Optimal Composition of the Public Spending and Economic Growth*

Jose Angelo Divino† Daniel T. G. N. Maciel‡ Wilfredo Sosa§

Abstract

The objective of this paper is to investigate the relationship among size of the government, composition of public spending, and economic growth. We expand the theoretical model due to Devarajan et al (1996) by including technological progress in a more general constant elasticity of substitution (CES) production function. In addition, we use a balanced panel data for the Brazilian states to estimate the model’s structural parameters and compute optimal ratios derived from the theoretical modeling. We found that private capital and government spending are substitute inputs in the production, while productive and unproductive expenditures are combined in a fixed ratio in the aggregate government spending. The public spending in investment is considerably lower than in costing, as occurs in developing countries with low economic dynamism. Finally, the average tax burden from the data is below the estimated optimal level, meaning that there is space for increasing taxation without hurting economic growth for some Brazilian states.

Keywords: Public spending; Optimal taxation; Economic growth.

JEL Codes: O41; H50.

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1 Introduction

Several countries around the world have recently faced episodes of fiscal crises due to the incapacity of their governments to bridge a deficit between public expenditures and tax revenues.¹ These crises share some common features, given that they are usually accompanied by economic, social, and political distresses and the recovery is painful to the society as hole because it simultaneously requires to cut government spending and to increase tax on individuals and firms. Due to the relevance of the fiscal policy to a country’s economic performance, it is important to keep an eye on both the relationship between the size of government and economic growth and the effects of the composition of the public expenditure on the country’s growth rate. The latter issue rests on the fact that some public expenditures are seen as productive while others are considered unproductive in terms of their impacts on the economic activity. Thus, under this perspective, a country would be able to improve its economic performance by changing the mix between these two kinds of public expenditures.

The empirical and theoretical literatures have devoted a considerable amount of work to analyze the relationship among the size of the government, composition of the public expenditure, and economic growth. Aschauer (1989), Lindauer and Velenchik (1992), Barro (1990, 1991), and Gupta et al. (2005), for instance, investigated the impacts of government spending on economic growth and productivity. In a pioneer study, Devarajan et al (1996) analyzed the relationship between composition of public expenditure and economic growth using both theoretical and empirical frameworks. Davoodi and Zou (1998), Xie et al (1999), and Zhang and Zou (1998) examined the growth effects of aggregate public expenditure by different levels of government in a fiscal-federalism environment. Finally, Zhang and Zou (2001) unified the previous literature by focusing on the growth impacts of the allocation of public expenditure among multiple sectors (such as health, education, transportation, among others) with multiple levels of government (such as local, state, and federal).

A further debate on the relationship between government size and economic growth relates the quality of government services and the efficiency of the tax systems to the optimal government size. The empirical results, however, are rather controversial. Agell et al (1999) and Agell et al (2006),

¹The cases of Greece, Portugal, Italy, Spain, and more recently Brazil, for instance, are well documented by the general media.
for instance, find a non-significant relationship between government size and economic growth while Fölster and Henrekson (1999, 2001, 2006) and Bergh and Henrekson (2011) argue that there is a negative relationship between government expenditure and growth. According to Bose et al. (2007), the share of government spending in capital in GDP is positively correlated with economic growth, but current expenditure is insignificant. Ghosh and Gregoriou (2008), on the contrary, find that current government spending increases growth while capital spending decreases it for a sample of developing countries. It is still missing in the literature, however, studies on the optimal size of the government and the optimal composition of the public expenditure.

For the Brazilian case, Rocha and Giuberti (2007) and Divino and Silva Jr. (2012) provide empirical evidence on the optimal composition of the public expenditure for states and municipal districts, respectively. Both authors do not impose any a priori restriction on the productivity of the public expenditure and find that the optimal share of the current spending should range from 61 to 81% of the total public spending. Rodrigues and Teixeira (2010) argue that government expenditure in capital at the state level is more effective to affect economic growth than at the national or municipal district levels. Silva and Thiches (2014) desegregate the public spending and find that public expenditures on infrastructure, health, communication, and sanitation are positively correlated with the aggregate output. However, no attempt was made by any of these studies to model the relationship among optimal size of the government, composition of the public spending, and economic growth.

The objective of this paper is to fill this gap by investigating theoretical and empirically the relationship among optimal size of the government in the economy, optimal shares of productive and unproductive public expenditures in the aggregate government spending, and economic growth. We extend the framework proposed by Devarajan et al (1996) by including an exogenous technological progress in a more general constant elasticity of substitution (CES) production function and we show how those optimal shares and government size depend on the structural parameters of the economy. We focus on economic growth instead of other measure of welfare for comparison purposes with the results by Devarajan et al (1996) and because it is important to identify the contribution of different components of the public expenditure to the economic growth.

Our major contribution is to show that the optimal size of the government in the economy, defined as the level of aggregate public expenditure over GDP that maximizes consumption growth, depends on the model’s
structural parameters and technological progress. In addition, we find that
the growth rate of consumption is inversely related with both the individ-
uals’ degree of impatience and the aggregate tax rate. For the Brazilian
states, the average optimal size is around 19% under a general specification
of the model. Considering the special case of a Cobb-Douglas production
function, that optimal size negatively depends on the share of the capital in
the production function.

The optimal share of productive public expenditure relatively to the un-
productive spending, on its turn, depends on the share of the productive
spending and the elasticity of substitution between productive and unpro-
ductive public spending in the CES production function of the government.
Considering the case of a Cobb-Douglas, that optimal share depends ex-
clusively on the share of the productive expenditure on the aggregate gov-
ernment spending. Thus, it is crucial to have good estimates for these
parameters in order to calculate optimal ratios for any given country.

In the general case, however, the optimal taxation depends on the whole
set of structural parameters. In particular, the technological progress has a
direct effect on the optimal level of taxation while the share of private capital
in total production has a negative effect on taxation. This theoretical finding
coincides with the empirical results obtained for the Brazilian economy.

