Fertility decisions of families in an intergenerational exchange model

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Abstract
We analyze fertility decisions of families in an intergenerational exchange model that might be applied to less-developed economies. Parents who have children for elderly support increase the number of children when the probability of becoming dependent rises. Increases in the children’s wage rates decrease the fertility rate when the elasticity of marginal utility of family care is low. The latter implies that the fertility rate declines with (especially, unexpected) rises in wages along with economic development. Possible fertility declines do not derive from the quantity-quality tradeoff of children but come from decreases in children’s needs for bequests with wage rises.

Keywords: bequests, elderly care, fertility, intergenerational exchange
JEL Classification: D13, D64, D74, J13, J14

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1. Introduction

Fertility rates have declined not only in economically developed but also even in economically developing areas of the world during recent decades. However, different reasons might exist for fertility declines between such developed and developing areas. In economically developed areas, the opportunity costs of child-rearing have been increasing because the wage rate, especially the female wage rate, rises as the economy develops. Higher opportunity costs induce parents to reduce child-rearing time and therefore rear fewer children. By contrast, in developing areas, and especially those without unfunded social security system, children might be regarded as parents’ “investment” for their own old age. Kagitcibasi (1982) reports that in Asian countries such as Indonesia, the Philippines, and Thailand, the proportions of parents who consider old age security as a reason for having children were about 80% and higher (see also Leroux and Pestieau, 2014). In contrast, the respective proportions in the U.S. and Germany were only about 8%. Horioka et al. (2018) report that the proportion of such parents is higher than 20% even in Japan (see also Bernheim et al., 1985). Therefore, the intergenerational exchange model might be applicable to developing economies, especially in Asia. We analyze the fertility decisions of families in such an exchange model.

In a seminal paper, Becker and Tomes (1976) formalize a model in which children are “consumption goods” with a tradeoff between the quality and quantity of children. Most works in the literature of population economics have followed this strand of research. By contrast, few reports describe studies that have examined family fertility decisions in intergenerational exchange models, although an exchange model has been developed theoretically by Bernheim et al. (1985) and more recently, for example, by Chang and Weisman (2005).

Although fertility decisions are analyzed in an intergenerational exchange model, we do not assume intergenerational altruism of parents and children. As described in this paper, parents’ strategic transfers and children’s transfer-seeking competition are modeled using a contest success function (CSF) à la Chang and Weisman (2005). Presuming that these competitions between siblings and between parents and the children take place after children grow up, we analyze the fertility decisions of parents in this paper. Our main concern is intergenerational exchange. Therefore, we assume that a couple of a woman and a man behaves unitarily as if they were a single economic

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1 Total fertility rates have declined since about 1980, even in low-income countries (UN, 2017). In some high income economies, however, fertility rebounds were observed recently (Myrskylä et al., 2009).
unit. For expositional simplicity, we call a couple “a parent” and a pair of a girl and a boy “a child” in this paper. The main conclusion is that increases in the probability of parents’ becoming dependent increase the fertility rate. However, the result also predicts that rises in the children’s wage rates decrease the fertility rate if the marginal utility of family elderly care declines moderately. The latter implies that the fertility rate declines along with economic growth, even in less-developed areas, if the parents desire family attention and care during old years persistently.

The next section introduces a model of the game and analysis of a subgame perfect equilibrium. Section 3 presents analysis of the fertility decisions of families. Sections 2 and 3 assume away market long-term care insurance, another old age security. To clarify the role of intergenerational exchanges in fertility decisions Section 4 considers the availability of private long-term care insurance. The last section concludes the paper.

2. Model

The benchmark problem can be formalized in the following way. It might be considered plausible that parents play a game with their children after they grow up and that grown-up children compete for transfers from the parent. Therefore, in stage 0, parents decide how many children they have. Then, a two-stage game between elderly parents and adult children will take place: Parents choose the allocation of income between their own consumption and bequests to their children in stage 1. Parents promise their children to transfer bequests if they are cared for by the children when becoming dependent. Then, with the promised bequest, children decide how much attention and care they provide when their parents become dependent in old age, competing for parental transfers among siblings in stage 2. Only after the provision of attention and care by children are revealed, the bequests are actually transferred. Transfers of bequests can be a threat as assuring children’s attention and care for parents. However, even though parents do not become independent, children will

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2 We can instead consider women and men explicitly. In that case, spouses have the same probability of becoming dependent. The number of children must be considered as the sum of the numbers of girls and boys. Issues of marriage are assumed away. However, with specific examination of collective decisions between parents and their children in each family, we assume a parent in a family.

