Child mortality, fertility, child labor, and economic growth*

Kei Takakura†
February 24, 2019

Abstract.

This paper analyzes how child mortality affects fertility, child labor, and the investment in education in the economy where skilled and unskilled workers coexist, employing a three overlapping generations model. Improvement in child mortality has different effects on skilled workers and unskilled ones. The paper compares taxation policy and a ban on child labor policy, which demonstrates that while taxation policy decreases the proportion of skilled workers in the economy, even though a ban on child labor decreases the welfare of the economy, it helps the economy converge to the high proportion of skilled people and hence that get to high output per capita.

Keywords: Child mortality, Health, Fertility, Education, Child labor, Overlapping generations model

JEL Classification Numbers: I15, I25, J13, J20, O11

---

*I thank all who help me to finish this thesis. I especially give deep thanks to Koichi Futagami and Kohei Okada. They always give me valuable and warm-hearted comments in seminar at Osaka University. Without their comments and instructions, I would not complete this paper. All remaining errors are mine.

†Graduate School of Economics, Osaka University. Address: 1-7 Machikaneyama, Toyonaka, Osaka 560-0043, Japan. E-mail: takakura.kei.0520@gmail.com
1. Introduction

Poor countries have distinctive features that are characterized by high fertility, high mortality, abundant child labor, and low output per capita. On the other hand, rich countries can be expressed as decline in fertility, low mortality, slight child labor, and high output per capita. Chacraborty (2004) shows us that high mortality from adulthood to old age creates persistent poverty. Hazan and Berdugo (2002) state there exists a poverty trap where fertility is high and child labor is abundant. Strulik (2004) elucidates that high child mortality gives rise to high fertility, parents’ choice that they do not invest in education to their children, and abundant child labor. Chacraborty (2004) and Strulik (2004) observe the fact there is a strong link between high mortality of adult and child and low income.

This paper focuses on how child mortality affects fertility, child labor supply, and parents’ investment in education for their each child in the economy where skilled and unskilled workers coexist. The paper firstly shows us increase in child survival probability has different effects on different workers. When child survival probability increases, skilled workers increase the investment in education and it invariably decreases fertility and child labor supply, however, unskilled workers strictly increase child labor supply and have a possibility that they increase fertility. Secondly, if fertility rate of unskilled workers is higher than that of skilled ones, in a long term the economy converges to it where many unskilled workers and less skilled workers exist, which creates low output per capita. Thirdly, we consider economic policies: taxation policy and a ban on child labor. Taxation policy gives light on how the increase of taxation affects different economic agents. The result is that it is likely to decrease the proportion of skilled workers rather than increase. We next consider a ban on child labor according to Strulik (2004). We conclude that even though a ban on child labor decreases the welfare of the economy, it increases the proportion of skilled workers and hence it follows that output per capita can be improved in the economy.

The paper is organized as follows. Section 2 develops the basic model and considers how improvement in child mortality affects different workers’ fertility, child labor supply, and the investment in education. In section 3, we demonstrate differential fertility crucially determines the graph of population dynamics and hence it yields low output per capita economy or high one. Section 4 compares taxation policy and a ban on child labor policy. Section 5 provides several concluding remarks.
2. The model

2.1. Environments

We consider a three-period overlapping generations model: school age, adulthood, and old age. Time is infinite and discrete. Individuals in school age do not make decisions. Individuals in adulthood give birth to children, raise and educate them, and determine a supply of child labor. Individuals in old age retire and only consume goods. An individual dies at the beginning of school age with probability \(1 - \pi^c \in [0, 1]\) and lives through school age with probability \(\pi^c \in (0, 1]\).

We employ a small open economy. Therefore, the interest rate faces given the world interest rate. International capital movements are free and do not generate any cost. Domestic country can buy and sell capital in a perfect competition and so can foreign countries do. On the other hand, labor is immobile. Domestic residents cannot work abroad and foreigners cannot work in the domestic country.

The economy produces a single good and there exist skilled and unskilled workers in this economy. Hence, we regard two sectors on the production side of the economy. Production takes place in both sectors. Physical capital, skilled labor, and unskilled labor are used to produce goods.