In order to estimate the structural parameters and find optimal ratios,
we applied the model to a panel data for the Brazilian states in the re-
cent period. A general CES production function, which combines private
capital, composition of public expenditure, and technological progress, was
estimated at the state level. We found that private capital and government
spending are substitute inputs in the production, while government expendi-
tures in investment and costing are combined in a fixed ratio due to the rigid-
ity of the public budget imposed by legal requirements. We computed the
composition of public spending, level of taxation, and economic growth im-
plied by the estimated structural parameters for the Brazilian states. Then,
we calculated the level of taxation and composition of the public expendi-
ture that maximize the economic growth. We also performed a sensitivity
analysis of the private capital productivity and average economic growth
with respect to changes in the composition of public expenditure and tax
rate.

The paper is organized as follows. The next section presents the model
economy and derives the theoretical results. The empirical evidence for the
Brazilian economy is reported and discussed in the third section. Finally,
the fourth section is dedicated to the concluding remarks.
2 The model

The theoretical framework is based on Devarajan et al (1996), whose model is extended to consider a general CES (constant elasticity of substitution) function for both the government aggregate expenditure and the economy production technology under a minimal set of restrictions on the parameter values. In addition, we add technological progress to the aggregate CES production function. Thus, our production function depends on private capital stock, $k$, aggregate government government spending, $x$, and exogenous technological progress, $A$. They are combined into a CES function expressed as:

$$y_t = A_t \left[ \alpha k_t ^{-\zeta} + (1 - \alpha) x_t ^{-\zeta} \right]^{-\frac{1}{\zeta}}$$

with $1 \geq \alpha > 0, \ \zeta \in (-1, 0) \cup (0, +\infty)$.

The aggregate government spending, $x$, is also given by a CES function, which combines two types of government spending, $g_1$ and $g_2$. As in Devarajan et al (1996), these expenditures are considered as productive, $g_1$, and unproductive, $g_2$, to reflect empirical findings in the literature. The government finances its spending by levying a flat-rate income tax, $\tau$. The following equations represent these relationships:

$$x_t = \left[ \alpha_1 g_{1t} ^{-\zeta_1} + (1 - \alpha_1) g_{2t} ^{-\zeta_1} \right]^{-\frac{1}{\zeta_1}}$$

and

$$\tau y_t = g_{1t} + g_{2t}$$

where $1 \geq \alpha_1 \geq 0, \ \zeta_1 \in (-1, 0) \cup (0, +\infty), \ 0 < \tau < 1$.

The private capital stock, $k$, follows a standard law of motion:

$$\dot{k}_t = (1 - \tau) y_t - c_t$$

The representative consumer chooses consumption, $c$, and capital, $k$, to maximize the discounted value of utility, which has the isoelastic form of a

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2. As in Devarajan et al (1996), labor does not enter directly in the production function.

3. A major find is that output growth is negatively correlated with the share of government consumption in GDP, usually classified as unproductive government spending, and positively correlated with public investment, which is viewed as productive government spending. See, for instance, Bose et al. (2007), Aschauer (1989), Barro (1990, 1991), and Devarajan et al (1996). Ghosh and Gregoriou (2008) find results in the opposite direction. But still, the aggregate government spending might be represented by a combination of productive and unproductive government expenditures.
constant relative risk aversion (CRRA) function:

\[
\int_0^{+\infty} \frac{c^{1-\sigma} - 1}{1 - \sigma} e^{-\rho t} dt
\]

subject to equations (1) to (4) and with \( \sigma > 0, \sigma \neq 1, \rho > 0 \). The functions \( A, c, k, y, g_1 \) and \( g_2 \) are defined on \([0, +\infty)\) with positive values (i.e.: \([0, +\infty) \rightarrow (0, +\infty)\)).

By substituting equation (2) in (1), we have that:

\[
y_t = A_t \left[ \alpha k_t^{-\zeta} + (1 - \alpha) \left( \alpha_1 g_1 t^{-\zeta_1} + (1 - \alpha_1) g_2 t^{-\zeta_1} \right) \right]^{-\frac{1}{\zeta}} \tag{6}
\]

which is still a CES function.

### 2.1 Theoretical Results

The solution of the consumer’s problem under the previous parametrization, and also in the special case of a Cobb-Douglas production function, allows us to derive our major findings. They are reported in the following sequence of Lemmas and Theorems.

**Lemma 1.** There exists \( \phi : [0, +\infty) \rightarrow [0, 1] \) such that \( g_{1t} = \phi_t \tau y_t \) and \( g_{2t} = (1 - \phi_t) \tau y_t \) \( \forall t \in [0, +\infty) \).

*Proof.* Follows from the fact that \( \tau y_t = g_{1t} + g_{2t} \) \( \forall t \in [0, +\infty) \). \( \square \)

**Theorem 1.** If \( \text{Im}(\phi) \subset (0, 1) \), then there exists \( \delta : [0, +\infty) \rightarrow (0, 1) \cup (1, +\infty) \) defined by

\[
\delta_t = \left[ \alpha_1 \phi_t^{-\zeta_1} + (1 - \alpha_1)(1 - \phi_t)^{-\zeta_1} \right]^{-\frac{1}{\zeta_1}} \tag{7}
\]

such that:

1. \( x_t = \tau \delta_t^{-\frac{1}{\zeta}} y_t \).
2. \( \delta_t \in (0, 1) \) when \( \zeta \in (-1, 0) \).
3. \( \delta_t > 1 \) when \( \zeta > 0 \).

*Proof.* See the Appendix. \( \square \)
The aggregate public spending, $x_t$, is generated by $\tau \delta_{t-1}^\frac{1}{\zeta}$ multiplied by the output of the economy. A special case emerges when $\zeta_1 \rightarrow 0$, which implies that $\delta = 1$, $x_t = \tau y_t$, and $x_t = g_1t + g_2t$. In addition, when $\zeta \in (-1, 0)$, $k_t$ and $x_t$ are substitutes inputs in the production. When $\zeta > 0$, they are complimentary inputs.

