Tying, Market Entry and R&D Investment in a Mixed Oligopoly

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Abstract

This paper explores the idea that an incumbent firm may employ tying to successfully fend off potential entrants in a market where there is mixed public and private ownership of companies. It is well known that when the incumbent and entrants are profit maximizing firms, tying can lead to market foreclosure and under investment in R&D. By contrast, this paper shows that when the incumbent is a profit maximizing firm and the entrants are public firms, with a mandated requirement to maximize profit and consumer surplus, market entrants have more incentive to invest in R&D. This makes the prospect of successful market entry more likely. Moreover, tying by the private incumbent improves welfare by reducing the incentive of public entrants to over invest in R&D. However, the situation is reversed when the incumbent is a public firm and entrants are profit maximizers, so that the results are consistent with the case where the incumbent and entrants are profit maximizing firms and tying leads to foreclosure and under investment in R&D.

Key Words: Tying; Market Foreclosure; Investment Uncertainty; Public Firms; Mixed Oligopoly.

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1 Introduction

Within the field of industrial organization the tying literature has focused on three broad questions: its impact on efficiency, its effect on consumer surplus via price discrimination and its potential for market foreclosure. In the study of its impact on efficiency, tying is considered to improve welfare by reducing the search cost of consumers and the packing and shipping cost of firms (Kenney & Klein, 1983; Malella & Nahata, 1980; Carlton & Waldman, 2003; Salinger, 1995). In the study of its use in price discrimination, tying has been shown to increase the monopolist’s profit by reducing consumer heterogeneity in reservation utility (Stigler, 1963; Adams & Yellen, 1976; Schmalensee, 1984; Bakos & Brynjolfsson, 1999; Geng et al., 2005). If we focus on the strategic effects of tying, then a number of papers have shown the possibility for an incumbent monopolist to use tying as a defence against entry and to extend its market power into adjacent markets (Whinston, 1990; Choi & Steffanides, 2001; Carlton & Waldman, 2002; Nalebuff, 2004; Peitz, 2008).

However, so far no single paper in the tying literature has explored the possibility of the use of tying as a means of market foreclosure in a mixed oligopoly. This is despite the presence of semi-privatised companies and public enterprises in industries such as telecommunications and electricity, where the practice of tying and the bundling of access with other services persists. Furthermore, many of these public enterprises have mandated service requirements, where they are required to service or cross-subsidize access in unprofitable market segments. Our paper develops a model of tying and market entry within a mixed oligopoly. The underlying motivation is to understand the role of mandated service requirements in determining the outcome of market entry when incumbents have the opportunity to tie the consumption of complementary goods that are supplied separately by market entrants.

Hence, our objectives are consistent with those in the mixed oligopoly literature, where Merrill & Schneider (1966) were the first to consider the use of public firms as a means of regulating oligopolistic behaviour. Their paper shows that the presence of public firms within an oligopolistic market is sufficient to improve short-term market performance, lowering price and increasing output at the market equilibrium. It is important to note that this result is not robust and depends on both market size and the nature of oligopolistic competition.¹ From the perspective of our paper, Heywood and

¹For example De Frajia & Delbono (1989) extend Merrill & Schneider (1966) to consider the case of Cournot competition between one public firm and n private firms. They find that full privatisation improves welfare when only when the number of private firms is large; when the market size is small, they show that the public firm has an important role in reducing the effect of market power on price and output. Cramer et al. (1991) examines pricing and location decision within the linear Hotelling model, and show that the presence of a public firm actually harms welfare when the total number of firms is more than two and less than six. However, this result is not robust. However, Matushima & Matsumura (2002) have shown, within the spatial model for quantity setting firms, that the presence of a public firm is sufficient to induce spatial agglomeration among private firms, and is therefore welfare improving. Hence, whether firms are quantity or price setters is an important determinant of market outcomes.
Ye (2009) have shown within the spatial model that timing of entry matters in the sense that the presence of the public firm, as either a leader or a follower, limits the extent to which private firms can locate asymmetrically.

Our paper shows that public enterprises have an important role in limiting tying as a means of market foreclosure and exclusion. However, the impact in terms of exclusion and welfare depends on whether the public firm is the incumbent or the entrant. In both cases, the incentive for the incumbent to use tying as a means of foreclosure is always present. However, when the entrants are public enterprises we find that they will invest more in market entry, even though this investment is risky; this lead to an over investment problem. We show that if the consumer surplus requirement on entrants is large enough, then tying can ease the over-investment problem and increase social welfare. We show that this situation is reversed when the incumbent is the public enterprise and the entrant firms are private profit maximizing firms. In this case tying discourages investment by market entrants, making entry more difficult and resulting in reduced social welfare.

In the tying literature the most similar paper to ours is Choi & Stefanides (2001). Their paper introduces a risky upfront R&D investment to study the impact of tying of market foreclosure. Unlike the Monopoly-Oligopoly market structure of Whinston (1990), Carlton & Waldman (2002), Nalebuff (2004) and Peitz (2008), where there is a single entrant, the incumbent faces the threat of entry by more than one firm. They show that tying reduces the probability of successful entry and discourages entrants from investing in R&D when entrants can supply only one of the complementary goods offered by the incumbent. The model in Choi & Stefanides (2001) corresponds to the case of full privatisation in the setting of our paper. Our findings are opposite to the case of what happens when all firms are profit maximizers, where tying always reduces social welfare.

The rest of this paper is organised as follows. Section two provides an outline of the model. Section three examines the equilibrium investment decision of the entrant firms and the tying decision of the incumbent when the entrant firms are public firms and the incumbent is a profit maximiser. Section four investigates the case when the incumbent is the public enterprise and entrants are private firms. Section five provides analysis of social welfare. Section six is the conclusion.

2 Model

The setting of the model is given as follows. Firm 1 is the incumbent monopolist; it produces two complementary intermediate goods that are respectively denoted as goods $A$ and $B$. These two goods are purchased by the final consumer and combined in fixed proportions, on a one-to-one basis, to form a final consumption good. There are $n$ consumers, each of whom requires one unit of this final good. The reservation utility that each consumer derives from consuming the final good, is equal to $V > 0$. There are
no fixed costs of production. The marginal cost for the incumbent producing each good is identical and is given by $\bar{c}$, where $0 < \bar{c} < V$. The incumbent firm has the option to bundle the sale of its two complementary goods. However, it must commit to the tying of the sale of these to goods prior to the entry decision of outside firms.

There are two potential market entrants, who can each produce and sell only one of the complementary goods. These firms will be denoted as $A_2$ and $B_2$, respectively. Once these firms enter into their respective markets they then compete on price against the incumbent firm. As the products offered in each market by the incumbent and the entrant firms are perfect substitutes, in equilibrium consumers will purchase the product with the lower marginal cost of production. To obtain this lower marginal cost, the entrants must pre-commit to a risky R&D investment prior to entry. If an entrant firm’s R&D investment is successful, then the entrant gets a superior technology to the incumbent firm, so that its marginal cost of production $0 < c < \bar{c}$. However, if the R&D investment fails, then the entrant’s marginal cost of production is higher than the consumers’ reservation utility and there will be no market entry.

