter analyzes the tax interactions among two-tier governments, composed of several regional governments and a federal government, which both share two tax bases, that are capital and energy. The particularity of the energy is that it influences the quality of environment. But as capital and energy are two interdependent production factors, the taxation of capital also indirectly affects the quality of environment. We find that the federal trade-off between the taxation of capital versus the taxation of energy only depends on the relative exogenous supply of energy and capital. The regional trade-off between the taxation of capital versus the taxation of energy is much more complex as it also depends on the comparison between the distortive effects of these taxes on both the regional public good and the environmental quality, as well as on the interdependence between the two taxes. We also find a substitutability among federal and regional taxes, whatever the interdependence between the two taxes, implying that the federal government increases a given tax rate in response to a decrease of one of the two regional tax rates.

Several extensions can be considered to improve our understanding of the architecture of energy fiscal federalism. Firstly, we can make some ad-hoc assumptions to simplify our results instead of general functional forms of the utility function and the production function. We also plan to do a numerical investigation of optimal energy tax policies. Finally, the symmetry assumption could also be relaxed by considering that jurisdictions differ in terms of population.
6 Appendices

6.1 Appendix B1: Profit maximization and market clearing conditions

Profit maximization

Differentiating with respect to \( k_i \) and energy \( e_i \) the first-order conditions of profit maximization

\[
\begin{align*}
&\{ f_{k_i} = r^k_i \\
&f_{e_i} = r^e_i \\
\end{align*}
\]

, which represent the implicit demands for respectively, capital and energy inputs, gives:

\[
\begin{align*}
&\{ f_{kk}dk_i + f_{ke}de_i = dr^k_i \\
&f_{ek}dk_i + f_{ee}de_i = dr^e_i \\
\end{align*}
\]

Using Cramer’s rule gives:

\[
\begin{align*}
dk_i = & \frac{f_{ee}dr^k_i - f_{ke}dr^e_i}{f_{kk}f_{ee} - f_{ek}f_{ke}} \quad (7) \\
de_i = & \frac{f_{kk}dr^e_i - f_{ek}dr^k_i}{f_{kk}f_{ee} - f_{ek}f_{ke}} \quad (8) \\
\end{align*}
\]

Since production factors are perfectly mobile, \( \rho_{k_i} = \rho_k, \rho_{e_i} = \rho_e \forall i. \)

The demand functions of capital and energy depend on all remunerations of production factors \( (\rho_k, \rho_e, t_j, \tau_j) \forall j. \)

Assuming that \( f_{ek} = f_{ke}, \) we derive that:

\[
\begin{align*}
k_{r,k} = & \frac{dk_i}{dr^k_i} = \frac{f_{ee}}{f_{kk}f_{ee} - f_{ek}f_{ke}} \quad (9) \\
\end{align*}
\]

\[
\begin{align*}
e_{r,e} = & \frac{de_i}{dr^e_i} = \frac{f_{kk}}{f_{kk}f_{ee} - f_{ek}f_{ke}} \quad (10) \\
\end{align*}
\]

\[
\begin{align*}
k_{r,e} = & \frac{dk_i}{dr^e_i} = \frac{-f_{ke}}{f_{kk}f_{ee} - f_{ek}f_{ke}} = e_{r,k} \equiv \frac{de_i}{dr^k_i} = \frac{-f_{ek}}{f_{kk}f_{ee} - f_{ek}f_{ke}} \quad (11) \\
\end{align*}
\]
Market clearing conditions

Differentiating with respect to $\rho_k$, $\rho_e$, $t_i$, $\tau_i$, $T_k$ and $T_e$ the following market-clearing conditions

\[ \sum_{i=1}^{n} k(\rho_k + t_i + T_k, \rho_e + \tau_i + T_e) = n\bar{k} \]  
\[ \sum_{i=1}^{n} e(\rho_k + t_i + T_k, \rho_e + \tau_i + T_e) = n\bar{e} \]  

yields, at the symmetric equilibrium:

\[ \sum_{i=1}^{n} k_{ri}^k d\rho_k + \sum_{i=1}^{n} k_{ri}^e d\rho_e + k_{ri}^k dt_i + k_{ri}^e d\tau_i + \sum_{i=1}^{n} k_{ri}^k dT_k + \sum_{i=1}^{n} k_{ri}^e dT_e = 0 \]  
\[ \sum_{i=1}^{n} e_{ri}^k d\rho_k + \sum_{i=1}^{n} e_{ri}^e d\rho_e + e_{ri}^k dt_i + e_{ri}^e d\tau_i + \sum_{i=1}^{n} e_{ri}^k dT_k + \sum_{i=1}^{n} e_{ri}^e dT_e = 0 \]  