Government collects a lump-sum tax on each consumer at the end of adulthood every period. Collected money is called public health expenditure, which is used to improve new medical facilities, sanitation improvements, disease control, and inoculation programs. Public health expenditure augments private health capital and improves child mortality.

2.2. Production

The production function of the unskilled sector is

\[ Y_t^u = w^u L_t^u. \]  

(1)

Let \( Y_t^u, w^u, \) and \( L_t^u \) represent unskilled worker’s final good, the wage, and labor. The production function of the skilled sector is

\[ Y_t^s = F(K_t, L_t^s). \]  

(2)

Let \( Y_t^s, K_t, \) and \( L_t^s \) express skilled worker’s final good, physical capital, and labor. We assume the production function of the skilled sector satisfies all
neoclassical assumptions. We solve the following firm’s profit maximization problem:
\[
\max_{\{K_t, L_t^s\}} F(K_t, L_t^s) - \bar{r}K_t - w_t^sL_t^s,
\]  
(3)
where $\bar{r}$ and $w_t^s$ are given the world interest rate and skilled worker’s wage. Maximizing the objective function (3) by using first order conditions with respect to $K_t$ and $L_t^s$ yields the following solutions:
\[
k_t = f'^{-1}(\bar{r}) \equiv \bar{k},
\]  
(4)
\[
w_t^s = f(\bar{k}) - f'(\bar{k})\bar{k} \equiv w^s,
\]  
(5)
where we define $k_t \equiv \frac{K_t}{L_t^s}$. Here, we notice $k_t$ and $w_t^s$ are constant from now on. We assume $w^s > w^u$.

2.3. Household behavior

Since there exist skilled workers and unskilled workers in the economy, we need to solve different household's utility maximization problems separately. Individuals derive utility from surviving children, their children’s leisure, their investments in education for their each child, their own consumption in adulthood, and their future consumption in old age. The utility function of an individual $i \in \{s, u\}$ who belongs to generation $t$ is
\[
u_t^i = \gamma \{\log(\pi_t^i n_t^i) + \phi \log(1 - \ell_t^i) + \beta \log(e_t^i)\}
+ (1 - \gamma)\{\log(c_t^i) + \delta \log(c_{t+1}^i)\}, \quad \gamma, \phi, \beta, \delta \in [0, 1), d \in (0, 1),
\]  
(6)
where a parameter $\gamma$ measures the extent to which an individual $i$ values their value to their own lifetime consumption. $\phi$ and $\beta$ are parameters, which express a preference for leisure and investments of children that their parents have compared to the preference for deriving their utility from surviving children. The utility function in our model implies that having more surviving children is more important than leisure and investments of children. $\delta$ is the rate of discount that they give priority to present consumption more than future consumption. We follow Hazan (2002) and children can work only in the unskilled sector due to inferior their bodies and capabilities. $d$ indicates children can earn a small income compared to unskilled workers.

Individuals and children have one unit of time. After being raised and educated by their parents in school age, individuals in adulthood educate their
children and determine the quantity of supply of child labor while working. Parents collect an earned income by their children. They also make a payment to government for a lump-sum tax \( \tau \). Here, all individual’s income is called a family income, which is described as 

\[
(1 - z n_i^t - e_i^t \pi_i^t n_i^t)w^s + d\ell_{c,t}^i \pi_i^c n_i^t w^u - \tau.
\]

Individuals also divide their family income into present consumption and saving. Hence, the budget and time constraint of individual \( i \) at time \( t \) is expressed as follows:

\[
\begin{align*}
    c_i^t &= (1 - z n_i^t - e_i^t \pi_i^c n_i^t)w^i + d\ell_{c,t}^i \pi_i^c n_i^t w^u - \tau - s_t, \\
    c_{i+1}^t &= (1 + \bar{r})s_t, \\
    e_i^s > e_i^u = \bar{e} > 0.
\end{align*}
\]

where \( z \) is the fixed amount time to bear and raise a child. We follow Fan and Zhang (2013) and assume unskilled parental investments in education has a lower bound \( e_i^s > \bar{e} > 0 \). In accordance with this reason, while skilled individuals can choose \( e_i^s > \bar{e} \), unskilled ones always choose \( \bar{e} \). Maximizing (6) subject to (7) and (8) for skilled workers yields the following optimal solutions:

\[
\begin{align*}
    s_i^s &= \frac{\delta(1 - \delta)(w^s - \tau)}{(1 - \gamma)(1 + \delta) + \gamma} \equiv s^s, \\
    n_i^s &= \frac{\gamma(\phi + \beta - 1)(w^s - \tau)}{(1 - \gamma)(1 + \delta) + \gamma (d\pi_i^c w^u - z w^s)}, \\
    \ell_{c,t}^e &= 1 - \frac{\phi (d\pi_i^c w^u - z w^s)}{(\phi + \beta - 1)d\pi_i^c w^u}, \\
    e_i^s &= \frac{\beta (d\pi_i^c w^u - z w^s)}{(\phi + \beta - 1)d\pi_i^c w^u}, \\
    c_i^s &= \frac{(1 - \gamma)(w^s - \tau)}{(1 - \gamma)(1 + \delta) + \gamma} \equiv c^s,
\end{align*}
\]
Maximizing (6) subject to (7) to (9) for unskilled workers yields the following optimal solutions:

\[
\begin{align*}
    s_t^u &= \frac{\delta(1 - \gamma)(w^u - \tau)}{(1 - \gamma)(1 + \delta) + \gamma} \equiv s^u, \quad (16) \\
    n_t^u &= \frac{\gamma(1 - \phi)(w^u - \tau)}{\{(1 - \gamma)(1 + \delta) + \gamma\} \{z - (d - \bar{e})\pi_t^u\} w^u}, \quad (17) \\
    f_{c,t}^u &= 1 - \frac{\phi\{z - (d - \bar{e})\pi_t^u\}}{(1 - \phi)d\pi_t^u}, \quad (18) \\
    c_t^u &= \frac{(1 - \gamma)(w^u - \tau)}{(1 - \gamma)(1 + \delta) + \gamma} \equiv c^u. \quad (19)
\end{align*}
\]

Since all variables are required to be non-negative, we obtain the following conditions:

\[
\begin{align*}
    d\pi_t^u w^u - z w^s &> 0, \quad (20) \\
    z - (d - \bar{e})\pi_t^u &> 0, \quad (21) \\
    w^s &> \tau, \quad (22) \\
    w^u &> \tau, \quad (23) \\
    \phi + \beta &> 1. \quad (24)
\end{align*}
\]

It is because our paper would focus on the poor countries rather than other countries. We assume both of skilled and unskilled workers depend on child labor income to a family income in a certain extent and unskilled workers’ investments in education is sufficiently low. In other words, the following inequality holds.

**Assumption 1.** We assume

\[
d > \bar{e}. \quad (25)
\]

If inequality (24) holds, we see child survival probability increase has a positive effect on fertility of unskilled workers. Then, we acquire the following proposition.

**Proposition 1.** Improvement in child mortality has different effects on different workers. It encourages skilled workers to decrease fertility, child labor supply, and to invest in education. However, it urges unskilled workers to increase fertility and to enlarge child labor supply.
Proof. We take the partial derivative of (11), (12), (13), (16), and (17) with respect to $\pi_{c,t}$ respectively:

\[
\frac{\partial n_{c,t}^s}{\partial \pi_{c,t}^s} = -\frac{\gamma(\phi + \beta - 1)(w^s - \tau)d\bar{w}}{(1 - \gamma)(1 + \delta) + \gamma}(d\pi_{c,t}^s w^u - zw^s)^2 < 0, \quad (26)
\]
\[
\frac{\partial \ell_{c,t}^s}{\partial \pi_{c,t}^s} = -\frac{\phi z w^s}{(\phi + \beta - 1)d\bar{w} w(\pi_{c,t})^2} < 0, \quad (27)
\]
\[
\frac{\partial e_{c,t}^s}{\partial \pi_{c,t}^s} = \frac{\beta z}{(\phi + \beta - 1)(\pi_{c,t})^2} > 0, \quad (28)
\]
\[
\frac{\partial n_{u,t}^s}{\partial \pi_{c,t}^s} = \frac{\gamma(\phi + \beta - 1)(1 - \phi)(d - \bar{e})}{(1 - \gamma)(1 + \delta) + \gamma}(1 - \gamma + \gamma w^u)w(z - (d - \bar{e})\pi_{c,t})^2 > 0, \quad (29)
\]
\[
\frac{\partial \ell_{c,t}^u}{\partial \pi_{c,t}^s} = \frac{\phi z}{(1 - \phi)d(\pi_{c,t})^2} > 0. \quad (30)
\]