The basic setting of our model is therefore modelled in the following three-stage game, which is solved by backward induction:

**Stage** 1: The incumbent firm, Firm 1, decides whether or not to tie the sale of products $A$ and $B$.

**Stage** 2: The entrant firms, Firm $A_2$ and $B_2$, decide on the amount of their respective R&D investments, $I_{A_2}$ and $I_{B_2}$.

**Stage** 3: Price competition occurs in the product market between the successful entrant firm or firms and the incumbent.

### 3 Private Incumbent vs. Public Entrants

This section considers the case where the incumbent monopolist is a profit maximising (privately-owned) firm and the potential entrants are public enterprizes. We begin deriving the Bertrand Nash equilibrium for the product market stage game. First we consider the situation where only one of the entrants makes a successful R&D investment. Without loss of generality (due to the symmetric structure of firms), we assume that only Firm $A_2$’s R&D investment is successful. This firm maximises a linear combination of its own profit and consumer surplus:

$$W_{A_2} = \alpha n(x_A - c) + (1 - \alpha) n(V - x_A - x_B), \quad \alpha \in [0, 1]$$

where $\alpha$ represents the degree of privatisation and $x_A$ and $x_B$ denote the prices charged in markets $A$ and $B$. Note that the closer that $\alpha$ is to 1, the higher the degree of
privatisation, with $\alpha = 1$ corresponding to the entrant acting as a profit maximising firm.

When there is no-tying, the complementarity between products $A$ and $B$ implies that there exists a continuum of price equilibria ranging between $x_{A1} = x_{A2} = x_A \in [\underline{c}, \bar{c}]$ in Market $A$ and $V - x_A$ in Market $B$. If the surplus (i.e., cost saving) generated from Firm $A2$’s successful R&D investment is given by $S = n(\bar{c} - \underline{c})$, then the equilibrium price that Firm 1 charges for products $A$ and $B$ is an interval depending on $\lambda \in [0, 1]$, which is the share of the surplus $S$ captured by the incumbent. This implies that when Firm $A2$ is the sole market entrant, the equilibrium prices in Markets $A$ and $B$ are given by

$$x_A = \underline{c} + (1 - \lambda)(\bar{c} - \underline{c}) \quad \text{and} \quad x_{B1} = V - \underline{c} - (1 - \lambda)(\bar{c} - \underline{c}), \quad \lambda \in [0, 1]$$

respectively. At the upper extremal equilibria, Firm 1 charges price $\bar{c}$ and $V - \bar{c}$ for products $A$ and $B$ respectively, while Firm $A2$ charges price $\bar{c}$ in Market $A$. In this equilibrium, Firm $A2$ captures all of the surplus $S$ resulting from its successful R&D investment. At the lower extremal equilibrium, Firm 1 charges price $\underline{c}$ and $V - \underline{c}$ for products $A$ and $B$, respectively, while Firm $A2$ charges $\underline{c}$ in Market $A$. Under this equilibrium all of the surplus $S$ is acquired by Firm 1 through its monopoly in Market $B$. Hence, as $\lambda \to 1$, the degree of price squeeze that the incumbent firm can exert on the entrant increases.

Now we consider the case of tying. When there is only one successful market entrant, then the market entrant will be unable to sell its good. Thus if the incumbent ties, then market entry is only possible if both Firms $A2$ and $B2$ successfully innovate. If $2\bar{c}$ is the price that the incumbent charges for the bundled product, then at the Bertrand equilibrium any price can be sustained that satisfies

$$x_{A2} + x_{B2} = 2\bar{c} \quad \text{and} \quad x_{A2} \geq \underline{c}, x_{B2} \geq \underline{c}.$$ 

Hence, there will be multiple Bertrand Nash equilibria. It can be seen from equation (1) that $W_{A2}$ is monotonically increasing (resp. decreasing) in $x_{A2}$, when $\alpha > (\text{resp.} <) 1/2$. This implies that if $\alpha > 1/2$, then $x_{A2} = x_{B2} = \bar{c}$. By contrast, if $\alpha < 1/2$, the unique symmetric equilibrium is $x_{A2} = x_{B2} = \underline{c}$. When $\alpha = 1/2$, for simplicity of analysis we will assume that $x_{A2} = x_{B2} = \bar{c}$.

### 3.1 Investment Decisions of Entrants

Recall that the market entrants can only enter if they invest in R&D and their innovation is successful. If the R&D investment expenditures of the entrant firms are given by $I_{A2}$ and $I_{B2}$, respectively, then the corresponding probability of R&D success is given $p(I_{A2})$ and $p(I_{B2})$. It is assumed that probability function $p(I)$ is identical for both entrants,
and increasing and concave with respect to the R&D investment $I$.\footnote{The concavity of probability function is important for ensuring the existence of equilibrium in the R&D investment stage game. In order to ensure the uniqueness and stability of equilibrium, we also assume that $p''(I_2)p''(I_2)p'(I_2)^2 > 0$.}

If the incumbent decides on no-tying, then Firm $A_2$’s profit of successful R&D investment is related to whether or not Firm $B_2$’s R&D investment is successful. If Firm $B_2$ succeeds, then the surplus generated by Firm $A_2$’s innovation is $S = n(\bar{c} - \zeta)$. However, if Firm $B_2$’s investment fails, then the surplus generated from Firm $A_2$’s innovation is given by $(1 - \lambda)S = (1 - \lambda)n(\bar{c} - \zeta)$, where $\lambda$ measures the price squeeze that the incumbent firm can exert on the entrant. As for consumer surplus, it is only greater than 0 when both Firms $A_2$ and $B_2$ succeed in their R&D investment and when $\alpha < 1/2$, in which case the total consumer surplus generated for the composite good is equal to $n(V - 2\zeta)$.

Thus, in Stage 2 of the game, Firm $A_2$’s optimal investment decision is expressed as follows when $\alpha < 1/2$:

$$
\max_{I_{A2}} W_{A2} = \alpha [p(I_{A2})(1 - p(I_{B2}))(1 - \lambda)S + p(I_{A2})p(I_{B2})S - I_{A2}] \\
+ (1 - \alpha)p(I_{A2})p(I_{B2})n(V - 2\bar{c}).
$$

(2)

Due to the symmetry of Firms $A_2$ and $B_2$ and the first order condition must satisfy

$$
p'(I^*) = \frac{1}{[1 - \lambda + \lambda p(I^*)]S + \frac{1 - \alpha}{\alpha} p(I^*)n(V - 2\bar{c})}.
$$

(3)

By totally differentiating the equation (3) and rearranging terms, we can make a comparative static analysis of how the degree of price squeeze $\lambda$ and the degree of privatisation $\alpha$ affects the equilibrium R&D investment expenditure level $I^*$ made by entrants.