By dividing the system formed by equations (14) and (15) by $dt_i$, we obtain at symmetric equilibrium:

\[ \left\{ \begin{array}{l}
\sum_{i=1}^{n} k_{ri}^k \frac{d\rho_k}{dt_i} + \sum_{i=1}^{n} k_{ri}^e \frac{d\rho_e}{dt_i} = -k_{ri}^k \\
\sum_{i=1}^{n} e_{ri}^k \frac{d\rho_k}{dt_i} + \sum_{i=1}^{n} e_{ri}^e \frac{d\rho_e}{dt_i} = -e_{ri}^k
\end{array} \right. \]  

which gives:

\[ \frac{d\rho_k}{dt_i} = \frac{e_{ri}^k \sum k_{ri}^e - k_{ri}^k \sum e_{ri}^e}{\sum k_{ri}^k \sum e_{ri}^e - \sum k_{ri}^e \sum e_{ri}^k} = \frac{n(f_{ek}f_{ke} - f_{kk}f_{ee})}{n^2(f_{kk}f_{ee} - f_{ke}f_{ek})} = -\frac{1}{n} \]  
\[ \frac{d\rho_e}{dt_i} = \frac{k_{ri}^k \sum e_{ri}^e - e_{ri}^k \sum k_{ri}^e}{\sum k_{ri}^k \sum e_{ri}^e - \sum k_{ri}^e \sum e_{ri}^k} = \frac{-n f_{ee}f_{ek} - (-n f_{ee}f_{ek})}{n^2(f_{kk}f_{ee} - f_{ke}f_{ek})} = 0 \]  

By dividing the system formed by equations (14) and (15) by $d\tau_i$, we obtain at symmetric equilibrium:
\begin{align*}
\left\{ \sum_{i=1}^{n} k_{r_i}^i \frac{dp_k}{dT_i} + \sum_{i=1}^{n} k_{r_i}^i \frac{dp_e}{dT_i} = -k_{r_i}^i \right. \\
\left. \sum_{i=1}^{n} e_{r_i}^i \frac{dp_k}{dT_i} + \sum_{i=1}^{n} e_{r_i}^i \frac{dp_e}{dT_i} = -e_{r_i}^i \right. \\
\right.
\end{align*}

which gives:

\begin{equation}
\frac{dp_k}{dT_i} = \frac{e_{r_i}^i \sum k_{r_i}^i - k_{r_i}^i \sum e_{r_i}^i}{\sum k_{r_i}^i \sum e_{r_i}^i - \sum k_{r_i}^i \sum e_{r_i}^i} = \frac{-nf_{kk}f_{ke} - (-nf_{ke}f_{kk})}{n^2(f_{kk}f_{ee} - f_{ke}f_{ek})} = 0
\end{equation}

\begin{equation}
\frac{dp_e}{dT_i} = \frac{k_{r_i}^i \sum e_{r_i}^i - e_{r_i}^i \sum k_{r_i}^i}{\sum k_{r_i}^i \sum e_{r_i}^i - \sum k_{r_i}^i \sum e_{r_i}^i} = \frac{-nf_{ke}f_{ek} - (-nf_{kk}f_{ee})}{n^2(f_{kk}f_{ee} - f_{ke}f_{ek})} = \frac{-1}{n}
\end{equation}