**Theorem 2.** If $\text{Im}(\phi) \subset (0,1)$, then there exists $\theta : [0, +\infty) \rightarrow (0, +\infty)$ defined by

$$\theta_t = \left[\frac{(A_t \tau)^\zeta - (1 - \alpha) \delta_t}{\tau^c \alpha}\right]^{-\frac{1}{\zeta}}$$

such that $k_t = \theta_t y_t \forall t \in [0, +\infty)$. If $A_t = 1$, then $\theta_t > 1$.

**Proof.** See the Appendix. □

Thus, each unit of private capital generates $\frac{1}{\theta}$ units of output in the economy. In other words, $\frac{1}{\theta}$ represents the private-capital productivity. Notice that $(A_t \tau)^\zeta - (1 - \alpha) \delta_t > 0$, because functions $k$ and $y$ are positive.

This means that this model collapses to a traditional $Ak$ growth model. In the special case of a Cobb-Douglas production function, where $\zeta = 0$ and $\zeta_1 = 0$, we have that both $\delta_t = 1$ and $\theta_t = 1$.

**Theorem 3.** If $\text{Im}(\phi) \subset (0,1)$ and $c : [0, +\infty) \rightarrow (0, +\infty)$ is a continuously differentiable function, then the the growth rate of consumption is:

$$\lambda_t = \frac{\dot{c}_t}{c_t} = \frac{1 - \tau}{\sigma \theta_t} - \frac{\rho}{\sigma}$$

**Proof.** See the Appendix. □

**Corollary 1.** If $\frac{1 - \tau}{\rho} \leq \theta_t$, then $\lambda_t \leq 0$.

**Corollary 2.** If $\frac{1 - \tau}{\rho} > \theta_t$, then $\lambda_t > 0$.

Theorem 3 states that the growth rate of consumption in this economy might be positive or negative, depending on $\frac{1 - \tau}{\rho}$ and $\theta_t$ according to Corollaries 1 and 2. In order to keep a positive growth rate of consumption, $\lambda_t > 0$, the economy should be characterized by low degree of impatience and low tax rate, such that $\frac{1 - \tau}{\rho} > \theta_t$. In this environment, the representative agent would be willing to transfer consumption across time and the after-tax income would be sufficiently high to allow for this transference. In the special case of a Cobb-Douglas, we have that $\lambda_t > 0$ only if $\tau + \rho < 1$, given that $\lambda_t = \frac{(1 - \tau - \rho)}{\sigma}$ in this case.
Next, in Lemma 2, we define the optimal share for productive (and unproductive) government spending also as a function of the economy structural parameters.

**Lemma 2.** If \( \text{Im}(\phi) \subset (0, 1) \), then \( \theta \) as a function of \( \phi \) attains its minimum value at \( \phi^* = \frac{\alpha_1 \frac{1}{\zeta_1+1}}{(1-\alpha_1)^{\frac{1}{\zeta_1+1}}+\alpha_1^\frac{1}{\zeta_1+1}} \).

**Proof.** See the Appendix.

Essentially, the optimal share of the productive public expenditure, \( \phi^* \), depends on \( \alpha_1 \) and \( \zeta_1 \). In the special case of a Cobb-Douglas production function, this optimal share simplifies to \( \phi^* = \alpha_1 \). This result is intuitive because the higher is the elasticity of the productive expenditure in the aggregate public spending, \( x_t \), the higher will be the optimal share of this productive expenditure.

**Theorem 4.** The maximizer of the growth rate consumption is the minimizer of \( \theta \) as a function of \( \phi \).

**Proof.** Since the growth rate consumption is

\[
\lambda_t = \frac{\dot{c}_t}{c_t} = \frac{1 - \tau}{\sigma \theta_t} - \frac{\rho}{\sigma} \tag{10}
\]

The statement follows because the maximum value of \( \lambda \) is achieved at the same \( \phi^* \) that \( \theta \) achieves its minimum value with respect to \( \phi \).

Now, we are able to find the optimal tax rate, \( \tau^* \), defined as the level of \( \tau \) that is compatible with the maximum growth rate of consumption. Notice that, by equation (3), this optimal \( \tau \) also corresponds to the optimal size of the government in the economy.

**Theorem 5.** Function \( k \) increases when \( \tau \) decreases.

**Proof.** Since \( k_t = \theta_t y_t \), the statement follows because

\[
\frac{\partial \theta}{\partial \tau} = -\frac{1 - \alpha}{\alpha} \left[ \frac{\theta_t}{\tau} \right]^{\zeta + 1} \delta_t \tag{11}
\]

**Theorem 6.** The maximizer of the growth rate consumption as a function of \( \tau \) is \( \tau^* = \left[ \frac{(1-\alpha) \delta_t}{\delta_t} \right]^{\frac{1}{\zeta+1}} \).
Proof. See the Appendix. □

Finally, we state a stylized relationship between consumption and technological progress, which also emerges in the current model.

**Theorem 7.** The growth rate of consumption, \( \lambda \), is an increase function of \( A \).

**Proof.** This follows from the fact that

\[
\frac{\partial \lambda}{\partial A} = \left[ \frac{1 - \tau}{\alpha \sigma} \right] (\theta_t A_t)^{\zeta_1 - 1} \tag{12}
\]

\[\square\]

## 3 Empirical Evidence

### 3.1 Econometric Model

We estimate, by nonlinear least squares, the aggregate CES function which results from the combination of equations (1) and (2), as given by equation (6). Our goal is to identify structural parameters of the model in order to compute elasticities of substitution, economic growth rate, optimal size of the government, and other optimal ratios implied by the theoretical modeling.\(^4\) In a panel data environment, considering the data set that will be used in the estimation, equation (6) might be written as:

\[
y_{jt} = A_{jt} \left[ \alpha k_{jt}^{\zeta_1} + (1 - \alpha) \left[ (1 - \eta_t) g_{1jt}^{\zeta_1} + \eta_t \eta_1^{\zeta_1} \right] \right]^{\frac{1}{\zeta_1}} + \varepsilon_{jt} \tag{13}
\]