The partial derivative of $I^*$ on $\lambda$ and $\alpha$ are given by

$$
\frac{\partial I^*}{\partial \lambda} = \frac{p'(I^*)[1 - p(I^*)]S}{[p''(I^*)p(I^*) + (p'(I^*))^2] \left[ \lambda S + \frac{1 - \alpha}{\alpha} n(V - 2\bar{c}) \right] + (1 - \lambda)p''(I^*)S}.
$$

(4)

and

$$
\frac{\partial I^*}{\partial \alpha} = \frac{p'(I^*)p(I^*)n(V - 2\bar{c})\alpha^{-2}}{[p''(I^*)p(I^*) + (p'(I^*))^2] \left[ \lambda S + \frac{1 - \alpha}{\alpha} n(V - 2\bar{c}) \right] + (1 - \lambda)p''(I^*)S}.
$$

(5)

Since the denominators of equation (4) and (5) are negative, while numerators are positive, $\partial I^*/\partial \lambda < 0$ and $\partial I^*/\partial \alpha < 0$.\footnote{The condition $p''(I_{A2})p''(I_{B2})p(I_{A2})p(I_{B2}) - [p'(I_{A2})p'(I_{B2})]^2 > 0$ and $I_{A2} = I_{B2} = I^*$, implies that we have $[p''(I^*)p(I^*)]^2 > [p'(I^*)]^4$ and hence $p''(I^*)p(I^*) + [p'(I^*)]^2 < 0$.} Equation (4) implies that at the equilibrium an increase in the price squeeze by the incumbent will reduce the incentive for potential entrants to invest in R&D. This is consistent with firm behaviour in Choi & Stefanides (2001) where both entrant and incumbent firms are profit maximisers.

Equation (5) implies that if we take $\lambda$ as given, then the equilibrium R&D expenditure
by potential entrants is always larger when the entrant is a public firm, that is $I^*(\alpha) > I^*_{pr}$ for $\alpha \geq 1/2$. Here we are denoting the equilibrium R&D investment expenditure in the basic model of Choi & Stefanides (2001) as $I^*_{pr}$; this corresponds to the case where the incumbent and entrants are private firms, i.e., $\alpha = 1$. Equation (5) implies that when entrants are pure profit maximizers there will be a lower investment in R&D. This is because the objective function of the public firm includes the consumers’ surplus, which is positive only when both entrants succeed in innovation. Hence, an increase in the marginal benefits of investment increases the incentive for a public firm to invest in R&D. This improves the likelihood of successful market entry.

If the incumbent decides on tying, then the Firms $A_2$ and $B_2$ must both be successful in their R&D investments and enter the market simultaneously. In this case firm $A_2$’s decision problem is when $\alpha \geq 1/2$:

$$\max_{I_{A2}} W_{A2} = \alpha [p(I_{A2})(1 - p(I_{B2}))S - I_{A2}] + (1 - \alpha)p(I_{A2})p(I_{B2})n(V - 2\bar{c}).$$ (6)

Because Firms $A_2$ and $B_2$ are symmetric, we have $\tilde{I}_{A2}^* = \tilde{I}_{B2}^* = \check{I}^*$. At the symmetric equilibrium, $\check{I}^*$ must satisfy the following first order condition

$$p'(\check{I}^*) = \frac{1}{p(\check{I}^*)S + \frac{1-\alpha}{\alpha}p(\check{I}^*)n(V - 2\bar{c})}. \quad (7)$$

As we have done in the case of no-tying, we will make a comparative-static analysis by totally differentiating equation (7) with respect to $\alpha$:

$$\frac{\partial \check{I}^*}{\partial \alpha} = \frac{p'(\check{I}^*)p(\check{I}^*)n(V - 2\bar{c})\alpha^{-2}}{[p''(\check{I}^*)p(\check{I}^*) + (p'(\check{I}^*))^2][S + \frac{1-\alpha}{\alpha}n(V - 2\bar{c})]^2}. \quad (8)$$

As in the case of tying $\partial \check{I}^*/\partial \alpha < 0$, which implies that the equilibrium R&D investment expenditure of entrant decreases as the degree of privatization increases. This implies that $\check{I}^*(\alpha) > \tilde{I}^*_{pr}$ for $\alpha \geq 1/2$. Furthermore, comparing equations (7) and (8) with equations (3) and (5), we can see that these equations are identical when $\lambda = 1$ and the incumbent is able to extract all the surplus derived from the entrant’s innovation. This implies that that in equilibrium $\check{I}^*(\alpha) = I^*(\alpha, \lambda = 1)$ and for all $\lambda < 1$, $\check{I}^*(\alpha) < I^*(\alpha, \lambda)$.

For the case where $\alpha < 1/2$, Firm A2’s optimal decision problem is:

$$\max_{I_{A2}} W_{A2} = -\alpha I_{A2} + (1 - \alpha)p(I_{A2})p(I_{B2})n(V - 2\bar{c}).$$ (9)

At the symmetric equilibrium $\check{I}^*$ satisfies the following equation

$$p'(\check{I}^*) = \frac{1}{\frac{1-\alpha}{\alpha}p(\check{I}^*)n(V - 2\bar{c})}. \quad (10)$$
We make a comparative-static analysis by totally differentiating equation (10) with respect to $\alpha$:

$$\frac{\partial \tilde{I}^*}{\partial \alpha} = \frac{p'(\tilde{I}^*)p(\tilde{I}^*)n(V-2\bar{c})\alpha^{-2}}{\left[p''(\tilde{I}^*)p(\tilde{I}^*) + (p'(\tilde{I}^*))^2\right] \frac{1-\alpha}{\alpha} n(V-2\bar{c})}.$$  (11)

As in the case of no-tying, $\partial \tilde{I}^*/\partial \alpha < 0$ the equilibrium R&D investment expenditure decreases as the degree of privatization increases. This implies that $\tilde{I}^*(\alpha) > \tilde{I}^*(\alpha = 1/2)$ for $0 < \alpha < 1/2$. We also have $\tilde{I}^*(\alpha) = I^*(\alpha, \lambda = 1)$ and for all $\lambda < 1$, $\tilde{I}^*(\alpha) < I^*(\lambda)$.

These results can now be stated in the following proposition:

**Proposition 1.** The equilibrium R&D investment expenditure level by entrant firms always is higher when public entrants’ consumer surplus obligation is higher (i.e., as $\alpha \to 0$). This is independent of the incumbent’s commitment to tying. However, if the incumbent decides not to tie, then a larger $\lambda$ will reduce the equilibrium R&D investment of market entrants.