Therefore, Proposition 1 is proved. \qed

3. Equilibrium

Total health expenditure is expressed as,

\[ G_t = \tau L_t, \]

where $L_t$ denotes total workers in the economy.

Let us define $g_t \equiv \frac{G_t}{L_t}$ as private health capital:

\[ g_t = \tau \equiv g. \quad (31) \]

We follow Kalemli-Ozcan (2002) and Chakraborty(2004), and assume child survival probability is the monotonically increasing function of private health capital. In other words, child survival probability satisfies the following properties:

\[
\frac{\partial}{\partial g_t} \{ \pi(g_t) \} > 0, \quad \frac{\partial^2}{\partial g_t^2} \{ \pi(g_t) \} < 0, \quad \lim_{g_t \to 0} \pi(g_t) = 0, \quad \lim_{g_t \to \infty} \pi(g_t) \leq 1, \quad (32)
\]
\[
\pi_{c,t}^c = \pi^c(g_t) = \pi^c(\tau) \equiv \pi^c.
\]
Substituting (31) into (11), (12), (13), (16), and (17) yields

\[ n^s_t = \frac{\gamma(\phi + \beta - 1)(w^s - \tau)}{(1 - \gamma)(1 + \delta) + \gamma} \{d \pi^c w^u - zw^s\} \equiv n^s, \]  

(33)

\[ \ell^s_{c,t} = 1 - \frac{\phi(d \pi^c w^u - zw^s)}{(\phi + \beta - 1)d \pi^c w^u} \equiv \ell^s_c, \]  

(34)

\[ e^s_t = \frac{\beta(d \pi^c w^u - zw^s)}{(\phi + \beta - 1)\pi^c w^s} \equiv e^s, \]  

(35)

\[ n^u_t = \frac{\gamma(1 - \phi)(w^u - \tau)}{(1 - \gamma)(1 + \delta) + \gamma} \{z - (d - \bar{e})\pi^c\} w^u \equiv n^u, \]  

(36)

\[ \ell^u_{c,t} = 1 - \frac{\phi(z - (d - \bar{e})\pi^c)}{(1 - \phi)d \pi^c} \equiv \ell^u_c. \]  

(37)

According to Fan and Chang (2013), the dynamics in this model is described as,

\[ \lambda_{t+1} = \frac{p(e^s_t)\pi^s_t n^s_t \lambda_t L_t + q(e^u_t)\pi^u_t n^u_t (1 - \lambda_t) L_t}{\pi^s_t n^s_t \lambda_t L_t + \pi^u_t n^u_t (1 - \lambda_t) L_t}, \]  

(38)

where \( \lambda_t, p = p(e^s_t) \in (0, 1), q = q(e^u_t) \in (0, 1) \) are the proportion of skilled workers, the probability children whose skilled parents become skilled workers at time \( t + 1 \), and the probability children whose unskilled parents become skilled workers at time \( t + 1 \) respectively. Therefore, \( p(e^s_t)\pi^s_t n^s_t \lambda_t L_t + q(e^u_t)\pi^u_t n^u_t (1 - \lambda_t) L_t \) and \( \pi^s_t n^s_t \lambda_t L_t + \pi^u_t n^u_t (1 - \lambda_t) L_t \) represent the total number of skilled workers at time \( t + 1 \) and the total number of working population at time \( t + 1 \). Thus, substituting (9), (32), (34), and (35) into (37), we can rewrite the dynamics in our model as follows:

\[ \lambda_{t+1} = \frac{p(e^s)n^s \lambda_t + q(e^u)n^u (1 - \lambda_t)}{n^s \lambda_t + n^u (1 - \lambda_t)}. \]  

(39)

We consider whether the dynamics (39) has at least one stable steady state. Before we check out stability, according to Fan and Chang (2012) we mainly focus on the economy where \( n^u > n^s \) and \( p(e^s) > q(e^u) \) hold. We discuss other regions later.