3 This is analogous to a convention adopted in Basu and Van (1998). This assumption enables us to avoid conflict with the recent empirical rejection of a unitary model. We also assume that economic behaviors of a girl and a boy are identical.

4 If a parental couple has only one child, then transferring a bequest cannot be a credible
receive the bequests.

The subgame perfect equilibrium game of the last two stages is solvable through backward induction in this section. Decision-making in stage 0 will be analyzed in the next section.

2.1 Attention and care provision of children in stage 2

In this stage, we first assume that a parent bequeaths transfers to \( n(>1) \) children according to a contest success function (CSF). Regarding fertility decisions, we also assume that children are identical. The expected after-transfer income of child \( i \), \( y_i \), can be written as

\[
y_i = \pi(1-a_i)w_i + (1-\pi)w_i + p_iB,
\]

where \( \pi \) denotes the probability that parents become dependent, \( a_i \) stands for her attention and care toward the parent, \( w_i \) represents the wage rate, and \( B \) signifies the bequest from the parent. \( p_i = a_i / \sum_h a_h \) represents the contest success function.

The third term \( p_iB \) on the right-hand side of (1) denotes the transfer which the child receives from the parent. The time endowment of children is assumed to be one. If the parent becomes dependent in old age, then the child’s labor supply is \( 1-a_i \) because she provides attention and care time to her parent. If the parent is autonomous, then the child does not provide attention and care. In these analyses, children receive bequests from parents even when parents are autonomous.

For a given bequest \( B \), the child chooses the amount of attention and care to maximize the after-transfer income (1). The first-order condition is

\[
\frac{dy_i}{da_i} = -\pi w_i + \frac{dp_i}{da_i} B = 0 .
\]

threat to the child. The child knows that the parent, if being dependent, must need the child’s attention and care in this model. Therefore, we assume that more than two children compete for transfers from the parent by promising to provide attention and care to them. Bernheim et al. (1985) provide a similar argument. However, because of wage income, it might be only an enticement in this paper.
Therein, we have \( dp_i / da_i = \frac{[(\sum^n h a_h) - a_i]}{(\sum^n h a_h)^2} \). From (2) we obtain
\[
a_i = (\sum^n h a_h) - \frac{\pi w_i (\sum^n h a_h)^2}{B}. \tag{2'}
\]
The amount of attention and care of each child depends on the received bequest, the parent’s probability of becoming dependent, the wage rate, and the siblings’ amounts of attention and care.

The Nash equilibrium of transfer-seeking competition among children is obtainable from (2') as
\[
a_i = \frac{(n-1)B (\sum^n h w_h) - (n-1)w_i}{\pi (\sum^n h w_h)^2}. \tag{3}
\]
Assuming that children are identical, one can obtain attention and care per child as
\[
a = \frac{(n-1)B}{\pi wn^2} = a(B, n; \pi, w). \tag{4}
\]
From (4) we have the following inequalities:
\[
\frac{\partial a}{\partial B} > 0, \quad \frac{\partial a}{\partial \pi} < 0, \quad \frac{\partial a}{\partial w} < 0, \quad \text{and} \quad \frac{\partial a}{\partial n} = \frac{B(2-n)}{\pi wn^2} > 0 \quad \text{as} \quad 2 = n. \quad \tag{5}
\]
The population growth rate is plausibly less than 100%. Therefore, we can readily assume that \( n < 2 \). Therefore, we have \( \partial a / \partial n > 0 \) from (6). More attention and care will be provided with a greater bequest received. Children will at least partly offset the decrease in the expected after-transfer income by reducing attention and care because the increased probability reduces the expected after-transfer income. Children will reduce attention and care when the wage rate rises because the need for bequest, i.e., the relative weight of bequest in post-transfer income, becomes small. Also, an increase in the number of siblings might enable each child to reduce attention and care per child without reducing the total attention and care to the parent. The increased number of

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5 Although \( n \) represents the number of children, it is natural to consider that \( n = 2 \) means 4 children, 2 girls and 2 boys, when a couple consists of a woman and a man as in this paper. That is, \( n \geq 2 \) means that the total fertility rate is higher than 4. It is apparently too high to be plausible because, except for low-income countries, total fertility rates in the world have been below 4 after 1995-2000 (UN, 2017). Therefore, condition \( n < 2 \) seems plausible, although it is not necessarily so.
siblings itself decreases a per-sibling receipt of bequests, giving each child an incentive to increase attention and care.