### 3.2 Tying Decision of the Incumbent Firm

The incumbent firm faces a trade-off when tying. On the one hand, if the incumbent firm chooses not to tie and there is a single successful market entrant, then it captures $\lambda S$ share of the surplus generated from the entrant’s innovation. Hence, the profit of Firm 1, when it chooses no-tying, is given by

$$\Pi_1(\lambda, \alpha) = \left[1 - p\left(I^*(\lambda, \alpha)\right)^2\right] n(V-2\bar{c}) + 2p\left(I^*(\lambda, \alpha)\right) [1-p\left(I^*(\lambda, \alpha)\right)] \lambda S;$$  (12)

On the other hand, if the incumbent chooses to tie, then by Proposition 1 the size of the equilibrium R&D investment made by market entrants is always smaller than if the incumbent were to choose tying. As a consequence, the likelihood of successful market entry is small (relying on successful innovations by both entrants). In this case its profit when choosing tying is given as follows:

$$\tilde{\Pi}_1(\lambda, \alpha) = \left[1 - p\left(\tilde{I}^*(\alpha)\right)^2\right] n(V-2\bar{c}).$$  (13)

Thus the net benefit of tying is given by

$$\Delta(\lambda, \alpha) = \left[p\left(I^*(\lambda, \alpha)\right)^2 - p\left(\tilde{I}^*(\alpha)\right)^2\right] n(V-2\bar{c}) - 2p\left(I^*(\lambda, \alpha)\right) [1-p\left(I^*(\lambda, \alpha)\right)] \lambda S.$$(14)

The first part of equation (14)

$$\left[p\left(I^*(\lambda, \alpha)\right)^2 - p\left(\tilde{I}^*(\lambda, \alpha)\right)^2\right] n(V-2\bar{c}) > 0.$$
This inequality represents the positive effect of tying on the incumbent firm’s profit. This positive effect occurs because the probability of Firm 1 being displaced is lower under tying, so that \( p(I^*(\lambda, \alpha)) > p(\tilde{I}^*(\lambda, \alpha)) \). The second part of equation (14) is the entrants opportunity cost from tying. In the case of no-tying, the incumbent firm can capture \( \lambda S \) of the surplus generated by the entrant’s innovation. This will disappear if Firm 1 chooses tying, hence when no-tying occurs \( 2p(I^*(\lambda, \alpha))[1 - p(I^*(\lambda, \alpha))]\lambda S > 0 \).

We can see that when \( \Delta(\lambda, \alpha) > 0 \), the positive effect generated from tying dominates, and Firm 1 will bundle the sale of products A and B. If \( \Delta(\lambda, \alpha) < 0 \), then no-tying should be chosen as the loss of the entrant’s surplus will have an adverse impact on the incumbent firm’s profit. Our objective is to find \( \lambda^* \), the critical level of surplus extraction where \( \Delta(\lambda^*, \alpha) = 0 \) and the incumbent firm is indifferent between the two tying regimes. Note that when \( \lambda = 0 \), the incumbent cannot extract surplus from the entrant firm and this implies that \( \Delta(0, \alpha) > 0 \). While at \( \lambda = 1 \), the equilibrium investment is identical across tying regimes, so that \( \tilde{I}^*(\alpha) = I^*(1, \alpha) \). This indicates that when \( \lambda = 1 \) and \( \Delta(1, \alpha) < 0 \) there will be no positive effect from reducing the probability of displacement. Since \( \Delta(\lambda, \alpha) \) is continuous with respect to \( \lambda \in (0, 1) \), this implies that there exists at least one \( \lambda^* \) such that \( \Delta(\lambda^*, \alpha) = 0 \).

Following Choi & Stefanides (2001), we only consider the range where \( \pi_1^*(\lambda, \alpha) \) is monotonically increasing with respect to \( \lambda \); thus there exists a unique \( \lambda^* \) such that \( \Delta(\lambda^*, \alpha) = 0 \). By Proposition 1, the entrant’s equilibrium R&D investment expenditure increases as \( \alpha \) decreases. If the probability of investment success is small (i.e., \( p(I^*) \leq 1/2 \)), then an increase in the R&D investment expenditure level by the public entrants will lead to an increase in the probability of there being only one successful entrant. This leads to the negative impact of tying on the incumbent’s profit. This will outweigh the positive impact of tying, as \( \alpha \to 0 \). Thus, if \( p(I^*) \leq 1/2 \), as \( \alpha \) decreases, the interval \([0, \lambda^*]\) in which tying is optimal for the incumbent firm will also decrease. However, when \( p(I^*) > 1/2 \), the impact of \( \alpha \) on the incumbent’s incentive to tie is uncertain (see Figure 1). The main result is expressed formally in the following proposition:

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4Generally speaking, the sign of the derivative of \( \partial \pi_1^*(\lambda, \alpha) / \partial \lambda \) depends on the specific form of the probability function \( p \). When \( p(I^*) > 1/2 \), we have

\[
\frac{\partial \pi_1^*(\lambda, \alpha)}{\partial \lambda} = 2p'(I^*) \frac{\partial I^*}{\partial \lambda} (\lambda S (1 - 2p(I^*)) - n(V - 2\tilde{e})p(I^*)) + 2p(I^*)[1 - p(I^*)]S > 0,
\]

which implies that \( \pi_1^*(\lambda, \alpha) \) is monotonically increasing with respect to \( \lambda \). However, when \( p(I^*) \leq 1/2 \), the sign of \( \partial \pi_1^*(\lambda, \alpha) / \partial \lambda \) is indeterminate.

5The result depends on the relative change in positive effect when compared to the change in the negative effect. Specifically, when

\[
[p(I^*)p'(I^*)] \frac{\partial I^*}{\partial \alpha} - [I^*]p'(I^*) \frac{\partial I^*}{\partial \alpha} \lambda S \geq p'(I^*) \frac{\partial I^*}{\partial \alpha}[1 - 2p(I^*)] \lambda S,
\]

the positive effect dominates, and the interval of \( \lambda \) in which the incumbent chooses tying narrows. Otherwise, the negative effect dominates and the interval of \( \lambda \) in which tying is optimal expands.
Proposition 2. Under the conditions of $p'(I) > 0$, $p''(I) < 0$, and $p'''(I) \leq 0$:  

1. if $p(I^*) \leq 1/2$, the interval of $[0, \lambda^*)$ in which tying dominates no-tying narrows when the potential entrants change from private firms to public firms; and  

2. if $p(I^*) > 1/2$, the change in the size of the interval $[0, \lambda^*)$ is uncertain and depends on the relative size of the positive and negative effect of tying.  

3. If $p(I^*) \leq 1/2$ and $p(I^*)$ is small enough and satisfies  

$$p(I^*) \leq 1/\left[2 + n(V - 2\bar{c})/\lambda S\right],$$  

then the interval $[0, \lambda^*)$ narrows without the condition of $p'''(I) \leq 0$.  

Proof. See Appendix for complete proof.

4 Public Incumbent vs. Private Entrants

This section considers the case where the incumbent is a publicly owned company and the potential entrants are privately owned, profit maximising firms. We first consider the Bertrand Nash equilibrium in the product market stage game. When there is no-tying by the incumbent and only one successful entrant, then equilibrium price in Market $A$ is given by $x_A = \zeta + (1 - \lambda)(\bar{c} - \zeta)$, where $\lambda \in [0, 1]$ denotes the share of the surplus $S$ captured by the incumbent. The price set by the incumbent in market $B$ is given by $x_{B1} = V - x_A$.  

In the case of tying, the incumbent charges price $2\bar{c}$ for the bundled product, and the entrants’ prices must satisfies $x_{A2} + x_{B2} = 2\bar{c}$, where $x_{A2} \geq \zeta$ and $x_{B2} \geq \zeta$. The symmetric equilibrium is unique and given by $x_{A2} = x_{B2} = \bar{c}$.  