\textbf{Assumption 2.} We assume

\[ n^u > n^s, \]  

(40)

\[ p(e^s) > q(e^u). \]  

(41)
\[
\frac{\partial}{\partial e^s} \{p(e^s)\} > 0, \quad \frac{\partial^2}{\partial (e^s)^2} \{p(e^s)\} < 0, \quad \lim_{e^s \to 0} p(e^s) = 0, \quad \lim_{e^s \to \infty} p(e^s) \leq 1,
\]
\[
\frac{\partial}{\partial e^u} \{q(e^u)\} > 0, \quad \frac{\partial^2}{\partial (e^u)^2} \{q(e^u)\} < 0, \quad \lim_{e^u \to 0} q(e^u) = 0, \quad \lim_{e^u \to \infty} q(e^u) \leq 1.
\]

(42)

We need to confirm the existence of the region such that \( n^u > n^s \). Then, the following lemma is necessary.

**Lemma 1.** There exists such sphere where fertility rate of unskilled workers is greater than that of skilled workers.

**Proof.** By using (32) and (35), we have

\[
n^u - n^s = \frac{\gamma (1 - \phi) (w^u - \tau)}{(1 - \gamma)(1 + \delta) + \gamma \{z - (d - \bar{e} \pi e)w^u\}} - \frac{\gamma (\phi + \beta - 1) (w^s - \tau)}{(1 - \gamma)(1 + \delta) + \gamma \{(d \pi e w^u - z w^s)\}}.
\]

If the following condition holds:

\[
(1 - \phi)(w^s - w^u) z \tau + (1 - \phi)(w^s - \tau) \pi e w^u > (1 - \phi)(w^s - w^u) d \pi e w^u + (w^s - \tau) \{z - (d - \bar{e})\} \beta w^u,
\]

inequality \( n^s < n^u \) holds. Therefore, Lemma 1 is proved.

(43)

If (39) and (40) hold, we get the following proposition.

**Proposition 2.** We suppose \( p > q \) and \( n^u > n^s \) hold. If \( p, q, n^s, \) and \( n^u \) are independent of \( \lambda_t \), then equation (39) has a stable steady state that satisfies \( q < \lambda^L < p \).

**Proof.** We take the first and second order derivatives of (38),

\[
\frac{\partial \lambda_{t+1}}{\partial \lambda_t} = \frac{(p - q)n^u n^s}{\{n^s \lambda_t + n^u(1 - \lambda_t)\}^2} > 0,
\]

(44)

\[
\frac{\partial^2 \lambda_{t+1}}{\partial \lambda_t^2} = \frac{2n^u n^s(p - q)(n^u - n^s)}{\{n^s \lambda_t + n^u(1 - \lambda_t)\}^3} > 0.
\]

(45)

We see when \( \lambda_t = 0 \), then \( \lambda_{t+1} = q \), when \( \lambda_t = 1 \), then \( \lambda_{t+1} = p \). Hence, since the slope \( \frac{\partial \lambda_{t+1}}{\partial \lambda_t} \) is smaller than 1 at the steady state, equation (38) has a stable steady state that satisfies \( q < \lambda^L < p \). Therefore, Proposition 2 is proved.
Lemma 2. The steady state that satisfies $q < \lambda^L < p$ is given by,

$$\lambda^L = \frac{(1 + q)n^u - pn^s - (\Delta)^{\frac{1}{2}}}{2(n^u - n^s)},$$

(46)

where $\Delta = [(1 + q)n^u - pn^s]^2 - 4q(n^u - n^s)n^u > 0$.