In the same way, we obtain:

\begin{equation}
\frac{dp_k}{dT_k} = \frac{\sum_{i=1}^{n} k_{r_i}^i \sum e_{r_i}^i - \sum_{i=1}^{n} k_{r_i}^i \sum e_{r_i}^i}{\sum k_{r_i}^i \sum e_{r_i}^i - \sum k_{r_i}^i \sum e_{r_i}^i} = \frac{-nf_{ke}(-nf_{ek}) - (-nf_{kk}nf_{ee})}{n^2(f_{kk}f_{ee} - f_{ke}f_{ek})} = -1
\end{equation}

\begin{equation}
\frac{dp_e}{dT_k} = \frac{\sum_{i=1}^{n} k_{r_i}^i \sum e_{r_i}^i - \sum_{i=1}^{n} k_{r_i}^i \sum e_{r_i}^i}{\sum k_{r_i}^i \sum e_{r_i}^i - \sum k_{r_i}^i \sum e_{r_i}^i} = 0
\end{equation}

\begin{equation}
\frac{dp_k}{dT_e} = \frac{\sum_{i=1}^{n} k_{r_i}^i \sum e_{r_i}^i - \sum_{i=1}^{n} k_{r_i}^i \sum e_{r_i}^i}{\sum k_{r_i}^i \sum e_{r_i}^i - \sum k_{r_i}^i \sum e_{r_i}^i} = 0
\end{equation}

\begin{equation}
\frac{dp_e}{dT_e} = \frac{\sum_{i=1}^{n} k_{r_i}^i \sum e_{r_i}^i - \sum_{i=1}^{n} k_{r_i}^i \sum e_{r_i}^i}{\sum k_{r_i}^i \sum e_{r_i}^i - \sum k_{r_i}^i \sum e_{r_i}^i} = -1
\end{equation}

6.2 Appendix B2: Intermediate details of maximization program derived at the symmetric equilibrium

Appendix B1: Impact of taxes on private consumption

\[ \pi_i = f(k_i, e_i) - r_i^k k_i - r_i^e e_i \]  

\[ \frac{d\pi_i}{dr_i} = \pi_i' = f_{k_i} \frac{dk_i}{dr_i} + f_{e_i} \frac{de_i}{dr_i} - k_i - r_i^k \frac{dk_i}{dr_i} - r_i^e \frac{de_i}{dr_i} \]
\[ \pi_{r_i}^k = (f_{k_i}^r - r_{i_k}) \frac{dk_i}{d\tau_i} + (f_{e_i}^r - r_{i_e}) \frac{de_i}{d\tau_i} - k_i \]

\[ \pi_{r_i}^e = -k_i. \]

\[ \pi_{r_i}^e = (f_{k_i}^e - r_{i_k}) \frac{dk_i}{d\tau_i} + (f_{e_i}^e - r_{i_e}) \frac{de_i}{d\tau_i} - e_i \]

\[ \pi_{r_i}^e = -e_i. \]

\[ C_i = \pi_i + \rho_{k_i}k + \rho_{e_i}e \quad (25) \]

\[ \frac{dC_i}{dt_i} = -k_i \quad (26) \]

\[ \frac{dC_i}{d\tau_i} = -e_i(1 + \frac{d\rho_e}{d\tau_i}) + \bar{e} \frac{d\rho_e}{d\tau_i} \]

\[ \frac{dC_i}{d\tau_i} = -\bar{e} \quad (27) \]

\[ \frac{dC_i}{dT_k} = -k_i \quad (28) \]

\[ \frac{dC_i}{dT_e} = -\bar{e} \quad (29) \]

6.3 Appendix B3: Impact of taxes on regional public good

\[ g_i = t_i k_i + \tau_i e_i \quad (30) \]
\[ \frac{dg_i}{dt_i} = k_i + t_i \frac{dk_i}{dr_i} \frac{dr_k}{dr_i} + \tau_i \frac{de_i}{dr_i} \frac{dr_k}{dr_i} = k_i + (t_i.k_i + \tau_i.e_i) \frac{dr_k}{dr_i} \]

\[ \frac{dg_i}{dt_i} = k_i + \frac{n - 1}{n} \frac{1}{f_{kk, fee} - f_{ke, fke}} (t_i.f_{ee} - \tau_i.f_{ek}) \]  