4.1 Investment Decision  

We now consider how the investment decision of the entrants impacts on the tying decision of the incumbent. First note that the objective function of the entrant firms is a special
case of when the entrants were public firms. Hence, when we set $\alpha = 1$ in Eqs. (3) and (10), we arrive at the first order condition for the symmetric equilibrium for the case of investment under no tying ($I^*$) and tying ($\tilde{I}^*$), respectively. These are shown below:

$$p'(C) = \frac{1}{[1 - \lambda + \lambda p(I^*)] S} \quad (15)$$

and

$$p'(\tilde{I}^*) = \frac{1}{p(\tilde{I}^*) S} \quad (16)$$

We will now consider the impact of a change in $\lambda$ on $I^*$, the amount the entrant invests if the incumbent decides not to tie:

$$\frac{\partial I^*}{\partial \lambda} = \frac{p'(1 - p)}{[p''p + (p')^2] \lambda + (1 - \lambda)p''} < 0. \quad (17)$$

This equation implies that as $\lambda$ increases it impacts on the incentive of entrants to invest in R&D. This reduces the likelihood of successful innovation and market entry.

4.2 Tying vs. No Trying

We can now explore the tying decision of the public incumbent. There are two cases to consider:

**Case 1: $\alpha < 1/2$.** When Firm 1 chooses **no-tying**, consumer surplus in the case where “both entrants fail”, “only one entrant succeeds” and “both two entrants succeed” are given by $n (V - 2\bar{c})$, $n (V - 2\bar{c}) + \lambda S$ and $n (V - 2\bar{c})$, respectively. Profit, it is always equal to 0 regardless of whether or not entrants succeed in their R&D investment. When Firm 1 chooses **tying**, its profit is also 0, and consumers’ surplus always is equal to $n (V - 2\bar{c})$, regardless of whether or not entrants succeed in investment. Hence, the objective function of Firm 1, when it chooses no-tying, is given by:

$$W_1^*(\lambda, \alpha) = (1 - \alpha) \{ n (V - 2\bar{c}) + 2p (I^*(\lambda)) [1 - p (I^*(\lambda))] \lambda S \} . \quad (18)$$

While its objective function when choosing tying is given as follows:

$$\tilde{W}_1^*(1, \alpha) = (1 - \alpha)n (V - 2\bar{c}) . \quad (19)$$

Thus the net benefit of tying is given by:

$$\Delta(\lambda, \alpha) = -2(1 - \alpha)p (I^*(\lambda)) [1 - p (I^*(\lambda))] \lambda S. \quad (20)$$
As for (20), ∀ \lambda \in [0, 1], we have \Delta(\lambda, \alpha) < 0, thus when \alpha < 1/2, firm 1 will choose not to tie, regardless of incumbent’s share of the innovation surplus \lambda. This is illustrated in Figure 2 a.

**Case 2:** \( \alpha \geq 1/2 \). As a public firm, the incumbent firm’s objective function must consider both profit and consumer surplus when making its tying decision. The consumer surplus component, it is always equal to \( n(V - 2\bar{c}) \) when Firms A2 and B2 succeed in entering. This is independent of whether or not the incumbent chooses to tie. In terms of the profit component, it faces the trade-off between the gains brought about from the entrant firms reducing their R&D investment and the loss resulting from giving up its \( \lambda S \) share of the surplus generated from the innovation of successful entrants. Hence, the objective function of Firm 1, when it chooses no-tying, is given by:

\[
W^*_1(\lambda, \alpha) = \left[ \alpha + (1 - 2\alpha)p(I^*(\lambda))^2 \right] n(V - 2\bar{c}) + 2\alpha p(I^*(\lambda)) [1 - p(I^*(\lambda))] \lambda S. \tag{21}
\]

While its objective function when choosing tying is given as follows:

\[
\tilde{W}^*_1(1, \alpha) = \left[ \alpha + (1 - 2\alpha)p(\tilde{I}^*)^2 \right] n(V - 2\bar{c}). \tag{22}
\]

Thus the net benefit of tying is given by:

\[
\Delta(\lambda, \alpha) = \left[ p(I^*(\lambda))^2 - p(\tilde{I}^*)^2 \right] (2\alpha - 1)n(V - 2\bar{c}) - 2p(I^*(\lambda)) [1 - p(I^*(\lambda))] \lambda S. \tag{23}
\]

As in the case with public entrants/private incumbent, the first part of Eq. (23) represents the positive net effect of tying on the incumbent firm’s benefit, while the second part is the opportunity cost from tying. Firm 1 will therefore face the trade-off between tying and not tying. To find \( \lambda^* \), the critical level of surplus where \( \Delta(\lambda^*, \alpha) = 0 \), we note that \( \Delta(0, \alpha) > 0 \), \( \Delta(1, \alpha) < 0 \) with \( I^*(1) = \tilde{I}^* \). Since \( \Delta(\lambda, \alpha) \) is continuous with respect to \( \lambda \in (0, 1) \), there still exists at least one \( \lambda^* \) such that \( \Delta(\lambda^*, \alpha) = 0 \). If \( p(I^*) \geq 1/2 \), it can be shown that \( W^*_1(\lambda, \alpha) \) is monotonically increasing with respect to \( \lambda \), thus there exists a unique \( \lambda^* \) such that \( \Delta(\lambda^*, \alpha) = 0 \).

\[^6\text{The sign of the derivative of } \partial W^*_1(\lambda, \alpha)/\partial \lambda \text{ depends on the specific form of the probability function } p. \text{ When } p(I^*) \geq 1/2, \text{ we have }
\]

\[
\frac{\partial W^*_1(\lambda, \alpha)}{\partial \lambda} = p'(I^*) \frac{\partial I^*}{\partial \lambda} [2(1 - 2\alpha)p(I^*)n(V - 2\bar{c}) + 2\alpha(1 - 2p(I^*))\lambda S] + 2\alpha p(I^*)(1 - p(I^*))S > 0.
\]

This implies that \( W^*_1(\lambda, \alpha) \) is monotonically increasing with respect to \( \lambda \). While, when \( p(I^*) < 1/2 \), the sign of \( \partial W^*_1(\lambda, \alpha)/\partial \lambda \) is undetermined.
Proposition 3. For the case where the incumbent firm is a public enterprise and entrants are profit maximizing firms:

1. If \( \alpha < \frac{1}{2} \), then for any \( \lambda \in [0, 1] \) the incumbent will never choose tying (Figure 2a).

2. If \( \alpha \geq \frac{1}{2} \), then there exists a unique \( \lambda^* \) such that the interval \([0, \lambda^*)\), under which tying generates positive returns in equilibrium; this contracts as \( \alpha \) decreases (i.e., as the weight attached to consumer surplus increases) (Figure 2b).

Figure 2: Impact of the incumbent firm’s decision to tie

Hence the situation for Firm 1, when it switches from a private firm to a public firm, has no effect on investment by entrant. However, it does affects Firm 1’s tying decision. Specifically, when Firm 1 pays more attention to profit (\( \alpha \geq \frac{1}{2} \)), it faces a trade-off between the positive and negative effect of tying. This is because consumers’ surplus is larger than zero only when both entrants succeed in innovation. This implies that tying has a negative effect on consumers’ surplus, since it decreases the probability of successful market entry. Recall that Firm 1’s profit is 0 in equilibrium, regardless of whether it chooses to tie or not. It follows that when \( \alpha < \frac{1}{2} \) the incumbent will choose not to tie, since tying can only have a negative effect on Firm 1 by reducing its consumer surplus requirement.