From (5) we also obtain

$$\frac{dy}{dB} = \pi w(-\frac{\partial a}{\partial B}) + 1 - \frac{1}{n} > 0,$$

where we use \( p = 1/n \). Therefore, each child has an incentive to acquire more bequests from the parent by providing more attention and care.

2.2 Bequests from parents in stage 1

Parents are assumed to choose allocation between their own consumption and bequest to children to maximize utility from both. The utility function of a parent is assumed to be linearly separable, following the literature such as Chang and Weisman (2005), as

$$U = [Y - B - \phi(n)] + \pi H(na).$$

Therein, \( Y \) represents the pre-transfer income of the parent, \( \phi(n) \) denotes the child-rearing cost of \( n \) children, and \( H(na) \) represents the utility from the total elderly attention and care provided by \( n \) children.\(^6\) We consider here that the child-rearing cost occurs before transferring bequest to the children (and after receiving bequests from their parents), i.e., in stage 0. Function \( H(.) \) is assumed to be strictly concave and \( H(na) \) goes to \(-\infty\) as \( na \to 0 \), whereas function \( \phi(.) \) can be either concave or convex. The welfare of an elderly parent is lower when she becomes dependent without attention and care from her children than when she is independent. We also assume here that the consumption level of the parent is independent of whether they becomes dependent or not in old age.\(^7\)

If the parent bequeaths \( B \) to transfer-seeking children, then the total elderly attention and care provided by children is obtained from (4) as

$$na = \frac{(n-1)B}{\pi wn}.$$  \(\text{(9)}\)

Substituting (9) into utility function (9), we obtain the first-order condition with respect to \( B \) as

$$1 = \frac{n-1}{nw} H'(\frac{(n-1)B}{\pi wn}),$$

from which it follows that

\(^6\) Child-rearing costs are measured in terms of consumption goods in this setting. We can instead assume that parents spend time in child rearing. In that case, the cost is measured in terms of foregone income i.e., the wage rate times the child-rearing time.

\(^7\) Unless parents are sure to be autonomous in old age, parents are assumed to have children for security during their old age.
The amount of bequest depends on the number of children, the children’s wage rate, and the probability of becoming dependent. From (10) we obtain

\[
\frac{\partial B}{\partial n} = -\frac{H'\pi w}{H''(n-1)}(1-\eta) > 0 \quad \text{as} \quad 1 > \eta, \quad (12)
\]

\[
\frac{\partial B}{\partial \pi} = \frac{B}{\pi} > 0, \quad (13)
\]

\[
\frac{\partial B}{\partial w} = \frac{H'\pi n}{H''(n-1)}(1-\eta) = 0 \quad \text{as} \quad 1 = \eta, \quad (14)
\]

where \( \eta \equiv \left[ \frac{(n-1)B}{\pi wn} \right]/H' > 0 \) is the elasticity of the marginal utility of elderly attention and care from children. The comparative static results can be interpreted as follows. Increases in the number children increase the total elderly attention and care if per-child attention and care remains constant. When the marginal utility of elderly care declines greatly with elderly care, i.e., if \( \eta \) is large, then parents reduce bequests, affecting the children’s provision of attention and care negatively, and increase their own consumption (see (12)). With a higher probability of dependence, parents increase bequests, thereby increasing children’s elderly care (see (13)). With a higher wage rate of children, when \( \eta \) is large, parents increase bequests to offset, at least partly, decreases in the children’s provision of attention and care because higher children’s wage rates reduce their needs for transfers from parents (see (14)).

3 Fertility decisions (stage 0)

In this section, we analyze parents’ fertility decisions.\(^8\) Parents choose the number of children knowing that they will play an intergenerational game with their children after children are born and grown up. The utility of a parent can be rewritten as

\[
U(n) = [Y - B(n) - \phi(n)] + \pi H\left(\frac{(n-1)B(n)}{\pi wn}\right). \quad (15)
\]

Bequests the children and elderly care provided by children are regarded as functions of

\(^8\) Mathematically, the optimization problem of parents can be solved for the number of children and bequests simultaneously. However, as described in the previous section, it is apparently natural to consider that a game with respect to transfers and care is played between elderly parents and grown-up children.
the number of children. The first-order condition of the parent’s utility maximization with respect to the number of children is given as
\[
\frac{dU}{dn} = -\frac{\partial B}{\partial n} - \phi' + \pi H'\left(\frac{B}{\pi wn^2} + \frac{n-1}{\pi wn}\right) = 0.
\] (16)