(31)

\[ \frac{dg_i}{dr_i} = e_i + t_i \frac{dk_i}{dr_i} \frac{dr_k}{dr_i} + \tau_i \frac{de_i}{dr_i} \frac{dr_k}{dr_i} = e_i + (t_i.k_i + \tau_i.e_i) \frac{dr_k}{dr_i} \]

(32)

\[ \frac{dg_i}{d\tau_i} = e_i + \frac{n - 1}{n} \frac{1}{f_{kk, fee} - f_{ke, fke}} (-t_i.f_{ke} + \tau_i.f_{kk}) \]

\[ \frac{dg_i}{dT_k} = \frac{dg_i}{dT_e} = 0 \]  

(33)

6.4 Appendix B4: Impact of taxes on federal public good

\[ G = \frac{1}{n} T_k \sum k(r^k_i, r^e_i) + \frac{1}{n} T_e \sum e(r^k_i, r^e_i) \]  

(34)

\[ \frac{dG}{dt_i} = \frac{1}{n} T_k (k_{ri} \frac{dr^k_i}{dr_i} + \sum_{j \neq i} k_{rj} \frac{dr^k_j}{dr_i}) + \frac{1}{n} T_e (e_{ri} \frac{dr^e_i}{dr_i} + \sum_{j \neq i} e_{rj} \frac{dr^e_j}{dr_i}) \]

\[ \frac{dG}{d\tau_i} = \frac{1}{n} T_k \left( \frac{-f_{kk}}{f_{kk, fee} - f_{ke, fke}} \right) \left[ \frac{n-1}{n} + (n-1)(\frac{-1}{n}) \right] + \frac{1}{n} T_e \left( \frac{-f_{kk}}{f_{kk, fee} - f_{ke, fke}} \right) \left[ \frac{n-1}{n} + (n-1)(\frac{-1}{n}) \right] \]

\[ dG \bigg/ d\tau_i = 0 \]  

(35)

\[ \frac{dG}{dT_k} = \frac{1}{n} T_k (k_{ri} \frac{dr^e_i}{dr_i} + \sum_{j \neq i} k_{rj} \frac{dr^e_j}{dr_i}) + \frac{1}{n} T_e (e_{ri} \frac{dr^e_i}{dr_i} + \sum_{j \neq i} e_{rj} \frac{dr^e_j}{dr_i}) \]

\[ \frac{dG}{dT_e} = \frac{1}{n} T_k \left( \frac{-f_{kk}}{f_{kk, fee} - f_{ke, fke}} \right) \left[ \frac{n-1}{n} + (n-1)(\frac{-1}{n}) \right] + \frac{1}{n} T_e \left( \frac{-f_{kk}}{f_{kk, fee} - f_{ke, fke}} \right) \left[ \frac{n-1}{n} + (n-1)(\frac{-1}{n}) \right] \]

\[ \frac{dG}{dT_e} = 0 \]  

(36)

\[ \frac{dG}{dT_k} = \frac{1}{n} \sum k(r^k_i, r^e_i) = k_i \]

(37)

\[ \frac{dG}{dT_e} = \frac{1}{n} \sum e(r^k_i, r^e_i) = e_i \]  

(38)
6.5 Appendix B5: Impact of taxes on environment

\[ E_i = h(e_i) \]

\[
\frac{dE_i}{dt_i} = \frac{dh}{de_i} \frac{de_i}{dr_i} \frac{dr_i}{dt_i} = E_e e_i \frac{n-1}{n} \]

\[
\frac{dE_i}{dt_i} = -E_e \frac{n-1}{n} \frac{f_{ek}}{f_{kk} f_{ee} - f_{ke} f_{ek}} \quad (39)
\]

\[
\frac{dE_i}{dr_i} = E_e e_i \frac{dr_i}{dt_i} \]

\[
\frac{dE_i}{dr_i} = E_e \frac{n-1}{n} \frac{f_{kk}}{f_{kk} f_{ee} - f_{ke} f_{ek}} \quad (40)
\]

\[
\frac{dE_i}{dT_k} = \frac{dE_i}{dT_e} = 0 \quad (41)
\]
References