5 Social Welfare

When the incumbent is a private firm and the entrants are public enterprises, the socially optimal R&D investment expenditure level is given as the solution to the following
firms, tying leads to an improvement in social welfare if Proposition 4. 

Proposition 1, we know that public firms have more incentive to invest than private firms, and that the size of the entrants’ R&D investment increases as the size of the service obligation $\alpha$ increases. In the case of $\alpha \geq 1/2$, we denote R&D investment expenditure by entrants in no-tying situation and tying situation as $I_{a \geq 1/2}^*$ and $\tilde{I}_{a \geq 1/2}^*$ respectively; and in the case of $\alpha < 1/2$, we denote R&D investment expenditure by entrants in no-tying situation and tying situation as $I_{a < 1/2}^*$ and $\tilde{I}_{a < 1/2}^*$, respectively. We have that $I_{a < 1/2}^* > I_{a \geq 1/2}^* > \tilde{I}_{a \geq 1/2}^*$ and $I_{a < 1/2}^* > \tilde{I}_{a < 1/2}^* > \tilde{I}_{a \geq 1/2}^*$.

Hence, when $\alpha \geq 1/2$, a sufficiently large increase in R&D investment expenditure makes $S < p(\tilde{I})S + p(\tilde{I})n(V - 2\bar{c})(1 - \alpha)/\alpha$, implying that $I_{a \geq 1/2}^* > \tilde{I}_{a \geq 1/2}^* > I^S$. By contrast, for the case where $\alpha < 1/2$, when $\alpha$ is close to 0, a sufficiently large increase in R&D investment will make $S < p(\tilde{I}^*)S + p(\tilde{I}^*)n(V - 2\bar{c})(1 - \alpha)/\alpha < p(\tilde{I}^*)n(V - 2\bar{c})(1 - \alpha)/\alpha$. Hence, $I_{a < 1/2}^* > \tilde{I}_{a < 1/2}^* > I^S$.\(^7\) This implies that when the potential entrants are public firms, R&D investment expenditure realized under the sub-game perfect equilibrium is always higher than the social optimum. In this case, tying improves welfare by decreasing investment (see Figure 3, Graph (a)).

This result is now stated below formally in the following proposition:

**Proposition 4.** When the incumbent is a private firm and the entrant firms are public firms, tying leads to an improvement in social welfare if

$$S < p(\tilde{I})S + p(\tilde{I})n(V - 2\bar{c})(1 - \alpha)/\alpha.$$ 

---

\(^7\)When $\alpha \geq 1/2$, because $p(\tilde{I})S + \frac{1 - \alpha}{\alpha} p(\tilde{I})n(V - 2\bar{c}) \leq p(\tilde{I})[S + n(V - 2\bar{c})]$, where $S + n(V - 2\bar{c}) > S$ and $p(\tilde{I}) < 1$, the situation satisfies $S < p(\tilde{I})S + p(\tilde{I})n(V - 2\bar{c})(1 - \alpha)/\alpha$. Hence $I_{a < 1/2}^* > I_{a \geq 1/2}^* > \tilde{I}_{a \geq 1/2}^* > I^*$ and $I_{a < 1/2}^* > \tilde{I}_{a < 1/2}^* > \tilde{I}_{a \geq 1/2}^* > I^*$. Furthermore, if $\forall \alpha \in [1/2, 1)$, we have $S \geq p(\tilde{I}^*)S + p(\tilde{I}^*)n(V - 2\bar{c})(1 - \alpha)/\alpha$. In the case of $\alpha < 1/2$, when $\alpha$ is small enough, we have $S < p(\tilde{I}^*)S + p(\tilde{I}^*)n(V - 2\bar{c})(1 - \alpha)/\alpha < p(\tilde{I}^*)n(V - 2\bar{c})(1 - \alpha)/\alpha$ for sure, hence $I_{a < 1/2}^* > \tilde{I}_{a < 1/2}^* > I^*$ is valid.
Otherwise, if \( \alpha \geq 1/2 \) and

\[
S \geq [1 - \lambda + \lambda p(I^*)]S + p(I^*)n(V - 2\bar{c})(1 - \alpha)/\alpha,
\]

or if \( \alpha < 1/2 \) and

\[
S \geq [1 - p(I^*)](1 - \lambda)S + p(I^*)n(V - 2\bar{c})(1 - \alpha)/\alpha,
\]

then tying reduces social welfare.

To understand the impact of tying Firm 1 when the incumbent is a public firm and the entrants are profit maximizers, we compare Eqs. (15) and (16) with the first order condition of socially optimal problem given in Eq. (25). From Proposition 3, we know that at \( \alpha < 1/2 \), tying does not have an effect on the R&D investment of market entrants in the symmetric equilibrium. However, when \( \alpha \geq 1/2 \), tying will discourage rivals from investing in R&D and if firm 1 chooses it in equilibrium, it will reduce social welfare. This result is now stated in the following proposition:

**Proposition 5.** When the incumbent is public firm \( \alpha \geq 1/2 \) and the entrants are private firms, if tying occurs it will reduce social welfare.

Hence, in this case tying always reduces social welfare. This corresponds with results from Choi & Stefanides (2001), in which the both the incumbent and potential entrants are profit maximizers. In this case tying behavior by the incumbent firm leads to under investment in R&D by potential entrants and a loss of welfare. It is therefore possible use a consumer surplus obligation on entrant firms as a means of mitigating and reducing the impact of tying behavior by a privatized incumbent.
6 Conclusion

This paper begins by exploring the case of a profit maximizing incumbent facing market entry from two potential entrants, who both maximize a convex combination of profit and consumer surplus. The paper then examines the opposite case, where the incumbent is the public firm and the entrant firms are profit maximizers. Our results show, when entrants are public firms and the incumbent maximizes profits, the R&D investment expenditure by potential entrants is always higher when the incumbent firm decides not to tie. This occurs regardless of the level of consumer surplus obligation imposed on the entrant. We also find that the R&D investment expenditure by potential entrants is always higher, regardless of the tying decision of the incumbent. Furthermore, if the probability of investment success is small \( p(I^*) \leq 1/2 \), then the range of values at which tying is optimal decreases for the incumbent firm regardless of service requirement imposed on entrants.

By contrast, when the incumbent is a public firm and the entrants are private firms we find that the overall effect of tying is negative from the perspective of social welfare. In this case tying behavior by the incumbent firm leads to under investment in R&D by potential entrants and a loss of welfare. It is therefore possible to use a public service obligation on entrant firms as a means of mitigating and reducing the impact of tying behavior by a privatized incumbent. This is consistent with results from Choi & Stefanides (2001), in which the both the incumbent and potential entrants are profit maximizers; in this case tying always reduces social welfare.

This welfare result is different from the case when the incumbent is a private firm and the entrants are public firms. Our results show tying can improve welfare by reducing the R&D investment of potential entrants, when \( \alpha > 1/2 \). By contrast, when the service requirement of the incumbent is small (i.e., \( \alpha \geq 1/2 \)), tying decreases welfare by reducing the incentive of entrants to invest in R&D, which is the expected outcome. Taken from the perspective of social welfare, our findings show, when the incumbent monopolist is a private firm and the entrants are public firms, tying can ease the over-investment problem and increase social welfare when the public service obligation is large.