Making use of (12), the optimal number of children must satisfy the following condition.
\[
\phi' = \frac{B}{wn^2} H'.
\] (17)

This condition implicitly gives the optimal number of children. However, we cannot solve for the optimal number of children explicitly because both sides of (17) include \(n\) explicitly and implicitly. Condition (17) means that the optimal fertility rate depends on the probability of parents’ dependence and children’s wage rates, i.e., we have \(n(\pi, w)\).

To explore some properties of the fertility rate, we have by differentiating (17) totally
\[
Ddn = \frac{BH'}{\pi wn^2} d\pi - \frac{1}{\eta} \frac{BH'}{w^2 n^2} (1 - \eta) dw,
\] (18)

where \(D = -d^2 U / dn^2\), which is positive because we assume that the second-order condition is satisfied.\(^9\) Therefore, from (18), we have
\[
\frac{dn}{dw} > 0 \quad \text{as} \quad \eta > 1, \quad \text{and} \quad \frac{dn}{d\pi} > 0.
\] (19a)

A rise in the children’s wage rate might raise or reduce the fertility rate, depending upon the elasticity of the marginal utility of family attention and care provided by children: an increase in the probability that parents become dependent in old age increases the number of children per parent.\(^10\) The result in (19a) has not been described in the literature. An increase in the children’s wage rate lowers the fertility rate if the elasticity of the marginal utility of family care by children is smaller than one. It is noteworthy that the children’s wage rates are measured in terms of parents’ consumable income. By contrast, condition (19b) is apparently foreseeable and not surprising. A higher

\(^9\) If \(H(.)\) is logarithmic and \(\phi(.)\) is convex, i.e., \(\eta = 1\) and \(\phi'' > 0\), then the condition is satisfied. Zhang and Zhang (1998) assume condition \(\phi'' > 0\) for the second-order condition for the individual’s lifetime utility maximization to hold in their model without elderly care provision.

\(^10\) The probability of being dependent can be considered to a proportion of the time period between the longevity and healthy life expectancy relative to longevity, although the length of lifetime is normalized in this paper. Mayhew (2011), among others, observes from UK data during 1980–2005 that healthy lifetime increases at a slower pace than longevity.
probability of elderly dependence induces parents to have more children.

These results can be interpreted in the following way. An increase in the children’s wage rate reduces per-child attention and care. Condition (19a) implies that if the elasticity of the marginal utility of family attention and care is one ($\eta = 1$), then the children’s wage rate does not affect the number of children. If the elasticity of the marginal utility of elderly care $\eta$ is small, i.e., if the marginal utility of family care decreases only moderately, then parents have less children with higher wage rates. A rise in the children’s wage rate reduces per-child attention and care provision for a parental bequest because the children’s needs for income transfers become small.$^{11}$ Therefore, parents might decrease the number of children to save child-rearing costs. By contrast, if $\eta$ is great, i.e., if the marginal utility of family attention and care decreases rapidly, then it is optimal for parents to increase the number of children for greater family attention and care during old age. The increased number of children tends to increase the total family attention and care through sibling competition for transfers.

A higher probability of becoming dependent increases the parent’s utility of elderly care provided by children. If the elderly care each child provides remains constant, then more children raise the parent’s utility from total elderly care because the marginal child-rearing cost is independent of the probability. However, increasing the number of children might not be good for parents: When the elasticity of the marginal utility of elderly care $\eta$ is great, more children might induce parents to reduce bequests (see (12)). Therefore, it is ambiguous whether changes in bequests consequently increase children’s elderly care or not. However, on balance, condition (19b) implies that a higher fertility rate must increase the total amount of children’s attention and care as a result of the game. It is noteworthy that in the present setting, parents decide the amount of bequest to children by taking the number of children as already determined.$^{12}$

4. Fertility decisions in intergenerational exchanges – availability of private long-term insurance

To clarify the parents’ fertility decisions in an intergenerational exchange setting, we assume that private long-term elderly care insurance is available to parents. Presuming that the insurance is actuarially fair, the utility of parents can be rewritten as

$^{11}$ See Appendix A1.