This equation is equivalent to (6) by making \( (1 - \eta_t) = \alpha_1 \) as the share of public spending in investment. The structural parameter \( \alpha \) defines the share of private capital in the output while \( (1 - \alpha) \) represents the share of the aggregate government spending in the output. In addition, \( \eta_t \) is the share of current spending and \( (1 - \eta_t) \) is the share of public investment in the aggregate government expenditure\(^5\). The parameters \( \zeta \) and \( \zeta_1 \) generate elasticities

\(^4\)We do not estimate alternative functional forms for the production function because they would not allow us to identify the parameters of interest. As stated in the text, the popular Cobb-Douglas production function emerges as a special case of (6). In addition, our key results from Theorems 1 and 2 follow if \( x_t = x_t(y_t) \) and \( k_t = k_t(y_t) \), and both Theorems need only the homogeneity property, which is satisfied by both Cobb-Douglas and Translog production functions.

\(^5\)(1 - \( \eta_t \)) is the share of public investment because \( g_t \) does not include expenses with interest rate and debt rollover.
of substitution\footnote{Because it is a CES function, the elasticities of substitution are given by $\psi = \frac{1}{1+\zeta}$ and $\psi_1 = \frac{1}{1+\zeta}$.} and $\varepsilon$ is an additive error term, assumed to be homoscedastic and independent and identically distributed.

The heterogeneous technological progress in each state is $A_{jt}$. Following Duffy and Papageorgiou (2000), it is modeled by:

$$A_{jt} = \exp(\gamma p_i + \nu t)$$

We use the cumulative distribution of patents across the Brazilian states to create a dummy variable for the $i-th$ group of the least productive ones in terms of number of registered patents. These states are considered as having low degree of technological progress. They will have an output smaller than the average whenever $\gamma < 0$.\footnote{We considered $i = 1\%$ and $i = 5\%$ in the estimation, but the results were similar. We also used other variables, such as number of patents, average of registered patents and average of registered patents in the last 5 years. However, we had converge problems. Thus, we chose to work with $i = 5\%$.} Given that the technological progress might change over time, $\nu$ estimates this potential time effect.

Finally, as a robustness check, we considered both level and per capita variables in the estimation. Thus, we estimate four regression models, described as follows:

1. Model (A): Without technological progress, such that $A_{jt} = 1$ for all states;
2. Model (B): With technological progress, such that $i = 5\%$ of the Brazilian states that least registered patents;
3. Model (C): Similar to model (A), but with per capita variables;
4. Model (D): Similar to model (B), but with per capita variables.

3.2 Data

We use a balanced panel composed by the 27 Brazilian states in the period from 2004 to 2010 with annual data, totalizing 189 observations for each variable.\footnote{The sample is limited to this time period due to data availability for computation of the private capital, $k$.} The nominal variables were deflated by the wide consumer price index (Indice Nacional de Preços ao Consumidor Amplo - IPCA), which is calculated by the Brazilian Institute of Geography and Statistics (IBGE).
and used as metric by the Central Bank of Brazil in the inflation targeting regime. The variables are described in sequence.

- \( y \): Gross domestic product (GDP) of each Brazilian state released by Ipeadata.9

- \( g_1 \): Government spending in investment by each Brazilian state, which consists of total capital spending minus payment of interest rate and debit rollover. The source was Ipeadata.

- \( g_2 \): Government spending on costing (or current spending) by each Brazilian state, also collected from the Ipeadata.

- \( k \): Stock of private capital of each Brazilian state, computed according to the methodology proposed by Sanches and Rocha (2010).

The technological progress of each Brazilian state was measured by the amount of registered patents provided by the National Institute of Intellectual Property (Instituto Nacional da Propriedade Industrial - INPI). We create a dummy variable, \( p_5 \), to represent 5% of the Brazilian states that least registered patents in each time period. Thus, \( p_5 = 1 \) when state \( j \) belongs to these 5% that least registered patents according to the cumulative distribution of patents and \( p_5 = 0 \) otherwise.

### 3.3 Empirical Results

Initially, we tested the panel data for the presence of unit root. We applied the first-generation tests due to Fisher-ADF, Levin-Lin-Chu (LLC), and Im-Pesaran-Shin (IPS). However, as these tests do not take into account cross-sectional correlations, which are likely in the case of sub-national governments, we also applied second-generation tests Hadri and PESADF due to Hadri (2000) and Pesaran (2007), respectively. These tests are robust to heterogeneous panels with cross-sectional dependence. The Hadri’s test has the null hypothesis of stationarity while all others have the null of unit root. The results, reported in Table 1 for both level and per capita variables, indicated that the panel is stationary at the standard 5% significance level.10 Thus, the non-linear estimation of equation (13) might be carried out with all variables in levels or per capita. Initial values of the parameters were set at 0.0001 in order to allow for convergence to either positive or negative values.

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9www.ipeadata.gov.br.

10 The exceptions are the tests IPS for \( g_1 \) in level and PESADF for \( k \) per capita, which did not reject the null of unit root. However, based on the results of the other four tests, we also considered these variables as stationary.
Table 1: Panel data unit root tests

<table>
<thead>
<tr>
<th>Tests</th>
<th>Level variables</th>
<th>Per capita variables</th>
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</thead>
<tbody>
<tr>
<td>ADF</td>
<td>-3.93**</td>
<td>-2.54**</td>
</tr>
<tr>
<td>LLC</td>
<td>-24.61**</td>
<td>-84.36**</td>
</tr>
<tr>
<td>IPS</td>
<td>-3.47*</td>
<td>-3.87*</td>
</tr>
<tr>
<td>Hadri</td>
<td>1.03</td>
<td>0.19</td>
</tr>
<tr>
<td>PESADF</td>
<td>-71.99**</td>
<td>-6.15**</td>
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</table>