Proof. We consider the steady state in equation (53):

$$(n^u - n^s)(\lambda^L)^2 + \{- (1 + q)n^u + pn^s\} \lambda^L + qn^u = 0.$$  (47)

The quadratic formula gives us

$$\lambda^L = \frac{(1 + q)n^u - pn^s \pm (\Delta)^{\frac{1}{2}}}{2(n^u - n^s)}.$$

We rewrite $\Delta$ as

$$\Delta = \{p(n^u - n^s) - (1 - p + q)n^u\}^2 + 4(1 - p)(p - q)n^u(n^u - n^s).$$
Hence, we obtain
\[ L = \frac{(1 + q)n^u - pn^s + (\triangle)^{\frac{1}{2}}}{2(n^u - n^s)} \]
\[ > \frac{p(n^u - n^s) + (1 - p + q)n^u + \{p(n^u - n^s) - (1 - p + q)n^u\}}{2(n^u - n^s)} \]
\[ = p. \]

We only consider the steady state that satisfies \( q < \lambda^L < p \). Therefore, Lemma 2 is proved.

\[ \text{Lemma 3. In the steady state, we have the following inequalities:} \]
\[ \frac{\partial \lambda^L}{\partial n^s} > 0, \quad \frac{\partial \lambda^L}{\partial n^u} < 0, \quad \frac{\partial \lambda^L}{\partial p} > 0, \quad \frac{\partial \lambda^L}{\partial q} > 0. \] (48)

\[ \text{Proof. We take the partial derivative of equation (46) with respect to } n^s, n^u, p, \text{ and } q \text{ respectively, then we acquire,} \]
\[ \frac{\partial \lambda^L}{\partial n^s} = \frac{(p - \lambda^L)\lambda^L}{(\triangle)^{\frac{1}{2}}} > 0, \]
\[ \frac{\partial \lambda^L}{\partial n^u} = \frac{(1 - \lambda^L)(q - \lambda^L)}{\lambda^L} < 0, \]
\[ \frac{\partial \lambda^L}{\partial p} = \frac{n^s\lambda^L}{(\triangle)^{\frac{1}{2}}} > 0, \]
\[ \frac{\partial \lambda^L}{\partial q} = \frac{n^u(1 - \lambda^L)}{\lambda^L} > 0. \]

Therefore, Lemma 3 is proved.

On the other hand, if \( n^s > n^u \) holds, it changes the slope of (38). We see
\[ \frac{\partial^2 \lambda_{t+1}}{\partial \lambda_t^2} = -\frac{2n^u n^s (p - q)(n^s - n^u)}{\{n^s\lambda_t + n^u(1 - \lambda_t)\}^3} < 0. \] (49)

Then, the steady state is as follows:
\[ \lambda^H = \frac{-(1 + q)n^u + pn^s + (\square)^{\frac{1}{2}}}{2(n^s - n^u)}, \] (50)

where \( \square = \{(1 + q)n^u - pn^s\}^2 + 4q(n^s - n^u)n^u \). Since \( \lambda^H \) must be smaller than 1, we need the following assumption.
Assumption 3.

\[(q - 1)n^u + (2 - p)n^s > (□)^{\frac{1}{2}}.\]  \hspace{1cm} (51)

By using the same calculation process, we observe \(p < \lambda^H\) and the same result of Lemma 3. Therefore, whether fertility rate of unskilled workers is greater than that of skilled ones affects the dynamics crucially. One creates low output per capita economy, the other does high output per capita economy.

![Figure 2: The transition process of \(\lambda_t\) where \(n^s > n^u\) holds](image)

4. Policy implications

4.1. Effect of taxation

We firstly confirm how a lump-sum tax increase affects fertility, child labor supply, and the investment in education.

Lemma 4. We have the following inequalities when government increases a lump-sum tax,

\[
\frac{\partial s^u}{\partial \tau} < 0, \quad \frac{\partial n^u}{\partial \tau} < 0, \quad \frac{\partial \ell_c^u}{\partial \tau} < 0, \quad \frac{\partial e^u}{\partial \tau} > 0, \quad \frac{\partial s^u}{\partial \tau} < 0, \quad \frac{\partial n^u}{\partial \tau} > 0, \quad \frac{\partial \ell_c^u}{\partial \tau} > 0.\]  \hspace{1cm} (52)
Proof. We take the partial derivative of (20), (48), (49), (50), (37), (51), and (52) with respect to \( \tau \) respectively. Then, we get,