References


Appendix

Proof of Proposition 2

Recall that for Eq. (14), \( \lambda^* \) is the point at which the incumbent is indifferent between choosing tying and no-tying. At this point \( \Delta(\lambda^*, \alpha) = 0 \). The derivative of this equation on \( \alpha \) is given by

\[
\frac{d\Delta(\lambda^*, \alpha)}{d\alpha} = \frac{\partial \Delta(\lambda^*, \alpha)}{\partial \alpha} \frac{d\lambda^*}{d\alpha} + \frac{\partial \Delta(\lambda^*, \alpha)}{\partial \alpha} = 0, \tag{26}
\]

Recall, from Proposition 2 we have that \( \partial I^*(\lambda, \alpha)/\partial \alpha < 0 \), which implies that the R&D investment expenditure of potential entrants increases when they change from private firms to public firms. Because \( \partial \Delta(\lambda^*, \alpha)/\partial \lambda < 0 \), \( d\lambda^*/d\alpha \) must have the same sign as \( \partial \Delta(\lambda^*, \alpha)/\partial \alpha \). Thus, if \( \partial \Delta(\lambda^*, \alpha)/\partial \alpha > 0 \), then the interval of \([0, \lambda^*]\) under which the incumbent firm chooses tying must contract. On the other hand, if \( \partial \Delta(\lambda^*, \alpha)/\partial \alpha < 0 \), the interval \([0, \lambda^*]\) must increase.

Hence, in order to pin down this effect we must judge the sign of \( \partial \Delta(\lambda^*, \alpha)/\partial \alpha \). To do this we decompose this partial derivative as follows:

\[
\frac{\partial \Delta(\lambda, \alpha)}{\partial \alpha} = \left[ p(I^*)p'(I^*) \frac{\partial I^*}{\partial \alpha} - p(\tilde{I}^*)p'(\tilde{I}^*) \frac{\partial \tilde{I}^*}{\partial \alpha} \right] 2n(V - 2\tilde{\epsilon}) + 2p'(I^*) \frac{\partial I^*}{\partial \alpha} [2p(I^*) - 1] \lambda S \tag{27}
\]

First, we will determine the sign of part 1 of Eq. (27). Let

\[
F(\lambda, \alpha) = p(I^*(\lambda, \alpha))p'(I^*(\lambda, \alpha)) \frac{\partial I^*(\lambda, \alpha)}{\partial \alpha}.
\]

The partial derivative of \( F(\lambda, \alpha) \) on \( \lambda \) is given by:

\[
\frac{\partial F(\lambda, \alpha)}{\partial \lambda} = \left[ p''(I^*)p(I^*) + (p'(I^*))^2 \right] \frac{\partial I^*(\lambda, \alpha)}{\partial \alpha} \frac{\partial I^*(\lambda, \alpha)}{\partial \lambda} + p(I^*)p'(I^*) \frac{\partial^2 I^*(\lambda, \alpha)}{\partial \alpha \partial \lambda} \tag{28}
\]

The sign of part 1 of Eq. (28) is negative. As for part 2, we solve the partial derivative of \( \partial I^*(\lambda, \alpha)/\partial \alpha \) on \( \lambda^* \)

i) When \( \alpha \geq 1/2 \):

\[
sign \left\{ \frac{\partial^2 I^*(\lambda, \alpha)}{\partial \alpha \partial \lambda} \right\} = \text{sign} \left\{ \frac{\partial I^*}{\partial \lambda} \left[ p''(I^*)p(I^*) + (p'(I^*))^2 \right] \right\} \\
\left\{ \left( p''(I^*)p(I^*) + (p'(I^*))^2 \right) \left( \lambda S + \frac{1 - \alpha}{\alpha} n(V - 2\tilde{\epsilon}) \right) + (1 - \lambda)p''(I^*)S \right\} \\
- p'(I^*) \left[ p''(I^*)p(I^*) + 3p'(I^*)p''(I^*) \right] \frac{\partial I^*}{\partial \lambda} \left( \lambda S + \frac{1 - \alpha}{\alpha} n(V - 2\tilde{\epsilon}) \right) \\
+ (p''(I^*)p(I^*) - 1) + (p'(I^*))^2 \right) S + (1 - \lambda)p''(I^*)S \frac{\partial I^*}{\partial \lambda} \right\} \tag{29}
\]

18
With \( \partial I^*(\lambda, \alpha)/\partial \lambda < 0 \), if \( p''(I) \leq 0 \), then Eq. (29) is negative, hence \( \partial F(\lambda, \alpha)/\partial \lambda < 0 \). Because \( \tilde{I}^*(\alpha) = I^*(1, \alpha) \), the sign of part 1 of Eq. (27) is positive, which implies the positive effect of tying decreases as \( \alpha \) falls and the service requirement of potential entrants increases.

\[ \text{ii) When } \alpha < 1/2: \]

\[
sign \left\{ \frac{\partial^2 I^*(\lambda, \alpha)}{\partial \alpha \partial \lambda} \right\} = \text{sign} \left\{ \frac{\partial I^*}{\partial \lambda} \left( p''(I^*)p(I^*) + (p'(I^*))^2 \right) \right\}
\]

\[
= \text{sign} \left\{ \left( p''(I^*)(1-p(I^*)) - (p'(I^*))^2 \right) (1-\lambda)S \right.
\]

\[
+ \left( p''(I^*)p(I^*) + (p'(I^*))^2 \right) \frac{1-\alpha}{\alpha} n(V-2\bar{c}) \right\}
\]

\[
- p'(I^*)p(I^*) \left[ p''(I^*)(1-\lambda)S \frac{\partial I^*}{\partial \lambda} \right.
\]

\[
+ (p''(I^*)p(I^*) + 3p'(I^*)p''(I^*)) \frac{\partial I^*}{\partial \lambda} \left( \frac{1-\alpha}{\alpha} n(V-2\bar{c}) - (1-\lambda)S \right)
\]

\[
+p''(I^*)(1-p(I^*)) - (p'(I^*))^2 S \right) \right\}
\]

Because, for all \( \lambda \in [0, 1] \)

\[
\frac{1-\alpha}{\alpha} n(V-2\bar{c}) - (1-\lambda)S = \frac{1-\alpha}{\alpha} n(V-\xi - \bar{c}) + n(\bar{c} - \xi) \left( \frac{1}{\alpha} - 2 + \lambda \right)
\]

\[
> n(V-2\bar{c}) - (1-\lambda)S = n(V-\xi - \bar{c}) + n(\bar{c} - \xi)\lambda > 0,
\]

we have \( \frac{1-\alpha}{\alpha} n(V-2\bar{c}) \geq (1-\lambda)S \). With \( \partial I^*(\lambda, \alpha)/\partial \lambda < 0 \), if \( p''(I) \leq 0 \) and \( \frac{1-\alpha}{\alpha} n(V-2\bar{c}) \geq (1-\lambda)S \), then Eq. (30) is negative, hence \( \partial F(\lambda, \alpha)/\partial \lambda < 0 \). Because \( \tilde{I}^*(\alpha) = I^*(1, \alpha) \), the sign of part 1 of Eq. (27) is positive, which implies the positive effect of tying decreases as \( \alpha \) falls.