<table>
<thead>
<tr>
<th>Tests</th>
<th>y</th>
<th>k</th>
<th>g1</th>
<th>g2</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF</td>
<td>-12.08**</td>
<td>-2.96**</td>
<td>-3.69**</td>
<td></td>
</tr>
<tr>
<td>LLC</td>
<td>-20.81**</td>
<td>-19.60**</td>
<td>-20.59**</td>
<td></td>
</tr>
<tr>
<td>IPS</td>
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<td>-1.11</td>
<td>-2.75**</td>
<td></td>
</tr>
<tr>
<td>Hadri</td>
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<td>1.38</td>
<td>0.66</td>
<td></td>
</tr>
<tr>
<td>PESADF</td>
<td>-44.14**</td>
<td>-1.84*</td>
<td>-9.25**</td>
<td></td>
</tr>
</tbody>
</table>

Source: Estimated by the authors. Notes: ** and * indicate statistical significance at the 1 and 5% levels, respectively. Hadri and PESADF account for cross-sectional dependence. The null hypothesis of the Hadri’s test is stationarity, while for all others is unit root.

Due to individual heterogeneity of the Brazilian states, which might lead to heteroscedasticity in the residuals, we performed a correction in the variance-covariance matrix for robust standard errors in clusters. However, for the results reported in Table 2, the coefficients that were statically significant at the 1% significance level maintained this significance with or without that correction.

The parameter $\nu$ in equation (14), which represents the deterministic component of the technological progress, was not statistically significant in any estimation. This might be due to short time horizon of the sample. Thus, the time effect was excluded from the estimations and only estimated values for $\gamma$ are reported in Table 2. As mentioned before, the first two estimated models refer to variables in levels while the last two use per capita variables. In addition, models A and C assume no technological progress across the Brazilian states ($A_{jt} = 1$), while models B and D estimates technological progress according to equation (14) with $\nu = 0$.

The share of private capital in the total output, $\alpha$, was estimated above 65% in all models, indicating that private capital is more important than the government’s compound spending in the aggregate production of the Brazilian states. This high estimated value might also be capturing the effect of labor, which was not explicitly included in the production function (1) based in Devarajan et al (1996). The negative estimated value for $\zeta$ points out that private investment and government spending are substitute inputs.
Table 2: Non-linear estimation

<table>
<thead>
<tr>
<th>Parameters</th>
<th>(A)</th>
<th>(B)</th>
<th>(C)</th>
<th>(D)</th>
</tr>
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<tbody>
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<td>( \alpha )</td>
<td>0.660***</td>
<td>0.659***</td>
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<td>0.689***</td>
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<td></td>
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<td>(0.072)</td>
<td>(0.075)</td>
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<td>( \zeta )</td>
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<td>-0.290***</td>
<td>-0.303***</td>
<td>-0.319***</td>
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<td>(0.065)</td>
<td>(0.068)</td>
<td>(0.049)</td>
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<tr>
<td>( \eta_1 )</td>
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<td>0.853***</td>
<td>0.989***</td>
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<td>(0.163)</td>
<td>(0.042)</td>
<td>(0.077)</td>
</tr>
<tr>
<td>( \zeta_1 )</td>
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<td>0.210</td>
<td>1.270</td>
<td>0.203</td>
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<td></td>
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<td>(0.708)</td>
<td>(1.528)</td>
<td>(0.340)</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>NA</td>
<td>-0.634***</td>
<td>NA</td>
<td>-0.680***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.086)</td>
<td></td>
<td>(0.101)</td>
</tr>
</tbody>
</table>

R-squared: 0.965 0.972 0.968 0.990
Adj. R-squared: 0.964 0.971 0.967 0.990

Source: Estimated by the authors. Notes: ***, **, and * indicate that the estimated coefficient is statistically significant at the 1, 5, and 10% levels, respectively.

in the production. This suggest that the ratio of these inputs utilization in the production changes more than proportionately to any change in the ratio of their marginal products, but we are not able to identify in which direction flows these changes.\(^\text{11}\)

Considering the composition of the government spending, \( \eta_1 \) is always above 85% meaning that the government spends a larger fraction of the public budget on current spending \((g_2)\) than in public investment \((g_1)\). This finding is in line with the fact that public investment is still a small fraction of public spending in the Brazilian economy. The coefficient \( \zeta_1 \) was not statistically significant at the 5% level, suggesting that the public spending composition, \( x_t \), might be represented by a Cobb-Douglas function, which is a special case of the general CES assumed in equation (2). This means that \( g_1 \) and \( g_2 \) are combined in a fixed ratio in the aggregate government spending, independently of the ratio of their marginal products. This results reflects the rigidity of the public budget of the Brazilian states, where the shares of several expenditures are fixed by law.

The Brazilian states classified as having lower technological progress according to the amount of registered patents presented a negative and statistically significant value for \( \gamma \). This means that they have a smaller output than the other states, which are in the group that registered more than 5% of the patents in each time period. Thus, the states contained in \( p_5 \) are

\(^{11}\)The elasticity of substitution, \( \psi \), ranges from 1.403 in model A to 1.468 in model D.
expected to have an output which is, on average, about 49 and 47% smaller than the states that are outside $p_5$ according to models (B) and (D), respectively.\textsuperscript{12} This finding is in line with the literature, according to which the less technologically developed economies are also the ones with lower levels of production.