\[
\frac{\partial s^s}{\partial \tau} = -\frac{\delta(1-\delta)}{(1-\gamma)(1+\delta) + \gamma} < 0,
\]

\[
\frac{\partial n^s}{\partial \tau} = -\frac{\gamma(\phi + \beta - 1)(d\pi^c w^u - z w^s) + (w^s - \tau) dw^u (\pi^c)' }{(1-\gamma)(1+\delta) + \gamma} \frac{(d\pi^c w^u - z w^s)^2}{(d\pi^c w^u - z w^s)^2} < 0,
\]

\[
\frac{\partial f^s_c}{\partial \tau} = \frac{\phi z w^s(\pi^c)'}{(\phi + \beta - 1) dw^u (\pi^c)^2} < 0,
\]

\[
\frac{\partial e^s}{\partial \tau} = \frac{\beta z w^s(\pi^c)'}{(\phi + \beta - 1) w^s(\pi^c)^2} > 0,
\]

\[
\frac{\partial s^u}{\partial \tau} = -\frac{\delta(1-\delta)}{(1-\gamma)(1+\delta) + \gamma} < 0,
\]

\[
\frac{\partial n^u}{\partial \tau} = \frac{\gamma(1-\phi)(w^u - \tau)(d-\bar{e})(\pi^c)' - \{ z - (d-\bar{e}) \}}{(1-\gamma)(1+\delta) + \gamma} \frac{w^u}{\{ z - (d-\bar{e})\pi^c \}^2} \leq 0,
\]

\[
\frac{\partial f^u_c}{\partial \tau} = \frac{\phi z (\pi^c)'}{(1-\phi)d(\pi^c)^2} > 0.
\]

Therefore, Lemma 4 is proved. \( \square \)

We consider how child mortality increase and a lump-sum tax one affect the proportion of skilled workers. We need the following assumptions to determine the sign of the unskilled workers.

**Assumption 4.** We assume,

\[
(w^u - \tau)(\pi^c)' > z - (d - \bar{e}) \tag{53}
\]

Then we obtain the following lemma.

**Lemma 5.** Child mortality increase has a negative effect on the proportion of skilled workers and so does a lump-sum tax one if (24) and (52) hold.

**Proof.** We take the partial derivative of (62) with respect to \( \pi^c \) substituting (48) and (51), then we have

\[
\frac{\partial \lambda}{\partial \pi^c} = \frac{\{(\lambda - 1) \frac{\partial n^u}{\partial \pi^c} + (p - \lambda) \frac{\partial n^s}{\partial \pi^c}\} \lambda + q(1-\lambda) \frac{\partial n^u}{\partial \pi^c}}{\Delta^{1/2}} < 0. \tag{54}
\]
Likewise, we obtain

\[
\frac{\partial \lambda}{\partial \tau} = \{(\lambda - 1)\frac{\partial n^u}{\partial \tau} + (p - \lambda)\frac{\partial n^s}{\partial \tau}\}\lambda + q(1 - \lambda)\frac{\partial n^u}{\partial \tau} < 0. \tag{55}
\]

Therefore, Lemma 5 is proved.

\[\square\]

4.2. Ban on child labor

We secondly investigate how a ban on child labor affects different workers according to Strulik (2004). When government imposes on the abolition of child labor \(c^s = 0\), we maximize (6) subject to (7) to (9) for skilled and unskilled workers again. We acquire the following optimal solutions with a subscript \(g\):

\[
s^s_{g,t} = \frac{\delta(1 - \gamma)(w^s - \tau)}{(1 + \delta)(1 - \gamma) + \gamma} \equiv s^s_g, \tag{56}
\]

\[
n^s_{g,t} = \frac{\gamma(1 - \beta)(w^s - \tau)}{\{(1 + \delta)(1 - \gamma) + \gamma\}zw^s} \equiv n^s_g, \tag{57}
\]

\[
e^s_{g,t} = \frac{\beta z}{(1 - \beta)\pi^c} \equiv e^s_g, \tag{58}
\]