$^{12}$ Mathematically, the optimization problem of parents can be solved for the number of children and bequests simultaneously. However, as described in the previous section, it is apparently natural to consider that a game with respect to transfers and care is played between elderly parents and grown-up children.
\[ U(n) = [Y - B(n) - I - \phi(n)] + \pi H\left(\frac{I}{\pi} + \frac{(n-1)B(n)}{\pi wn}\right), \]  
\hspace{1cm} (20)

where \( I \) denotes an insurance premium and \( I / \pi \) represents the insurance benefit.

The first-order condition for optimal choice of the insurance is

\[ \frac{\partial U}{\partial I} = -1 + H' \leq 0, \]  
\hspace{1cm} where the equality holds for \( I > 0 \). \hspace{1cm} (21)

From (10) and (17), if parents choose to have children and give them transfers, it holds

\[ 1 = \frac{n-1}{wn} H' = \frac{1}{\phi'} \frac{B}{wn^2} H' \quad \text{or} \quad H' = \frac{wn}{n-1} = \phi' \frac{wn^2}{B}, \]  
\hspace{1cm} (22)

from which one can say that the optimal \( n \) and \( B \) satisfy relation \( n(n-1)\phi' = B \).\(^{13}\)

Therefore, we have two cases:

(i) if \( \frac{wn}{n-1} \geq H' = 1 \), then \( I > 0 \), and

(ii) if \( \frac{wn}{n-1} = H' < 1 \), then \( I = 0 \) \hspace{1cm} (23a)

\hspace{1cm} (23b)

In case (i), we have \( B = n = 0 \) in the present setting because parents do not obtain any benefit from having children. Parents are provided long-term care solely by the insurance. In other words, there is no exchange between parents and children in this case.\(^{14}\) By contrast, in case (ii), parents do not purchase the long-term care insurance and rather have children and give them bequest in exchange for family long-term care. In this case we have \( w < (n-1)/n < 1 \), i.e., the wage rate is lower than unity because \( n-1 > 0 \).

At this stage of arguments, it is noteworthy that \( n \) in conditions (23a) and (23b) is endogenously determined, depending upon the model parameters. Therefore, the condition in (23a) ((23b)) should be read as that the wage rate is sufficiently high (low, respectively). The intergenerational exchanges might occur when the wage rate of children is sufficiently low. From (19a) we might conjecture that case (ii) holds, i.e., intergenerational exchanges occur, for low wage rates when \( \eta < 1 \). However, after the wage rate becomes sufficiently high, such intergenerational exchanges might not play any role in determining the fertility rate, described as in case (i).

\(^{13}\) In this paper, because parents have children only for their old support, condition (22) holds if they have children.

\(^{14}\) When \( wn / (n-1) = H' = 1 \), parents are indifferent between having children and purchasing private insurance. For exposition, we include this case in case (i).
5. Conclusion

We have analyzed the fertility decisions of families in an intergenerational exchange model presuming that parents have no utility of having children. Parents who choose to have children for support from them during old age increase the number of children when the probability of becoming dependent when old rises. Although this result is intuitively foreseeable, it is ambiguous whether elderly care provision per child increases along with parental bequest to children. Per-child attention and care might decrease with a bequest from the parent. By contrast, increases in the children’s wage rates decrease the number of children, i.e., the fertility rate, although each child’s attention and care might increase if the elasticity of the marginal utility of family care is sufficiently small.

The last possible negative effect of the wage increase implies that the fertility rate might decrease along with economic growth, especially unexpected economic growth, and therefore rising wages in developing economies where many parents have children as old age security, i.e., where parents greatly want to have family support. In fact, during the 1970s and 1980s where the fertility rates rapidly declined in Indonesia, the Philippines, and Thailand, their per-capita economic growth rates were rather high. The high growth rates might involve unexpected increases in per-capita incomes. In these countries, most parents have had children because of their elderly support (Kagitcibasi, 1982; Leroux and Pestieau, 2014). Therefore, the theoretical prediction of this paper implies that parents might have lower elasticities of the marginal utility of family elderly care in these countries.15

It is noteworthy that the possibility of fertility declines does not derive from a tradeoff between the quantity and quality of children but from declines in children’s needs for bequests from parents as income compensation to poor young people. The child labor literature also predicts that economic development reduces fertility (Hazan and Berdugo, 2002; Chakraborty and Das, 2005). However, the logic differs from ours. In the child labor literature, in which parents typically control children’s time, economic development induces parents to substitute child education for child labor, i.e., from quantity to quality of children.