To find the growth rate of consumption, $\lambda$, and the other compound parameters implied by the theoretical model, we need to set values for the risk aversion coefficient $\sigma$ and the intertemporal discount factor, $\rho$, in addition to the estimated coefficients from Table 2. Notice that the preference of the government for investment spending is $\alpha_1 = (1 - \eta_1)$. We use the values of $\sigma = 4.89$ and $\rho = 0.123$, obtained from the empirical literature that estimates utility functions parameters for the Brazilian economy.\textsuperscript{13} With these values, we can compute the average growth rate of consumption, $\overline{\lambda}$, the ratio of private capital to output, $\overline{\theta}$, and the deviations from the predicted $\overline{\lambda}$ to the observed $\overline{\lambda}$ from the data, as reported in Table 3.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>(A)</th>
<th>(B)</th>
<th>(C)</th>
<th>(D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>0.113</td>
<td>0.147</td>
<td>0.011</td>
<td>0.139</td>
</tr>
<tr>
<td>$\overline{\theta}$</td>
<td>2.151</td>
<td>3.453</td>
<td>1.920</td>
<td>3.148</td>
</tr>
<tr>
<td>$\overline{\lambda}$</td>
<td>0.060</td>
<td>0.033</td>
<td>0.070</td>
<td>0.040</td>
</tr>
<tr>
<td>Mean deviation</td>
<td>-0.010</td>
<td>0.017</td>
<td>-0.033</td>
<td>0.003</td>
</tr>
<tr>
<td>Mean–square deviation</td>
<td>0.000</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Source: Calculated by the authors.

The inverse of $\overline{\theta}$ measures the productivity of the private capital in the economy. Thus, the lower $\overline{\theta}$ the more productive is the private capital. Table 3 indicates that this productivity ranges from 0.52 to 0.29, depending on the estimated model.

Comparing the estimated models with and without technological progress, model (A) yields the smallest mean-squared deviation from $\overline{\lambda}$ with respect to the observed consumption growth rate from the data. Except for model (C), the values of the estimated parameters are similar across the alternative models. This means that the estimated structural parameters are quite

\textsuperscript{12}These values were computed by making $A_{jt} = \exp(\gamma p_5)$, where $\gamma = -0.634$ and -0.680 for the states that belong to $p_5$ in models B and D, respectively, according to Table 2.

\textsuperscript{13}To find the intertemporal discount rate, $\rho$, we use $\beta = \frac{1}{1+\rho}$, with $\beta = 0.89$. Issler and Piqueira (2000), Catalão and Yoshino (2006), Costa and Gutierrez (2015), and Faria et al. (2016) report estimates that range from 0.9 to 10 for $\sigma$ and 0.8 to 0.99 for $\beta$, depending on the frequency of the data and functional forms of the utility. We used the average values of the estimates for annual data.
The optimal fraction of investment spending, $\phi^*$, and optimal tax rate, $\tau^*$, that maximize consumption growth, $\lambda$, are reported in Table 4. The short time horizon of the sample might have biased the impact of the government spending in investment over the economic growth of the Brazilian states, as measured by $\phi^*$.\footnote{Divino and Silva Jr. (2012) found that the optimal share of public spending in capital is 32% for high-income, 23% for middle-income, and 19% for low-income Brazilian municipal districts.} Given that the average total tax revenue is around 16% of the GDP for the Brazilian states, according to the data for the 2004 to 2010 period available from the Ipeadata, one might argue that $\bar{\tau} < \tau^*$ and some states have average taxation below the optimum level.

According to Theorem 6, the heterogeneity of the Brazilian states in terms of technological progress is inversely related to the optimal size of the government. Thus, heterogeneity in the technological progress generates different levels of optimal taxation. We find that states with less technological progress ($p_i = 1$) are characterized by a larger government size, since their $\tau^*$ ranges from 0.201 in model (D) to 0.236 in model (B). For the other states, with higher technological progress ($p_i = 0$), $\tau^*$ ranges from 0.146 to 0.182 in these same models. This result was expected, since states with smaller technological progress also present lower economic dynamism and, as argued by Devarajan et al. (1996), are more dependent to the participation of the government in the economy.

Table 4: Optimum levels of $\phi$ and $\tau$.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>(A)</th>
<th>(B)</th>
<th>(C)</th>
<th>(D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi^*$</td>
<td>0.113</td>
<td>0.147</td>
<td>0.011</td>
<td>0.139</td>
</tr>
<tr>
<td>$\tau^*$ (average)</td>
<td>0.186</td>
<td>0.212</td>
<td>0.166</td>
<td>0.177</td>
</tr>
<tr>
<td>$\tau^*$ ($p_i = 0$)</td>
<td>NA</td>
<td>0.182</td>
<td>NA</td>
<td>0.146</td>
</tr>
<tr>
<td>$\tau^*$ ($p_i = 1$)</td>
<td>NA</td>
<td>0.236</td>
<td>NA</td>
<td>0.201</td>
</tr>
</tbody>
</table>

Source: Calculated by the authors. Models (A) and (C) do not consider technological progress, making $\tau^*$ the same across all states.

Because $\zeta_1$ is not statistically different from zero, we have that $\phi^* = \alpha_1$. In this case, only with the optimal levels of public investment, $\phi^*$, and tax rate, $\tau^*$, it is not possible to access how the consumption growth rate depends on these parameters. To do so, we need to compute the partial derivatives reported in Tables 5 and 6.

The ratio of public spending in investment, $\phi$, according to Lemma 2 is above the optimal level because $\partial \theta / \partial \phi > 0$. This result implies that an increase in $\phi$ leads to a decrease in the productivity of the private capital.
and so to a decrease in the growth rate of consumption, $\lambda$. According to Theorem 4, the maximization of $\lambda$ as a function of $\phi$ occurs when $\theta$ is at the minimum. This might also mean that public spending in costing, $g_2$, is below the optimal level.

An increase in the tax rate, $\tau$, decreases the private capital relatively to the output of the economy. This is a classic result given that an increase in taxation will reduce the amount of private capital available in the economy. However, the effect of a higher $\tau$ on $\lambda$ is not obvious because the productivity of the private capital increases.

In fact, as shown in Table 6, $\partial \lambda / \partial \tau > 0$ for the Brazilian states. The effective taxation is below the optimal level, suggesting that, on average, there is space for a tax increase without harming economic growth of the economy. Another result is the direct effect of changes in technological progress to the consumption growth. This is expected since an increase in technology has a positive effect on output, which leads to a rise in consumption. The magnitude of the effect is very similar across the alternative estimated models.