\[
c^s_{g,t} = \frac{(1 - \gamma)(w^s - \tau)}{(1 + \delta)(1 - \gamma) + \gamma} \equiv c^s_g, \tag{59}
\]

\[
s^u_{g,t} = \frac{\delta(1 - \gamma)(w^u - \tau)}{(1 + \delta)(1 - \gamma) + \gamma} \equiv s^u_g, \tag{60}
\]

\[
n^u_{g,t} = \frac{\gamma(1 - \beta)(w^u - \tau)}{\{(1 + \delta)(1 - \gamma) + \gamma\}(z + \bar{e}\pi^c)w^u} \equiv n^u_g, \tag{61}
\]

\[
c^u_{g,t} = \frac{(1 - \gamma + \beta\gamma)(w^u - \tau)}{(1 + \delta)(1 - \gamma) + \gamma} \equiv c^u_g \tag{62}
\]

**Lemma 6.** When government imposes on a ban on child labor, child survival probability increase decreases fertility of unskilled workers and it also does the investment in education for skilled workers.

**Proof.** We take the partial derivative of (57) and (60) with respect to \(\pi^c\).
respectively. Then, we obtain,
\[
\frac{\partial e^s_{g,1}}{\partial \pi^c} = -\frac{\beta z}{(1 - \beta)(\pi^c)^2} < 0, \tag{63}
\]
\[
\frac{\partial n^u_{g,1}}{\partial \pi^c} = -\frac{\bar{e}}{(z + \bar{e}\pi^c)^2} < 0. \tag{64}
\]
Therefore, Lemma 6 is proved.

When government imposes on a ban on child labor, the population dynamics coincides on (38). Then, we need the following lemma to analyze population dynamics.

**Lemma 7.** A ban on child labor overturns fertility relationship between skilled and unskilled workers when inequality (39) holds. In other words, we have the following inequality:
\[
n^u_g < n^s_g. \tag{65}
\]

**Proof.** By using equation (56) and (60), we see
\[
n^u_g - n^s_g = \frac{\gamma(1 - \beta)(w^u - \tau)}{(1 + \delta)(1 - \gamma) + \gamma(z + \bar{e}\pi^c)w^u} - \frac{\gamma(1 - \beta)(w^s - \tau)}{(1 + \delta)(1 - \gamma) + \gamma(z + \bar{e}\pi^c)w^s} \approx \frac{(w^u - w^s)z\tau - (w^s - \tau)\bar{e}\pi^cw^u}{1 - (1 + \delta)(1 - \gamma) + \gamma(z + \bar{e}\pi^c)w^u} < 0.
\]
Therefore, Lemma 7 is proved.

Since inequality (64) yields \(\frac{\partial^2 \lambda_{g,t+1}}{\partial \lambda_{s,t}^2} < 0\), the steady state \(\lambda_g\) is quite similar to \(\lambda^H\). However, when government imposes on a ban on child labor, it obviously decreases the welfare of the economy. Hence, only if fertility rate of unskilled workers is greater than that of skilled ones, a ban on child labor is effective to convert low output per capita economy into high one. We compare taxation policy to a ban on child labor policy and we conclude the following proposition.

**Proposition 3.** We consider the economy where fertility of unskilled workers is greater than that of skilled ones and improvement in child mortality has a positive effect on fertility of unskilled workers. Then, taxation policy
decreases the proportion of skilled workers. On the other hand, even though a ban on child labor decreases the welfare of the economy, however, it over-turns fertility relationship between skilled and unskilled workers and hence it converts low output per capita economy to high output per capita economy.

Proof. According to Proposition 1, Proposition 2, Lemma 5, and Lemma 7, Proposition 3 follows.

5. Concluding remarks

We considered a three-period overlapping generations model to analyze how child mortality affects fertility, child labor, and the investment in education in the economy where skilled and unskilled workers coexist. In our model, improvement in child mortality had different effects on different economic agents. It increased child labor supply of unskilled workers and increased fertility of them. Then, improvement in child mortality by taxation decreased the proportion of skilled workers. On the other hand, a ban on child labor decreased the welfare of the economy, however, it helped the economy converge to the high proportion of skilled people and hence get to high output per capita compared to the situation where government did not enforce a ban on child labor.

References