So far we have not considered policy effects. Child allowances for instance can be regarded as affecting the fertility rate. In the present model setting, child allowances might affect only the fertility decision of parents in stage 0. The introduction or extension

15 Kagitcibasi (1982) reports that the proportions of people who have children for old age security were 7 or 8 % in the US and Horioka (1984) reports that it was 35.1% in Japan. Horioka (2014) reports that they became to 2.5% in the US and 20.5 % in Japan, respectively. The proportion in Japan remained relatively higher.
of the allowances increases the number of children in families if they are financed through lump-sum taxes on parents. The effect of increases in bequests depends on the elasticity of the marginal utility of elderly care. The increased number of children makes competition for transfer seeking severer. Therefore, it increases each child’s attention and care to parents. However, there can be a variety patterns of intergenerational exchanges in reality. Our setting is merely a model. Especially, policy design must take into account wider patterns of intergenerational exchanges.

Appendices
A.1 Effects on parental bequests and children’s elderly care

This appendix presents an analysis of the effects of the children’s wage rate and the probability of parental dependence on children’s attention and care and on parental bequest. In a Nash equilibrium, we can express a parental bequest and per-child elderly care as

\[ B = B(n(\pi, w); \pi, w) \quad \text{and} \quad a = a(B(\pi, w), n(\pi, w); \pi, w). \quad (A1) \]

Differentiating (A1) and from (5), (6), (12), (13), and (14), we obtain the following results:

\[ \frac{dB}{d\pi} = \frac{\partial B}{\partial n} \frac{\partial n}{\partial \pi} + \frac{\partial B}{\partial \pi} > 0 \quad \text{as} \quad \eta \geq 1, \quad \text{undetermined otherwise,} \quad (A2) \]

\[ \frac{dB}{dw} = \frac{\partial B}{\partial n} \frac{dn}{dw} + \frac{\partial B}{\partial \pi} \frac{\partial \pi}{\partial w} < 0 \quad \text{as} \quad \eta = 1, \quad (A3) \]

\[ \frac{da}{d\pi} = \frac{\partial a}{\partial B} \frac{\partial B}{\partial \pi} + \frac{\partial a}{\partial n} \frac{\partial n}{\partial \pi} + \frac{\partial a}{\partial \pi} \quad \text{undetermined,} \quad (A4) \]

\[ \frac{da}{dw} = \frac{\partial a}{\partial B} \frac{\partial B}{\partial w} + \frac{\partial a}{\partial n} \frac{\partial n}{\partial w} + \frac{\partial a}{\partial \pi} \frac{\partial \pi}{\partial w} < 0 \quad \text{as} \quad \eta \geq 1, \quad \text{undetermined otherwise.} \quad (A5) \]

In the case of a long-linear utility function of elderly care, we have \( dB/d\pi > 0 \), \( dB/dw = 0 \), and \( da/dw < 0 \) because \( \eta = 1 \).

A2. Child allowance policy

Presuming that a child-rearing subsidy per child is \( \beta \), then the subsidy is financed by lump-sum taxes on parents. Let \( T \) be a lump-sum tax. The utility of a parent can be written as

\[ U = Y - B - \phi(n) + \beta n - T + \pi H \left( \frac{(n-1)B}{\pi wn} \right). \quad (A6) \]

The first-order condition for utility maximization is

\[ \text{See Appendix A2.} \]
\[ \frac{\partial U}{\partial n} = -\phi' + \beta + \frac{BH'}{wn^2} = 0, \]  
(A7)

from which we can obtain

\[ \frac{dn}{d\beta} = D^{-1} > 0. \]  
(A8)

We also have

\[ \frac{dB}{d\beta} = \frac{\partial B}{\partial n} \frac{dn}{d\beta} > 0 \quad \text{as} \quad 1 = \eta, \]  
(A9)

\[ \frac{da}{d\beta} = \frac{\partial a}{\partial n} \frac{dn}{d\beta} > 0. \]  
(A10)

An increase in the child-rearing subsidy increases the number of children a parent has, intensifying the competition for transfers among children. Therefore, it increases per-child attention and care, thereby increasing the total family long-term care for a given bequest. However, the amount of bequest depends on the elasticity of the marginal utility of elderly care provided by children. If the elasticity is sufficiently great, then parents might increase consumption of their own, reducing their bequest to children.
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Conflict of interest
The author has no conflict of interest, financial or otherwise, related to this study.

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