### 4 Conclusion

The objective of this paper was to investigate both theoretical and empirically the relationship among optimal size of the government in the economy, optimal shares of productive and unproductive public expenditures in the aggregate government spending, and economic growth. We expanded the model by Devarajan et al (1996) to include exogenous technological progress in a more general constant elasticity of substitution (CES) production function. We showed how those optimal ratios depend on the structural
parameters of the economy and provided empirical evidence by using a balanced panel data for the Brazilian states in the recent period.

The Cobb-Douglas and $Ak$ production functions are obtained as special cases of the general CES specification. In the special case of Cobb-Douglas production function, economic growth requires that the sum of tax rate and intertemporal discount rate in the utility be strictly smaller than one. In addition, the optimal public spending that maximizes consumption growth is given by the fraction of public spending allocated to investment.

In the general case, the growth rate of consumption is inversely related with both the individuals’ degree of impatience and the tax rate. The optimal taxation, defined as the one that maximizes the consumption growth, depends on the whole set of structural parameters. In particular, the technological progress has a direct effect on the optimal level of taxation while the share of private capital in the output has a negative effect on the optimal taxation.

The estimated parameters for the Brazilian states suggest that, on average, the share of private capital in the total production is 0.66 while the share of total government spending is 0.34. The estimated elasticity of substitution between these two inputs ranged from 1.40 to 1.45, depending on the model specification. Thus, private capital and government spending are substitute inputs in the production and the latter has a significant share in the output of the Brazilian states.

Devarajan et al. (1996) argue that developing countries, due to the low economic dynamism, require a significant fraction of government spending allocated to costing. This claim was confirmed by our empirical evidence, were about 85% of the total public spending was in costing and only 15% was in investment. This result is in line with the widely spread consensus that public investment is still a small fraction of public spending in the Brazilian economy.

Taking as a whole, the average government size is below the optimal level. Thus, it is possible to increase the growth rate of consumption by rising the government spending under a balanced public budget. The average optimal taxation implied by the estimated models is around 19%, while the average taxation observed from the data for the Brazilian states is around 16%. Thus, there is space for increasing taxation, and so for rising government spending, without hurting the economic growth.

For future research, it would be interesting to include public debt in the government budget constraint along with its dynamics in an intertemporal framework. It would be also interesting to expand the empirical analysis to a panel of countries with data before and after the international financial
crisis. Some of these suggestions are object of ongoing research.
References


Appendix

Proof of Theorem 1

Proof. 1. By definition of the function $x$:

$$x_t = \left[\alpha_1g_{1t}^{-\zeta_1} + (1 - \alpha_1)g_{2t}^{-\zeta_1}\right]^{-\frac{1}{\tau_t}}$$
$$x_t^{-\zeta_1} = \alpha_1g_{1t}^{-\zeta_1} + (1 - \alpha_1)g_{2t}^{-\zeta_1}$$
$$x_t^{-\zeta_1} = \alpha_1\phi_t^{-\zeta_1} + (1 - \alpha_1)(1 - \phi_t)^{-\zeta_1} \tau_t y_t^{-\zeta_1}$$
$$x_t = \left[\alpha_1\phi_t^{-\zeta_1} + (1 - \alpha_1)(1 - \phi_t)^{-\zeta_1}\right]^{-\frac{1}{\tau_t}} \tau y_t$$
$$x_t = \tau \delta_t^{-\frac{1}{\zeta_1}} y_t$$

The aggregate public spending, $x_t$, is generated by $\tau \delta_t^{-\frac{1}{\zeta_1}}$ multiplied by the output of the economy. A special case emerges when $\zeta_1 \to 0$, which implies that $\delta = 1$, $x_t = \tau y_t$, and $x_t = g_{1t} + g_{2t}$. The third equation follows from Lemma 1.

2. If $\zeta_1 \in (-1, 0)$, then $0 < \phi_t^{-\zeta_1} < 1$ and $0 < (1 - \phi_t)^{-\zeta_1} < 1$. So, $0 < \alpha_1\phi_t^{-\zeta_1} + (1 - \alpha_1)(1 - \phi_t)^{-\zeta_1} < 1$. The statement follows because $\frac{\phi_t}{\zeta_1} > 0$. If not, $\zeta_1 > 0$. Here, $\phi_t^{-\zeta_1} > 1$ and $(1 - \phi_t)^{-\zeta_1} > 1$. So, $\alpha_1\phi_t^{-\zeta_1} + (1 - \alpha_1)(1 - \phi_t)^{-\zeta_1} > 1$. The statement follows because $\frac{\phi_t}{\zeta_1} < 0$.

3. If $\zeta_1 \in (-1, 0)$, then $0 < \phi_t^{-\zeta_1} < 1$ and $0 < (1 - \phi_t)^{-\zeta_1} < 1$. So, $0 < \alpha_1\phi_t^{-\zeta_1} + (1 - \alpha_1)(1 - \phi_t)^{-\zeta_1} < 1$. The statement follows because $\frac{\phi_t}{\zeta_1} < 0$. If not, $\zeta_1 > 0$. Here, $\phi_t^{-\zeta_1} > 1$ and $(1 - \phi_t)^{-\zeta_1} > 1$. So, $\alpha_1\phi_t^{-\zeta_1} + (1 - \alpha_1)(1 - \phi_t)^{-\zeta_1} > 1$. The statement follows because $\frac{\phi_t}{\zeta_1} > 0$.

Proof of Theorem 2
Proof. By definition of function $y$:

\[
\begin{align*}
    y_t &= A_t \left[ \alpha k_t^{-\zeta} + (1 - \alpha)x_t^{-\zeta} \right] y_t \zeta
    y_t^{-\zeta} &= \alpha (A_t k_t)^{-\zeta} + (1 - \alpha)(A_t x_t)^{-\zeta}
    y_t^{-\zeta} &= \alpha (A_t k_t)^{-\zeta} + (1 - \alpha)(A_t \tau)^{-\zeta} \delta_t y_t^{-\zeta}
    \alpha A_t^{-\zeta} k_t^{-\zeta} &= \left[ 1 - \frac{1-\alpha}{(A_t \tau)^{-1} \delta_t} \right] y_t^{-\zeta}
    k_t &= \left[ \frac{(A_t \tau)^{-\zeta} - (1-\alpha) \delta_t}{\alpha \delta_t} \right]^{-\frac{1}{\zeta}} y_t
    k_t &= \theta_t y_t
\end{align*}
\]

Thus, each unit of private capital generates $\frac{1}{\theta}$ units of output in the economy. In other words, $\frac{1}{\theta}$ represents the private-capital productivity.