To get the sign of part 2 of Eq. (27), we can seen that if \( p(I^*) \leq 1/2 \), the sign of part 2 is positive as the negative effect of tying is increasing. In contrast, if \( p(I^*) > 1/2 \), then then sign of part 2 is negative, since the negative effect of tying decreases.

To sum up, if \( p''(I) \leq 0 \) and \( p(I^*) \leq 1/2 \), then \( \partial \Delta(\lambda^*, \alpha)/\partial \alpha > 0 \). When \( \alpha \) changes from being equal to 1 (corresponding to private firm) to smaller than 1 (corresponding to public firm with a mandatory service requirement), the interval of \( \lambda \) satisfying tying in equilibrium \([0, \lambda^*]\) narrows. However, if \( p''(I) \leq 0 \) and \( p(I^*) > 1/2 \), both the positive and negative effect of tying decreases, so the sign of \( \partial \Delta(\lambda^*, \alpha)/\partial \alpha \) is uncertain and depends on the relative size of the positive effect when compared to the negative effect. When

\[
[p(I^*)p'(I^*)(\partial I^*/\partial \alpha) - p(\tilde{I}^*)p'(\tilde{I}^*)(\partial \tilde{I}^*/\partial \alpha)]2n(V-2\bar{c}) > p'(I^*)(\partial I^*/\partial \alpha)[1 - 2p(I^*)]\lambda S,
\]

the positive effect dominates and \( \partial \Delta(\lambda^*, \alpha)/\partial \alpha > 0 \). If this occurs, then the interval of
\( \lambda \) in which tying is optimal narrows. In contrast, when

\[
[p(I^*)p'(I^*)(\partial I^*/\partial \alpha) - p(\tilde{I}^*)p'(\tilde{I}^*)(\partial \tilde{I}^*/\partial \alpha)]2n(V - 2\bar{e}) < p'(I^*)(\partial I^*/\partial \alpha)[1 - 2p(I^*)]\lambda S,
\]

then \( \partial \Delta(\lambda^*, \alpha)/\partial \alpha < 0 \), the negative effect dominates, thus the interval in which tying is optimal expands. Moreover, if \( p(I^*) \) is small enough and satisfies \( p(I^*) \leq 1/[2 + n(V - 2\bar{e})/\lambda S] \), then the interval \([0, \lambda^*] \) decreases without the condition of \( p''(I) \leq 0 \).

### Proof of Proposition 3

There are two cases to consider:

**Case 1:** If \( \alpha < 1/2 \), then for all \( \lambda \in [0, 1] \), we have \( \Delta(\lambda^*, \alpha) < 0 \). Thus firm 1 chooses no-tying in equilibrium (see Figure 3, Graph (a)).

**Case 2:** If \( \alpha \geq 1/2 \), the first part of Eq. (23),

\[
\left[ p(I^*(\lambda))^2 - p(\tilde{I}^*)^2 \right](2\alpha - 1)n(V - 2\bar{e}) > 0,
\]

and the second part \([-2p(I^*(\lambda))[1 - p(I^*(\lambda))]\alpha \lambda S < 0 \). To find \( \lambda^* \), the critical level of surplus where \( \Delta(\lambda^*, \alpha) = 0 \), we note that \( \Delta(0, \alpha) > 0 \) and \( \Delta(1, \alpha) < 0 \) with \( I^*(1) = \tilde{I}^* \).

Since \( \Delta(\lambda, \alpha) \) is continuous with respect to \( \lambda \in (0, 1) \), there exists at least one \( \lambda^* \) such that \( \Delta(\lambda^*, \alpha) = 0 \). If \( p(I^*) \geq 1/2 \), then it can be shown that \( W^*_1(\lambda, \alpha) \) (and \( \Delta(\lambda, \alpha) \)) is monotonically increasing with respect to \( \lambda \). However, when \( p(I^*) < 1/2 \), there are multiple equilibria.

From the proof of Proposition 1, we know \( \partial \lambda^*/\partial \alpha \) has same sign with \( \partial \Delta(\lambda^*, \alpha)/\partial \alpha \). So we can get the relation between \( \lambda^* \) and \( \alpha \) from the sign of \( \partial \Delta(\lambda^*, \alpha)/\partial \alpha \):

\[
\frac{\partial \Delta(\lambda, \alpha)}{\partial \alpha} = \left[ p(I^*(\lambda))^2 - p(\tilde{I}^*)^2 \right]2n(V - 2\bar{e}) - 2p(I^*(\lambda))[1 - p(I^*(\lambda))]\lambda S (31)
\]

To get the sign of Eq. (31), we note that \( \lambda^*_{pr} \) satisfies \( \left[ p(I^*(\lambda^*_{pr}))^2 - p(\tilde{I}^*)^2 \right]n(V - 2\bar{e}) - 2p(I^*(\lambda^*_{pr}))[1 - p(I^*(\lambda^*_{pr}))]\lambda^*_{pr}S = 0 \). When this is substituted into Eq. (31), we have \( \partial \Delta(\lambda^*_{pr}, \alpha)/\partial \alpha > 0 \). Because \( \partial \Delta(0, \alpha)/\partial \alpha > 0, \partial \Delta(1, \alpha)/\partial \alpha < 0 \), and \( \partial \Delta(\lambda, \alpha)/\partial \lambda < 0 \), it exists unique \( \lambda^o \in (0, 1) \), which satisfies \( \partial \Delta(\lambda^o, \alpha)/\partial \alpha = 0 \) and \( \lambda^o > \lambda^*_{pr} \).

To sum up, we have if \( p(I^*) \geq 1/2 \):

1. For all \( \lambda < \lambda^o \), \( \partial \Delta(\lambda, \alpha)/\partial \alpha > 0 \). When \( \alpha \) decreases from 1 to the interval of \([1/2, 1) \), \( \Delta(\lambda, \alpha) \) decreases. Thus the interval of \( \lambda \) satisfying tying in equilibrium narrows as \( \alpha \) decreases. Some part of interval of \( \lambda \) satisfying tying in equilibrium turns to the interval satisfying no-tying in equilibrium when firm 1 changes from private firm to public firm.

2. For all \( \lambda \geq \lambda^o \), \( \partial \Delta(\lambda, \alpha)/\partial \lambda \leq 0 \). When \( \alpha \) decreases from 1 to the interval of \([1/2, 1) \), \( \Delta(\lambda, \alpha) \) increases. However, \( \lim_{\alpha \to 1/2} \Delta(\lambda^o, \alpha) < 0 \), for for all \( \lambda \). Hence, if
firm 1 is a private firm, it will still choose no-tying in equilibrium even when \( \alpha \) is decreasing and close to 1/2 (see Figure 3, Graph (b)).

Thus, if \( \alpha \geq 1/2 \), the interval of \( \lambda \) satisfying tying in equilibrium narrows and the degree increases when \( \alpha \) decreases.