The third equation follows from the substitution of $x_t$. Notice that $(A_t \tau)^{-\zeta} - (1 - \alpha) \delta_t > 0$, because functions $k$ and $y$ are positive.

Now, we consider the particular case in which $A_t = 1$.

If $\zeta \in (-1, 0)$, then

\[
\begin{align*}
    0 < \quad \delta_t < 1 < \tau^{-\zeta} \quad \forall t \geq 0
    0 < \quad (1 - \alpha) \delta_t < 1 - \alpha < (1 - \alpha) \tau^{-\zeta} \quad \forall t \geq 0
\end{align*}
\]

This implies that

\[
\begin{align*}
    0 < \quad \alpha \tau^{-\zeta} < \tau^{-\zeta} - (1 - \alpha) \delta_t \quad \forall t \geq 0
    0 < \quad 1 < \left[ \frac{\tau^{-\zeta} - (1-\alpha) \delta_t}{\alpha \tau^{-\zeta}} \right] \quad \forall t \geq 0
    0 < \quad 1 < \theta_t \quad \forall t \geq 0
\end{align*}
\]

If not ($\zeta > 0$), then

\[
\begin{align*}
    0 < \quad \tau^{-\zeta} < 1 < \delta_t \quad \forall t \geq 0
    0 < \quad (1 - \alpha) \tau^{-\zeta} < 1 - \alpha < (1 - \alpha) \delta_t \quad \forall t \geq 0
\end{align*}
\]

This implies that

\[
\begin{align*}
    0 < \quad \tau^{-\zeta} - (1 - \alpha) \delta_t < \alpha \tau^{-\zeta} \quad \forall t \geq 0
    0 < \quad \left[ \frac{\tau^{-\zeta} - (1-\alpha) \delta_t}{\alpha \tau^{-\zeta}} \right] < 1 \quad \forall t \geq 0
    1 < \quad \theta_t \quad \forall t \geq 0
\end{align*}
\]

Notice that all quantities are positive. The last inequality follows because $-\frac{1}{\zeta} < 0$. 

\[\square\]
Proof of Theorem 3

Proof. Following Lemma 2, we have that the dynamic equation for \( k \) might be rewritten as
\[
\dot{k}_t = \left[ \frac{1 - \tau}{\sigma t} \right] k_t - c_t.
\]
Thus, problem (P) is nothing else than a variational calculus problem, where:
\[
L(t, k_t, \dot{k}_t) = \left[ \frac{1 - \tau}{\sigma t} k_t - \dot{k}_t \right]^{1 - \sigma - 1} 1 - \frac{1}{1 - \sigma} e^{-\rho t}.
\] (15)

Here, \( \frac{\partial L}{\partial k} = \left[ \frac{1 - \tau}{\sigma t} \right] c_t - \sigma \theta_t \) and \( \frac{d}{dt} \frac{\partial L}{\partial \dot{k}} = \sigma c_t^{-\sigma} \dot{c}_t + \rho c_t^{-\sigma} e^{-\rho t} \). The statement follows from the Euler equation. \( \square \)

Proof of Lemma 2

Proof. Since,
\[
\theta_t = \left[ \frac{(A_t \tau)^\zeta - (1 - \alpha) \delta_t}{\alpha \tau^\zeta} \right]^{-\frac{1}{\zeta}}
\] (16)
then
\[
\frac{\partial \theta}{\partial \phi} = - \frac{1 - \alpha}{\alpha \tau^\zeta} \theta_t^{\zeta + 1} \delta_t^{1 - \frac{1}{\zeta}} \left[ \alpha_1 \phi_t^{-\zeta - 1} - (1 - \alpha_1)(1 - \phi_t)^{-\zeta - 1} \right]
\] (17)

So, \( \frac{\partial \theta}{\partial \phi} = 0 \) at \( \phi^* = \frac{\alpha_1^{-1 + \tau}}{(1 - \alpha_1) \alpha_1^{1 + \tau} + \alpha_1^{-1 + \tau}} \).

We point that if \( \phi \in (0, \phi^*) \), then \( \frac{\partial \theta}{\partial \phi} < 0 \). Analogously, if \( \phi \in (\phi^*, 1) \), then \( \frac{\partial \theta}{\partial \phi} > 0 \). Thus, the statement follows. \( \square \)

Proof of Theorem 6

Proof. Notice that
\[
\frac{\partial \lambda}{\partial \tau} = \frac{-\sigma \theta_t - (1 - \tau) \sigma \frac{\partial \theta}{\partial \tau}}{\sigma^2 \theta_t^2}
\] (18)

But, from Theorem 5 we have that \( \frac{\partial \theta}{\partial \tau} = -\frac{1 - \alpha}{\alpha} \left[ \frac{\theta_t}{\tau} \right]^{\zeta + 1} \delta_t \). So,
\[
\frac{\partial \lambda}{\partial \tau} = \frac{-A_t^{\zeta} \tau^{1 + \delta_t} + \delta_t (1 - \alpha)}{\sigma \tau \theta_t((A_t \tau)^\zeta - (1 - \alpha) \delta_t)}
\] (19)
The statement follows, because for each $\tau < \tau^*$ the function $\lambda$ increases. Analogously, for each $\tau > \tau^*$ the function $\lambda$ decreases. In the special case where $\zeta \in (-1, 0)$, a positive variation of $A_t$ leads to an increase in $\tau^*$. This suggests that a higher technological progress allows for a higher level of optimal taxation, provided that all other variables are kept unchanged. \qed